MATHEMATICAL PROBLEM SOLUTION PATHS OF TRAINEES IN A TEACHER TRAINING COLLEGE

by

KOAY CHEN YONG

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1.1 Koay, C. Y. & Fatimah Saleh, (2005) *Pre-Service Teachers' Mathematical Problem Solving Schemes: A Preliminary Finding.* Papers presented at the ICMI-EARCOME 3, East China Normal University, 7-12 August, 2005, Shanghai, China.

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1.3 Koay, C. Y. & Fatimah Saleh, (2006) *Comparison of Mathematical Problem Solving Heuristics Between Successful and Unsuccessful Solvers.* Papers presented at the XII IOSTE Symposium Science and Technology Education in the Service of Humankind, Universiti Sains Malaysia, 30 July – 4 August 2006, Penang, Malaysia.

LANGKAH PENYELESAIAN MASALAH MATEMATIK GURU PELATIH DI SEBUAH MAKTAB PERGURUAN

ABSTRAK

Kajian ini meneroka imej mental bagi penyelesaian masalah dan langkah penyelesaian masalah matematik guru pelatih di sebuah maktab perguruan. Langkah penyelesaian masalah matematik penyelesai berjaya dan tidak berjaya juga dikenal pasti. Penyelidikan ini dilaksana berasaskan perspektif konstruktivis, iaitu pembelajaran matematik adalah suatu proses membina perwakilan yang bermakna bagi individu dan bukan daripada orang lain.

Kajian menggunakan teknik koding terbuka dan pendekatan pentafsiran. Tujuh belas guru pelatih Kursus Diploma Perguruan Malaysia (KDPM) dan Kursus Perguruan Lepas Ijazah (KPLI) pengkhususan matematik telah dipilih secara pemilihan bertujuan (purposive sampling). Rakaman audio dan video digunakan untuk mengumpul data guru pelatih semasa mereka menyelesaikan enam masalah secara individu dengan menggunakan teknik menyuarakan fikiran (think-aloud) diikuti dengan retrospektif dan temu bual klinikal. Analisis melibatkan langkah penyelesaian dan transkrip sesi penyelesaian masalah secara menyuarakan fikiran guru pelatih. Transkrip retrospektif dan temu bual klinikal menjadi asas untuk triangulasi bagi menentukan imej mental dan langkah penyelesaian masalah matematik guru pelatih.

Dapatan kajian menunjukkan bahawa guru pelatih mempunyai imej mental komposit bagi penyelesaian masalah. Imej mental utama yang muncul ialah ‘sebagai proses’, ‘sebagai matlamat’, dan ‘sebagai halangan’.

Dapatan menunjukkan bahawa guru pelatih KPLI lebih cenderung mengamalkan pendekatan aljebra manakala guru pelatih KDPM lebih cenderung mengamalkan pendekatan aritmetik dalam langkah penyelesaian masalah matematik. Imej mental guru pelatih bagi masalah yang diberi ada perkaitan dengan pendekatan

penyelesaian masalah yang diguna dalam langkah penyelesaian masalah matematik. Dapatan juga menunjukkan bahawa penyelesai berjaya membina pengetahuan dengan perkaitan yang baik, membina langkah penyelesaian yang bermakna, kerap mengawal usaha penyelesaian masalah, dan lebih anjal dalam proses penyelesaian masalah. Mereka lebih tabah dalam usaha untuk mencapai matlamat semasa menyelesaikan sesuatu masalah, dan mempunyai langkah penyelesaian masalah matematik yang lebih sistematik dan bermakna. Penyelesai tidak berjaya mempunyai perkaitan kandungan pengetahuan matematik yang agak longgar, menunjukkan kekurangan dalam kawalan diri dan kurang anjal dalam proses penyelesaian masalah. Mereka mempunyai langkah penyelesaian masalah matematik yang tidak lengkap, kurang bermakna dan terlalu ringkas. Dapatan kajian ini menyokong dapatan kajian lepas Wilson (1993) bahawa penyelesaian masalah adalah bersifat dinamik dan berkitaran.

MATHEMATICAL PROBLEM SOLUTION PATHS OF TRAINEES IN A TEACHER TRAINING COLLEGE

ABSTRACT

This study explores the mental images of problem solving and the mathematical problem solution paths of trainee teachers in a teacher training college. The mathematical problem solution paths of successful and unsuccessful problem solvers are also identified. This research adopts a constructivist perspective that views learning mathematics as a process of constructing meaningful representation and knowledge is constructed by the individual himself or herself.

This research employs the open coding techniques and the interpretative approach. Seventeen trainees majoring in mathematics undertaking the Malaysian Diploma in Teaching (KDPM) and the Post Graduate in Teaching (KPLI) courses were selected by purposive sampling. Data were collected by audio and video taping the trainee teachers while solving six tasks individually using the think-aloud technique, followed by retrospection and clinical interviews. The analysis involved the trainee teachers' solution paths and transcripts of the think-aloud problem solving sessions. The transcripts of the retrospection and clinical interviews provide the basis for triangulation to ascertain the trainee teachers' mental images of problem solving and mathematical problem solution paths.

