Final Examination
2018/2019 Academic Session

June 2019

JIM201 – Linear Algebra
(Aljabar Linear)

Duration : 3 hours
(Masa: 3 jam)

Please check that this examination paper consists of **NINE (9)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN (9)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions.

*[**Arahan** : Jawab **SEMUA** soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

1. Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

(a). Calculate

(i). $|A|$

(ii). $\text{adj}(A)$

(iii). A^{-1}

(iv). $|\text{adj}(A)|$

(v). $A\,\text{adj}(A)$

(vi). reduced row-echelon form of A

(vii). Solution of X if $AX = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

(60 marks)

(b). Let $B = [b_{ij}]_{3\times 3}$ be a 3×3 matrix and I is an identity matrix. Given that

$$E_1^3(-2)E_2^3(1)E_3\left(\frac{1}{2}\right)E_3^2(-1)E_1^2(-1)E_2^1(-1)B = I$$

Find B^{-1} and B.

(40 marks)

2. (a). Given the system of linear equations

$$x + 2y + 3z = 6$$
$$2x - 3y + 2z = 14$$
$$3x + y - z = -2$$

Solve the system using the method of

(i). Cramer's rule.

(ii). Gauss-Jordan elimination.

(60 marks)

(b). Determine the values of constant k if the system of linear equations

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + \left(k^2 - 14\right)z = k + 2$$

has

(i). a unique solution,

(ii). infinitely many solutions,

(iii). no solution.

(40 marks)

3. (a). State the conditions that a subset W of a vector space V is a subspace of V.

(20 marks)

(b). Show that each of the following vector is a subspace of V.

(i). $S = \left\{ \begin{pmatrix} 3y \\ y \end{pmatrix} \middle| y \in R \right\}, \quad V = R^2,$

(ii). $S = \left\{ \begin{bmatrix} a & -a \\ b & 0 \end{bmatrix} \middle| a,\ b \in R \right\}, V = M_{2\times 2}.$

(iii). $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| x + y = 0, \quad x, y \in R \right\}, \quad V = R^2.$

(40 marks)

(c). Let $T:V\rightarrow W$ be a linear transformation. State the definition for each of the following:

(i). Kernel of T.

(ii). Range of T.

(20 marks)

(d). Given that $T:\mathbb{R}^2\rightarrow\mathbb{R}$ where $T(x,y)=x+y$.

Find the kernel and the range of T.

(20 marks)

4. (a). Let matrix A be diagonalizable. Then there exist two matrices P and D such that

$$P^{-1}AP=D.$$

Show that

$$A^n=PD^nP^{-1}$$

by induction.

(30 marks)

(b). Given the matrix

$$A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}.$$

Find

(i). the characteristic polynomial,

(ii). eigenvalues,

(iii). eigenvectors,

(iv). the matrix P that diagonalizes A.

(70 marks)

5. (a). State the definition for each of the following:

   (i). orthogonal set.

   (ii). orthonormal set.

   (iii). orthogonal basis.

   (iv). orthonormal basis.

(40 marks)

(b). Determine whether each of the following set is orthogonal or orthonormal.

   (i). $S=\{(1,0,0),(0,1,0),(0,0,1)\}\subseteq R^3$.

   (ii). $S=\{(1,1,1),(-2,1,1),(0,-1,1)\}\subseteq R^3$.

(20 marks)

(c). Use Gram-Schmidt process to find an orthogonal basis and orthonormal basis from the set

$$S=\{(1,1,0),(-1,-1,1),(1,2,4)\}\subseteq R^3.$$

(40 marks)

1. Pertimbangkan matriks

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

(a). Hitung

(i). $|A|$

(ii). $\text{adj}(A)$

(iii). A^{-1}

(iv). $|\text{adj}(A)|$

(v). $A\,\text{adj}(A)$

(vi). bentuk eselon baris terturun A

(vii). penyelesaian bagi X jika $AX = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

(60 markah)

(b). Katakan $B = [b_{ij}]_{3\times3}$ adalah matriks 3×3 matrix dan I ialah matriks identiti. Diberi

$$E_1^3(-2)E_2^3(1)E_3\left(\frac{1}{2}\right)E_3^2(-1)E_1^2(-1)E_2^1(-1)B = I$$

Cari B^{-1} dan B.

(40 markah)

2. (a). Diberi sistem persamaan linear

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

Selesaikan sistem menggunakan kaedah

(i). petua Cramer.

(ii). penghapusan Gauss-Jordan.

(60 markah)

(b). Tentukan nilai pemalar k jika sistem persamaan linear

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + \left(k^2 - 14\right)z &= k + 2 \end{aligned}$$

mempunyai

(i). penyelesaian unik,

(ii). penyelesaian tidak terhingga,

(iii). tiada penyelesaian.

(40 markah)

3. (a). Nyatakan syarat bahawa subset W bagi suatu ruang vektor V adalah suatu subruang bagi V.

(20 markah)

(b). Tunjukkan bahawa setiap vektor berikut adalah suatu subruang bagi V.

(i). $S = \left\{ \begin{pmatrix} 3y \\ y \end{pmatrix} \middle| \, y \in R \right\}, \quad V = R^2,$

(ii). $S = \left\{ \begin{bmatrix} a & -a \\ b & 0 \end{bmatrix} \middle| \, a, \, b \in R \right\}, \; V = M_{2\times 2}.$

(iii). $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \, x + y = 0, \quad x, y \in R \right\}, \quad V = R^2.$

(40 markah)

(c). Katakan $T : V \to W$ adalah transformasi linear. Nyatakan definisi bagi setiap yang berikut:

(i). Kernel T.

(ii). Julat T.

(20 markah)

(d). Diberi $T : \mathbb{R}^2 \to \mathbb{R}$ dengan $T(x, y) = x + y$.

Cari kernel dan julat bagi T.

(20 markah)

4. (a). Katakan matriks A adalah terpepenjuru. Maka wujud dua matriks P dan D supaya

$$P^{-1}AP = D.$$

Buktikan bahawa

$$A^n = PD^nP^{-1}$$

secara aruhan.

(30 markah)

(b). Diberi matriks

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Cari

(i). polinomial cirian,

(ii). nilai eigen,

(iii). vektor eigen,

(iv). matriks P yang pepenjurukan A.

(70 markah)

5. (a). Nyatakan definisi bagi setiap yang berikut:

    (i). set ortogon.

    (ii). set ortonormal.

    (iii). asas ortogon.

    (iv). asas ortonormal.

(40 markah)

(b). Tentukan sama ada setiap set berikut adalah ortogon atau ortonormal.

(i). $S = \{(1,0,0),(0,1,0),(0,0,1)\} \subseteq R^3$.

(ii). $S = \{(1,1,1),(-2,1,1),(0,-1,1)\} \subseteq R^3$.

(20 markah)

(c). Guna proses Gram-Schmidt untuk mencari asas ortogon dan asas ortonormal daripada set

$$S = \{(1,1,1),(0,1,1),(0,0,1)\} \subseteq R^3.$$

(40 markah)

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