



Final Examination
2018/2019 Academic Session

June 2019

**JIM101 – Calculus
(Kalkulus)**

Duration: 3 hours
(Masa: 3 jam)

Please check that this examination paper consists of **SEVEN (7)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions.

Arahan : Jawab **SEMUA** soalan].

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

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1. (a). State the domain and range of the following functions:

(i). $f(x) = \sqrt{x-6}$

(ii). $f(x) = |x-3|$

(20 marks)

(b). Given $(g \circ f)(x) = \frac{1+2x}{2+x}$, $x \neq -2$ and $f(x) = 3+2x$. Find

(i). $(g \circ f)(3)$

(ii). $g(x)$

(iii). $f^{-1}(x)$

(20 marks)

(c). Evaluate the following limits:

(i). $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 9}$

(ii). $\lim_{x \rightarrow 0} \frac{\sin(4x)}{x}$

(iii). $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + \sqrt{x-1}}{\sqrt{x}}$

(iv). $\lim_{x \rightarrow 1^+} [\ln(x^2 - 1) - \ln(x^2 + x - 2)]$

(60 marks)

2. (a). Write the following complex numbers in polar form

(i). $z = \frac{3}{2-i} + 5 - 4i$,

(ii). $z = \alpha + \alpha i$, $\alpha > 0$.

(40 marks)

(b). Use DeMoivre's Theorem to find the 5th power of the complex number

$$z = 2(\cos 24^\circ + i \sin 24^\circ). \text{ Express the answer in the form } a + bi.$$

(30 marks)

(c). If $z = a + bi$, a and b are real numbers, prove that

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$$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = \frac{2(a^2 - b^2)}{a^2 + b^2}$$

(30 marks)

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3. (a). Solve the following integrals:

(i). $\int x^8(1-x) dx$

(ii). $\int \frac{t^2 + e^{\sqrt{t}}}{\sqrt{t}} dt$

(iii). $\int (\ln x)^2 dx$

(50 marks)

(b). The function $f(x)$ is defined as

$$f(x) = \begin{cases} 6^{x+2}, & x \leq 0 \\ x^3 e^{x^4}, & x > 0 \end{cases}$$

Evaluate $\int_{-1}^1 f(x) dx$.

(30 marks)

(c). Given $y = \cos^2 4x$. Show that

$$\frac{1}{32} \frac{d^2 y}{dx^2} + y = \sin^2 4x$$

(20 marks)

4. (a). Consider the function $g(x) = 3x^4 + 16x^3 + 24x^2 + 3$.(i). Find all the critical points of $g(x)$.

(ii). Classify each of the critical point you obtained in part (i). as a maximum or a minimum (or neither) by using the first derivative test.

(40 marks)

(b). Given an equation of a conic section

$$3x^2 - 6x + y^2 + 2y = 5.$$

(i). Is this a circle, ellipse, parabola or hyperbola?

(ii). Form the standard equation of that conic section.

...4/-

- (iii). Find the centre, foci, vertices and the length of latus rectum.
(iv). Sketch its graph.

(60 marks)

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5. (a). Find the area of the region bounded by the curves $f(x) = x^2 + 2$, $g(x) = \sin(x)$, $x = -1$ and $x = 2$.

(30 marks)

- (b). Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{25 - x^2}$, $x = 0$ and $y = 3$ about the x -axis.

(30 marks)

- (c). The equation of a circle is $x^2 + y^2 - 2x - 4y - 20 = 0$. The line with equation $4x - 3y + \beta = 0$ is a tangent to this circle.

- (i). State the center and radius of the circle.
(ii). Find the possible values of β .

(40 marks)

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1. (a). Nyatakan domain dan julat untuk fungsi berikut:

(i). $f(x)\sqrt{x-6}$

(ii). $f(x)|x-3|$

(20 markah)

(b). Diberi $(g \circ f)(x) = \frac{1+2x}{2+x}$, $x \neq -2$ dan $f(x) = 3+2x$. Cari

(i). $(g \circ f)(3)$

(ii). $g(x)$

(iii). $f^{-1}(x)$

(20 markah)

(c). Nilaikan had berikut:

(i). $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 9}$

(ii). $\lim_{x \rightarrow 0} \frac{\sin(4x)}{x}$

(iii). $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + \sqrt{x-1}}{\sqrt{x}}$

(iv). $\lim_{x \rightarrow 1^+} [\ln(x^2 - 1) - \ln(x^2 + x - 2)]$

(60 markah)

2. (a). Tuliskan nombor kompleks berikut dalam bentuk kutub

(i). $z = \frac{3}{2-i} + 5 - 4i$,

(ii). $z = \alpha + \alpha i$, $\alpha > 0$.

(40 markah)

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- (b). Gunakan teorem DeMoivre untuk mencari kuasa 5 untuk nombor kompleks $z = 2(\cos 24^\circ + i \sin 24^\circ)$. Tuliskan jawapan dalam bentuk $a + bi$.

(30 markah)

- (c). Jika $z = a + bi$, a dan b merupakan nombor nyata, buktikan bahawa

$$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = \frac{2(a^2 - b^2)}{a^2 + b^2}$$

(30 markah)

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3. (a). Selesaikan kamiran berikut:

(i). $\int x^8(1-x) dx$

(ii). $\int \frac{t^2 + e^{\sqrt{t}}}{\sqrt{t}} dt$

(iii). $\int (\ln x)^2 dx$

(50 markah)

- (b). Fungsi $f(x)$ ditakrifkan sebagai

$$f(x) = \begin{cases} 6^{x+2}, & x \leq 0 \\ x^3 e^{x^4}, & x > 0 \end{cases}$$

Cari nilai $\int_{-1}^1 f(x) dx$.

(30 markah)

- (c). Diberi $y = \cos^2 4x$. Tunjukkan bahawa

$$\frac{1}{32} \frac{d^2 y}{dx^2} + y = \sin^2 4x$$

(20 markah)

4. (a). Pertimbangkan fungsi $g(x) = 3x^4 + 16x^3 + 24x^2 + 3$.

(i). Cari semua titik genting bagi fungsi $g(x)$.

(ii). Klasifikasi setiap titik genting yang anda dapati dalam bahagian (i) sebagai maksimum atau minimum (atau tidak) dengan menggunakan ujian terbitan pertama.

(40 markah) ...7/-

- (b). Diberi suatu persamaan keratan kon

$$3x^2 - 6x + y^2 + 2y = 5.$$

- (i). Adakah ini bulatan, elips, parabola atau hiperbola?
- (ii). Bentukkan persamaan piawai bagi keratan kon tersebut.
- (iii). Cari pusat, titik fokus, bucu dan panjang latus rectum.
- (iv). Lakarkan graf tersebut.

(60 markah)

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5. (a). Cari luas kawasan yang dibatasi oleh lengkung $f(x) = x^2 + 2$, $g(x) = \sin(x)$, $x = -1$ dan $x = 2$.

(30 markah)

- (b). Cari isipadu pepejal yang terjana oleh putaran rantau yang dibatasi oleh $y = \sqrt{25 - x^2}$, $x = 0$ dan $y = 3$ pada satah- x .

(30 markah)

- (c). Satu persamaan bulatan ialah $x^2 + y^2 - 2x - 4y - 20 = 0$. Satu garis dengan persamaan $4x - 3y + \beta = 0$ ialah tangen pada bulatan tersebut.

- (i). Nyatakan pusat dan jejari bagi bulatan tersebut.
- (ii). Cari nilai β .

(40 markah)

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