The findings of this study show that the trainee teachers have composite mental images of problem solving. The most common mental images that emerged are 'as a process', 'as a goal', and 'as an obstacle'.

The findings reveal that KPLI trainee teachers are more inclined to adopt the algebraic approach while the KDPM trainee teachers tend to adopt the arithmetical approach in their mathematical problem solution paths. The trainee teachers' mental images of the task have some connections to the problem solving approaches

employed in their mathematical problem solution paths. Findings also indicate that successful solvers are able to construct well-connected knowledge, construct meaningful solution paths and regularly monitor their problem solving efforts, and exhibit greater flexibility during their problem solving process. They are more persistent in their pursuit in working towards the goal of solving a task and have a more systematic and meaningful mathematical problem solution paths. The unsuccessful solvers have loosely related mathematical content knowledge, lack self-monitoring and are less flexible in their problem solving process. They possess incomplete, less meaningful, and over simplified mathematical problem solution paths. The findings of this study support previous findings of Wilson (1993) that problem solving is dynamic and cyclical in nature.

CHAPTER ONE
INTRODUCTION

1.1 Problem Solving in Mathematics Education

Mathematics has always been an important subject in education from the pre-school right up to tertiary level. It is also one of the core subjects in the school curriculum for the Integrated Primary School Curriculum (KBSR or Kurikulum Bersepadu Sekolah Rendah) and the Integrated Secondary School Curriculum (KBSM or Kurikulum Bersepadu Sekolah Menengah) in Malaysia. Mathematics and problem solving skills is the foundation for many fields of study including science, engineering, medicine, business, economics, and many other subjects. Problem solving skills are vital to individuals' everyday life. The ability to solve problems is not only an asset to oneself but is also an important asset to a nation such as in strategic management and the progress in the field of science and technology.

Mathematical problem solving has been an important part of mathematics education for the past decades. The first major call to focus on problem solving was issued by the National Council of Supervisors of Mathematics (NCSM) in 1977. The NCSM (1977) points out that the principal reason for studying mathematics is learning to solve problems. The National Council of Teachers of Mathematics (NCTM, 1980) later followed this call in its *Agenda for Action,* which recommends that "problem solving be the focus of school mathematics" (p.1). As such, 'problem solving' was declared as the theme of the 1980s for school mathematics (Krulik, 1980). Finally, this was followed by the publication of *Everybody Counts* (National Research Council, 1989) and the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), both of which emphasize problem solving as one of the major parts of the curriculum.

The National Council of Teachers of Mathematics (NCTM), in their *Professional Standards for Teaching Mathematics*, states boldly that "Problem solving, reasoning and communication are processes that should pervade all mathematics instruction and should be modelled by teachers" (NCTM, 1991, p.23). In the *Principles and Standards for School Mathematics*, the NTCM also emphasizes that instructional programs should enable all students in all grades to:

> (1) build new mathematical knowledge through problem solving;
> (2) solve problems that arise in mathematics and in other contexts;
> (3) apply and adapt a variety of appropriate strategies to solve problems; and
> (4) monitor and reflect on the process of mathematical problem solving.
>
> (NCTM, 2000, p.52)

Hence, mathematics educators and teachers are advice to employ problem solving as a method of inquiry and application throughout the Standards to provide a consistent context for learning and applying mathematics. This is because problem solving is the process by which students experience the power and usefulness of mathematics in the world around them.

In order to keep pace with society's expectation and needs for a rapid changing world of the twenty-first century, schools must graduate students who are prepared to be problem solvers. This necessitates a pedagogical shift from transmitting a whole mass of expected knowledge that is largely memorized to one that is more problem solving oriented. Problem solving is not only a skill to be taught in mathematics, but also a skill that will be carried over to everyday 'problems' and serve a person well throughout life (Posamentier & Krulik, 1998). According to Schoenfeld (1985), the understanding and teaching of mathematics should be approached as a problem-solving domain. Begle (1979) stated, "The real justification for teaching mathematics is that it is a useful subject and, in particular, that it helps in solving many kinds of problems" (p.143). The National Council of Supervisors of Mathematics (1977) in their position paper expresses the same view on basic skills that sees problem solving as a useful skill that should be taught in mathematics. Similar statements describe mathematics as essentially a problem-solving endeavour were found in many literatures for example, Braunfeld (1975); Carpenter, Corbitt, Kepler, Lindquist, and Reys (1980).

In their historical review of problem solving, Stanic and Kilpatrick (1988) identify three main themes regarding problem solving:

> (1) 'problem solving as context' – problems are employed as vehicles in the service of other curricular goals.
> (2) 'problem solving as skill' – which is rather narrowly defined as being able to obtain solutions to the problems assigned.
> (3) 'problem solving as art' – which holds that real problem solving (that is, working problems of the 'perplexing' kind) is the heart of mathematics.
>
> (Stanic & Kilpatrick, 1988, p.338)

Problem solving is dependent on a large store of knowledge and capabilities and it requires what is called 'thinking'. 'Problem' means a question that requires some originality on the part of the learner for its solution. It requires the learner to construct elements of prior learning together in a new way and having solved such a problem, something has been learned. Hence, one of the major challenges for mathematics educators is to understand how knowledge structures that students' construct would have a significant effect on the progress made during solution attempt in mathematical problem solving. Research of this nature that target on knowledge construction and application in mathematics are essential if mathematics educators are to understand the problem of failure to apply previously learnt mathematical knowledge to the solution of new problems (Board of Senior Secondary Studies, 1995).

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), emphasize the importance of learners being actively involved in their learning, that is, they should "construct, modify, and integrate ideas by interacting with the physical world, materials, and other children" (p.17). Currently, one major theoretical orientation in the mathematics community of the west is constructivism (von Glasersfeld, 1987). Some of the trends in science and mathematics education in the 1990's from the west are evident in the Malaysian educational scenario. For instance, there has been effort in introducing constructivism into Malaysian education especially in the teacher training colleges (Ibrahim, 1995). The constructivist approach in the teaching and learning of mathematics has been incorporated as one of the components in the mathematics syllabus for the pre-service teacher education programme (refer to Appendix E). The theoretical framework in this study will look into the constructivist perspective, derived from the work of Piaget (1970; 1980) who asserts that conceptual knowledge cannot be transferred and carefully packaged from one person to another. Rather, each learner solely based on his or her own experience (Nik Azis, 1987; 1999; Steffe & Gale, 1995; von Glasersfeld, 1994) must construct it. This study will explore the mathematical problem solution paths and thought processes of trainee teachers in mathematical problem solving.

1.1.1 Problem Solving in the Mathematics Teacher Education Curriculum

The two main courses offered in the pre-service teacher education programme in Malaysian teacher training colleges are Malaysian Diploma in Teaching, (KDPM or Kursus Diploma Perguruan Malaysia) and the Post Graduate Diploma in Teaching, (KPLI or Kursus Perguruan Lepas Ijazah). The KDPM is a three-year course comprising six semesters, and the entry requirement for Mathematics Studies is a credit in mathematics at the Malaysia Certificate of Education (SPM or Sijil Pelajaran Malaysia) level. Whereas, KPLI is a one-year course comprising of two semesters. The entry requirement for the Mathematics Studies for KPLI is a first degree in related areas such as Mathematics, Science or Statistics and also a strong credit in mathematics at the Malaysian Certificate of Education (SPM or Sijil Pelajaran Malaysia) or Higher School Certificate (STPM or Sijil Tinggi Pelajaran Malaysia) level.

Initially, the mathematics curriculum in the KPLI was planned to produce competent mathematics graduate teachers for secondary schools and matriculation colleges. However, in January 2003, the KPLI was also introduced for primary education in mathematics with the aim of producing primary school mathematics teachers of high quality and competency in teaching mathematics in the Integrated Primary School Curriculum (KBSR) (Bahagian Pendidikan Guru, 2002).

The mathematics curriculum for trainee teachers in the teacher training colleges comprises of two main sections, namely the mathematical content and the pedagogical content. The mathematics syllabus emphasizes the use of higher order thinking skills such as innovative, critical as well as creative thinking. Priority is also given to the learning process based on the constructivist approach coupled with the use of strategies such as problem solving, cooperative, collaborative, contextual learning, self-paced and reflective learning (Bahagian Pendidikan Guru, 2001; 2002). In addition, one important feature in the mathematics syllabus is the focus on the teaching and learning of mathematics that is based on the trainee teachers' understanding and solving mathematical problems which are integrated across the mathematics curriculum (Bahagian Pendidikan Guru, 2002).

The component of problem solving in the mathematics syllabus for KDPM Semester Two mathematics major is taught as 1 credit component or 15 hours out of the overall 7 credits or 105 hours (Bahagian Pendidikan Guru, 2001). However, the KPLI mathematics course comprise of KPLI for primary school and KPLI for secondary school. In the mathematics syllabus for KPLI for primary school, the component on problem solving is being taught in Semester One with a time allocation of 12 hours out of a total of 105 hours or 7 credits for Mathematics Studies in that semester (Bahagian Pendidikan Guru, 2002). In addition, there is also a new component, 'school-based experience' being incorporated into the mathematics syllabus for the KPLI for primary school. The time allocation for problem solving, specifically in the area related to difficulties in problem solving (refer to Table 1.1) for the component on 'school-based experience' is 10 hours out of the overall 45 hours or 1 credit. The aim of incorporating the school-based experience component is to allow trainee teachers to explore the learning situation in a real classroom environment and to relate their theoretical knowledge to the practice of teaching in an actual school setting. Thus, trainee teachers are required to make 10 visits to primary schools prior to their teaching practice to gain exposure and experience related to the actual school context. In the mathematics syllabus for the KPLI secondary school, the component on problem solving is taught in Semester One with a time allocation of 10 hours out of a total of 135 hours or 9 credits of mathematics studies in semester one (Bahagian Pendidikan Guru, 2001).

In order to have a better understanding of the components on problem solving in the Mathematics Studies for trainee teacher education course, an overview of the mathematics syllabus as stipulated by the Teacher Education Division in the Ministry of Education, Malaysia (Bahagian Pendidikan Guru, 2001; 2002) is shown in Table 1.1. The distribution of the various components in the KDPM and KPLI for primary and secondary school mathematics syllabus can be found in Appendix E.

An examination of the problem solving component in the mathematics syllabus for the KDPM and KPLI primary school shows that the focus is on the definition of problem and problem solving, the various problem solving strategies, Polya's Model of problem solving and application of Newman's Error Analysis Method in identifying the difficulties in problem solving. However, in the mathematics syllabus for KPLI secondary school, the focus is on the difficulties in problem solving and the various problem solving strategies. It can be seen that the component on problem solving is very much emphasized in the mathematics curriculum in the teacher education programme for the KDPM and the KPLI course, which is also in accordance with the teaching and learning of mathematics in the Integrated Primary School Curriculum (KBSR) and the Integrated Secondary School Curriculum (KBSM).

Table 1.1: An overview of the component of problem solving in the mathematics syllabus for KDPM and KPLI (Bahagian Pendidikan Guru, 2001, 2002)

Content	Learning Outcomes	KDPM Semester 2 (15 hrs)	KPLI Primary School (12 hrs)	KPLI Secondary School (10 hrs)
Topic: Problem Solving 1. Definition of Problem and Problem Solving	- To state and explain the meaning of problem and problem solving - To differentiate between routine and non-routine problem	1 hr	1 hr –	– –
2. Models of Problem Solving	- To explain the steps in Polya's Model	2 hrs	1 hr	–
3. Difficulties in Problem Solving 3.1 Reading 3.2 Comprehension	- To identify students' initial errors in solving word problems using Newman's Error Analysis procedure - To study and analyse various difficulties in problem solving	2 hrs	4 hrs	–
3.3 Transformation 3.4 Process Skills 3.5 Encode 3.6 Carelessness 3.7 Motivation	- To use general strategies in planning and implementing problem solving process effectively - To identify difficulties in a problem concerning mathematical language and symbol	–	–	4 hrs
School-Based Experience (Note: To be carried out as an assignment during the school visit.)	- To try out Newman's Error Analysis procedure - To suggest appropriate strategies to overcome the difficulties	–	10 hrs	–

Table 1.1 .... continued

4. Problem Solving Strategies 4.1 Guess and Check 4.2 Draw a Diagram	- To explain each strategies with examples - To solve any problem with various strategies - To create mathematical problems relevant to the strategies	8 hrs	–	–
4.3 Act Out Simulation 4.4 Working Backward 4.5 Analogy 4.6 Simplify the Problem	- To make and examine conjecture - To explore and investigate alternatives solutions - To evaluate and select appropriate problem solving strategies	–	6 hrs	–
4.7 Look for a Pattern 4.8 Construct a Table 4.9 Identify Subgoal 4.10 Experimenting	- To explain the meaning of problem and problem solving - To explain the steps in Polya's Model	–	–	6 hrs
5.0 Teaching Strategies for Problem Solving	- To translate the problem into one's own understanding - To select the appropriate problem solving strategy - To plan activities relevant to the chosen strategy - To prepare appropriate teaching and learning aids - To reflect on the effectiveness of the chosen strategy	2 hrs	–	–

1.2 The Epistemological Basis of the Study

In the constructivist perspective, knowledge is actively constructed by the individual through his interactions with the environment. Knowledge cannot be transferred ready-made from a teacher to the learner but must be constructed by the learner himself. Any kind of knowledge is constructed rather than perceived through the senses (von Glasersfeld, 1994).

Learning is a process of constructing meaningful representation, of making sense of one's experiential world. For the educators, the challenge is to be able to build a hypothetical model of the conceptual world of students since this world could be very different from what is intended by the educator (von Glasersfeld, 1995). In the constructivist view of mathematics learning, the teachers' challenge and emphasis is on creating learning environments that will help students create good schemes of mathematics understanding. In the process of learning, students' errors are seen in a positive light and as a means of gaining insight into how they are organizing their experiential world. The notion of doing something 'right' or 'correctly' is to do something that fits with 'an order one has established oneself' (von Glasersfeld, 1987, p.15). The way by which a student organizes and manages the flow of experience will help teachers understand the mental activity of the student is considered to be constructive.

The primary task in the development of problem solving abilities in mathematics learning can be viewed as the construction of schemes or mental activities (Steffe & Cobb, 1984). Schemes constitute mental activities that an individual take as being material for the processes of reflection and abstraction (Nik Azis, 1999; Steffe & Cobb, 1984). This implies that students acquire new knowledge through an active process of assimilation and accommodation, where new as well as existing knowledge is transformed as students construct more inclusive schemes of understanding.

According to von Glasersfeld (1995), "Learning is not a stimulus-response phenomenon. It requires self-regulation and the building of conceptual structures through reflection and abstraction" (p.14). Reflective abstraction is a very important part in the constructivist's 'world view' because all new knowledge presupposes some type of an abstraction (Steffe & Cobb, 1984). In other words, when a new experience or knowledge is included into the scheme from actions or operations of the person based on his prior experience. Reflective abstraction concerns patterns derived from actions or operations (von Glasersfeld, 1995). When students build up conceptual coordination, such as the abstract structures required in mathematics, pre-existing elements need to be combined and only a few possible combinations turn out to be viable. The elements that constitute the building blocks that the student must combine in the learning of mathematics is distinguished as operations which are products of reflection and abstraction.

From the constructivist perspective, it is important to note that reflection and abstraction are activities that must be carried out by the person. They involve the person's own experiences under all circumstances. The teacher's role is to foster these activities but they cannot be induced. It is the realization by the individual that one is recognizing the activity more than once. This realization arises from one's own mental operations and not induced by an external source.

It is in the act of knowing that the learner is active, and consequently faces an external disturbance. The learner will react and regulate the processes of assimilation and accommodation in order to compensate and consequently moves towards equilibrium. Equilibration is thus, an active process of self-regulation in enabling the growth of more complex schemes. It is this activity of equilibration that is central to the knowledge acquisition process.

This discussion on the knowledge acquisition process serves as a brief explanation of the basic epistemological points for the present study. von Glasersfeld remarks that:

> This feature of the Piagetian model, as I see it, constitutes its main basis as a constructivist theory of cognition in which "knowledge" is no longer a true or false representation of reality but simply the schemes of action and the schemes of operation that are functioning reliably and effectively.
>
> (von Glasersfeld, 1980, p. 83)

This study undertakes four basic assumptions based on the constructivist perspective on how trainee teachers construct meanings and solution paths in mathematical problem solving situations:

1. Knowledge on mathematical problem solving must be actively constructed by the individual based on his or her own experience.
2. Trainee teachers construct meanings by reflecting on their mental activity, and their conceptual reorganizations have genesis in problematic and goal setting situations.
3. The construction of that knowledge can be traced to the basic building block known as scheme.
4. Schemes constitute the mental activities that trainee teachers take as material for reflection and abstraction.

In summary, the subjects' mathematical problem solving knowledge can be viewed as coordinated schemes of actions and operations the subjects have constructed in their mathematical problem solution paths. The investigation to examine the mathematical problem solution path as the subjects attempt to solve mathematical problem tasks could yield an understanding of the possible difficulties they might encounter when solving mathematical problems. Moreover, it could also provide useful information to mathematics educators regarding what mathematical knowledge the individual might construct, how they might construct it, and aspects of previous mathematical knowledge that needs to be refined and emphasized.

1.2.1 The Notion of Schemes

According to Piaget (1980), a scheme is the fundamental building block in the construction of knowledge.

> A scheme for Piaget is the coordination and organization of adaptive action, considered as a behavioural structure within the organism, such that the organism can transfer or generalize the action to similar and analogous circumstances.
>
> (Furth, 1981, p. 44)

For Piaget (1970), a scheme is "whatever is repeated or generalizable in an action" (p.34).

> All action that is repeated or generalized through application to new objects engenders by this fact a 'scheme'.
>
> (Piaget, 1980, p. 24)

Von Glasersfeld (1980) relates the notion of scheme to a structure comprising of three basic parts: a situation (or situations) that serves as a trigger for an action or operation (conceptual or internalised activity); an action or operation; and finally, a result or a sequel of the activity.

These schemes are used in a process called adaptation, where adaptation consists of two processes namely assimilation and accommodation. In discussing the general and specific actions, Piaget relates the notion of scheme to these two processes:

> A scheme is an instrument of assimilation, hence a generalization, and it therefore intervenes in every problem of intelligence. But each scheme must accommodate itself to the given situation so that its exercise implies an equilibration between assimilation and accommodation.
>
> (Piaget, Inhelder, & Sinclair, 1968, p.476)

In other words, "an assimilation is the process of incorporating new experiences into the scheme, while an accommodation is any changes that results in either the creation of a new or more elaborated scheme or in the splitting of the scheme into sub-schemes" (von Glasersfeld, 1982, p.82). Thus, a scheme results simultaneously from the action of the learner and from his or her prior experience of accommodation to better fit the subject's experienced world. It derives from assimilation as a result of mental activity and seeks to incorporate experiential data and to recognize and generalize that which has been discovered.

1.3 Statement of the Problem

Problem solving is an important aspect in the teaching and learning of mathematics. Moreover, problem solving is a dominant element in the mathematics curriculum for it exists in three different modes, namely as content, ability, and learning approach (Ministry of Education, 2006). The Malaysian Mathematics Curriculum for primary and secondary school also include problem solving as one of the objectives in the KBSR and KBSM Mathematics Syllabus (Ministry of Education, 2000; 2003).

Despite the fact that problem solving has been the focus in Malaysian Mathematics Curriculum, the findings of the *Third International Mathematics and Science Study – Repeat* (TIMSS-Repeat, 2000) revealed that 55% of Malaysian students (61% at the international level) stated that the teachers' emphasis in reasoning and problem solving in mathematics was average. 23% of Malaysian students felt that the teachers' emphasis in reasoning and problem solving was high, while 22% stated it as low. This implies that in Malaysian schools, the teachers' focus in reasoning and problem solving was still below the international mark although problem solving is considered one of the important components in the mathematics curriculum. The findings in TIMSS-Repeat (2000) also indicate that at the international level, teachers who emphasize reasoning and problem solving in their teaching of mathematics tend to have students with higher mean score. According to the TIMSS-Repeat (2000), Malaysia was in the sixteenth position out of 38 participating countries in terms of mean score for mathematics. Other countries in the Pacific region such as Singapore, Korea, Chinese Taipei, Hong Kong and Japan were in the top five positions on the list. The TIMSS report raises concern among mathematics educators regarding the standards and qualities of the learning and teaching of mathematical problem solving in Malaysia. Thus, there is a need to encourage researchers to study on areas in the teaching and learning aspect of problem solving in mathematics education in the local context.

Over the years, problem solving has emerged as one of the major concerns at all levels of school mathematics from primary right up to tertiary level. Educators seem to agree that problem solving abilities and higher order thinking skills should be integrated into any good instructional program. However, the limited knowledge of teachers in the context of problem solving (Federal Inspectorate, 1993) is a setback where the planning of problem solving instruction is concerned. Teachers have to understand how their students are thinking when they solve problems and the thought processes involved in problem solving in order to enhance their teaching and to make learning meaningful to students (Aida, 2001; Fatimah, 1997; Lee, 2002). Reflection on the researcher's observations and experience in classroom teaching, as well as considering the views of other mathematics teachers, generally reveals that students are able to solve routine problems that are similar to the examples used by their teachers. However, when they are given problems that are different from the examples used by their teachers or are given novel problems they face difficulties and are unable to solve them. Hence, there is a need to conduct research in problem solving that focuses on students while they are solving problems in the classroom as stressed by Stacey (2000).

An important role of mathematics educators is also to enhance the problem solving capabilities of their students. Therefore, it is necessary to study and prepare our trainee teachers in this crucial art and skill of problem solving. If trainee teachers are to be an effective guide for the learning of problem solving skills, then they must become a problem solver (Krulik & Rudnick, 1982). Recent research on mathematical problem solving placed importance on the investigation of the quality of knowledge that students bring to the solution process (Byers & Erlwanger, 1985; Lester, 1994). Results from this stream of research are beginning to reveal the type of links that students are able to establish between the various components of mathematical knowledge that would have a significant effect on the progress made during solution attempt. As mentioned by Rowe (1991), what seems to be lacking in our education is the importance of knowing how students think. Comparitively very limited studies have been done to investigate the problem solving abilities and thought processes of students in developing countries such as Malaysia (Lee, 2002). This aspect in mathematics education specifically in areas on problem solving is important so that teaching methods and content can be adapted to the learning needs of individuals. Thus, there is no other better way to start than to conduct studies related to problem solving on the trainee teachers in the teacher education programme.

A major challenge faced by mathematics educators is to improve present levels of understanding as to why students fail to apply the knowledge and skills that their teachers take great effort to share with them (National Council of Teachers of Mathematics, 1989; Australian Education Council, 1990; Board of Senior Secondary

Studies, 1995). One cause of concern among mathematics educators that has received a great deal of interest is the study of students' knowledge construction and what effect, if any, this would have in helping students apply prior knowledge during task performance. Current investigation on mathematical problem solving tends to focus on issues concerning students' ability in accessing and making flexible use of previously learnt knowledge. Hence, studies of this nature that target knowledge construction and application in mathematics are essential if mathematics educators are to understand the problem of students' success or failure to apply previously learnt mathematical knowledge to new problems. As such, this study intends to investigate potential connections between mental images and mathematical problem solution paths constructed by trainee teachers in their problem solving attempts. This study also attempts to investigate the mathematical problem solution paths of how trainee teachers go about doing mathematical problem solving.

1.4 Purpose of the Study

Problem solving is generally recognized as one of the most important components of mathematics (Williams, 2003). In fact, problem solving is viewed as a process that provides students an opportunity to experience the power of mathematics in the world around them (Cai, 2000). Hence, the purpose of integrating problem solving in classroom instructions is to develop students' problem solving skills, help them acquire ways of thinking, form habits of persistence, and build their confidence in dealing with unfamiliar situations. These views are also advocated by the international mathematics education community (Cockroft, 1982; Hashimoto, 1987; NCTM, 2000; State Education Commission of China, SECC, 1993). In all these countries, mathematical problem solving in classroom activities are viewed as an important focus of instruction that provides opportunities for students to enhance their flexible, independent mathematical thinking and reasoning abilities.

In the recent *Ninth International Congress of Mathematics Education, ICME-9* (Pehkonen, 2000), one of the points stressed is that problem solving in teaching of school mathematics is getting more and more emphasis all over the world. In addition, the rise of constructivist approach in learning has further increased its importance in mathematics education. Recent studies as reported in the *Professional Standards for Teaching Mathematics* (NCTM, 1991) indicate that the current trend is the shift of teaching and learning of mathematics to the constructivist approach.

In the education arena in Malaysia, the importance of problem solving in mathematics instruction has been emphasized in the school mathematics curriculum (Abdul Rafie, 2002; Ministry of Education, 1988; 1989; 1990; 1994; 1996; 2000; 2003). Similarly, problem solving has been the focus in the mathematics curriculum for the teacher education programme (Bahagian Pendidikan Guru, 1998; 2001; 2003). Despite the emphasis in problem solving in the mathematics curriculum, research findings from the few studies reveal that students are lacking in problem solving abilities and are unable to solve problems that require higher order thinking (Abdul Razak, Alias Baba, Rashidi, Thamby Subahan, & Siti Fatimah, 1998; Federal Inspectorate, 1993; Nor Azlan & Lui, 2000). In addition, mathematics teachers have also expressed the need to enrich their knowledge to improve on problem solving instructions and to better understand students' problem solving abilities (Aida, 2001; Fatimah, 1997; Federal Inspectorate, 1993).

Research conducted on trainee teachers is rather limited, thus the findings concerning trainee teachers' mathematical problem solution path and thought processes in mathematical problem solving seeks to contribute further towards enriching the knowledge of mathematics educators regarding this area of study. As such, this study attempts to explore the mental images of the trainee teachers on problem solving and their understanding of problem solving. Secondly, this study intends to examine the mathematical problem solution path and thought processes of the problem solvers while attempting to solve mathematical problems. Finally, an attempt is made to ascertain the mathematical problem solution paths of successful and unsuccessful problem solvers.

1.5 Research Questions

This study attempts to answer the following research questions:

1. What are the mental images of trainee teachers regarding problem solving?
2. What are the mathematical problem solution paths used by the trainee teachers to solve the mathematical tasks?
3. What are the mathematical problem solution paths exhibited by successful and unsuccessful trainee teachers?

1.6 Significance of the Study

The findings of this study, would help mathematics educators have a better understanding on the mathematical problem solution paths and thought processes of trainee teachers in their attempt to solve mathematical tasks. In addition, knowing how trainee teachers perceive problem solving and their mental images of problem solving will help mathematics educators in teacher training colleges to understand better how problem solving should be taught and presented in the classroom. The study on the mathematical problem solution paths will be useful to mathematics educators in understanding how trainee teachers construct and connect mathematical knowledge in solving mathematical problems. Knowledge about the trainee teachers' mathematical problem solution paths will play an important role in helping educators and curriculum planners to plan better teacher development programme. This study may help to illuminate mathematics educators' awareness of the trainee teachers' problem solving process as well as to encourage the mathematic educators to incorporate problem solving skills in the planning of instructions to enhance mathematics learning in the classroom.

The findings may also contribute some useful information to curriculum planners and policy makers in any future restructuring of the mathematics teacher education curriculum. In this aspect, it will help the relevant authorities to evaluate the suitability and relevancy of problem solving in the present mathematics curriculum. The knowledge obtained from this study may enhance better pedagogical and teaching methods presently employed in the teacher training colleges specifically in the area of problem solving.

1.7 Limitation of the Study

This study takes into consideration several limitations. One of them is the small sample size comprising of seventeen trainee teachers as the researcher intends to explore in detail and in-depth each subject of the study. The method of data collection is a time consuming task as each individual session comprises of three sections. In view of the small sample size, the findings of this study may serve to provide some light in the area regarding trainee teachers' mathematical problem solution paths. However the findings of this study cannot be generalised to the population.

In this study, six problem solving tasks were given to each subject to solve within certain time constraints. The six problem solving tasks were categorised under the main areas of the mathematics curriculum that comprises relationships and patterns, representation, basic algebra, and geometry. According to Payne (1994), too many questions might tire the subjects and this would in turn affect their thought processes.

Part of the data analysis will be based on the think-aloud technique and retrospection of the subjects. However, the use of think-aloud has its own setback (Payne, 1994; Smagorinsky, 1994). Precautions need to be observed in order to minimise the inhibiting effects of the techniques while encouraging verbalisation. The setback is whether the verbatim is a true representation of the thought processes. It was also felt that talking while doing might interfere with cognition processes. Ericsson and Simon (1980; 1984) have focused on the possibilities of verbalisation affecting normal cognitive processes and they have found that generally verbalisation does not affect the thought processes.

Although the think-aloud technique provides a rich source of data for the study of problem solving and learning, they do not contain a complete description of the processes that are operating during these activities. As early as 1945, Drunker, a strong proponent of the think-aloud method warns that, "a protocol is relatively reliable for what it positively contains, but not for what it omits" (p. 11). Besides, there is also the danger of 'telling more than we can know' (Nisbett & Wilson, 1977). Some researchers have noted the possibilities that individuals might proceed differently when asked to think-aloud than they would if they were to work in silence (Kilpatrick, 1969; Neisser, 1963). Still there are others (Byers & Davison, 1967; Marks, 1951) who found that think-aloud actually had a facilitating effect on problem solving performance. However, Ericsson and Simon (1980, 1984) suggested that the effect of verbalisation on the actual thought processes is quite minimal. Despite these limitations, the protocol analysis remains the most appropriate data collection tool to examine and understand the problem solving behaviours of the subjects (Teong, 2002; Chinnappan & Lawson, 1996).

Several critiques of the clinical interview method can be found in Brainerd (1978), Flavell (1963), Ginsburg (1977), Ginsburg, Kossan, Schwartz, & Swanson

(1983), and Opper (1977). The clinical method has been criticized for lack of standardization of procedures; contributing false negatives (Brainerd, 1978); and lack of information for precise replication (Flavell, 1963). Nevertheless, the problem of lack of standardized procedures and lack of information for replication can be offset by the interview plans that are discussed in detail in section 3.4.2.

1.8 Definition of Terms

Knowledge construction refers to the individual learner's active building-up of meanings or understandings on concepts or content knowledge based on the learner's own experience or prior knowledge.

Scheme refers to a cluster of knowledge that contains information about core concepts, the relations between these concepts and knowledge about how and when to use these concepts. A scheme is whatever is repeated and generalizable in an action and is consider to be activated if a student mentions it and uses it in his or her solution path.

Mental image refers to the image or representation that forms in an individual's thought, abstraction of procedures or knowledge as a result of the immediate application of the knowledge scheme at a particular time in a specific context such as while the subject attempts to solve a particular task.

Thought processes refer to the individual's organization, flow, strategies and production of thought during problem solving.

Reflective abstraction concerns patterns or the realization that one is carrying out the same recognition procedure that arises from one's own actions or operations.

Understanding in this context is based on the assumption that knowledge is represented internally and it refers to the way an individual's internal representation are structured. More specifically, the understanding of problem solving relates to the mental representation of an individual as part of an individual's problem solving process.

A Problem is a situation or task for which the individual has a goal that cannot be achieved directly and he or she who confronts it has no particular method that will guarantee a solution at that particular time.

Problem solving in this context refers to a coordinated schemes of actions and operations the learner has constructed at a particular point of time to achieve a solution to a problem situation.

Mathematical problem solution path refers to the written solution or the written work that can be expressed in terms of mathematical content of the problem solvers in their attempts in solving problems. The solution path also encompasses the sequence of actions leading from one state to another or the path from the initial-state to the goal-state, it also encompasses the mathematical concepts, problem solving approaches or strategies employed by the solvers. The mathematical problem solution path is the result of the activity that involves a sequence of schemes of actions and operations of the solvers.

Problem solving behaviours refer to the desired learning outcome as an expression of relationship among rules and concepts, computation, generating conjectures, construction, drawing diagrams, graphs or tables in the problem solving process.

Problem solving strategies refer to the general techniques for handling a task when no specific solution method has yet been found. Some of the problem solving strategies are guess and check, use a model or diagram, simulation, working backwards, looking for a pattern and so on.

Problem solving task refers to problems at all levels in any mathematical context and is not restricted to any specific area of mathematics or sets of problems that are encountered in a problem solving class.

Heuristics refers to the mental processes to problem solving. It also means a general suggestion (independent of any particular topic or subject matter) that helps problem solvers approach and understanding to a problem and efficiently use their resources to solve it.

Trainee teachers refer to pre-service teachers who follow the teacher education courses in the Malaysian Diploma in Teaching (KDPM) and the Post Graduate Diploma in Teaching (KPLI) conducted in the teacher training colleges in Malaysia designed towards preparing teachers for primary and secondary schools. In this study the trainee teachers refer specifically to those who major in mathematics in their respective course.

Solution path refers to the written solution of the trainee teacher during his or her attempts to solve the task. Solutions for a problem are often viewed as searching for a "solution path" from the initial state to the goal-state. The essence of the problem solving task occurs in a context where the solution path is not readily known to the problem solver. A solution path is often useful in problem solving and different problems may have different possible solution paths. There is more that one path to the solution and some paths may or may not lead to the solution. Although the mathematical concepts used are correct it simply does not always lead to the solution. Good problem solvers become aware of what they are doing and frequently monitor or self access their progress or rely on their past experience in the problem solving process in choosing the correct solution path. Being able to solve problems is not through memorizing the solution path to the problem. It is to be able to understand the actions taken during the problem solving process and in making the right connections based on the solver's own understanding, knowledge construction and their past mathematical experience. The solvers have to construct the solution path to the problem themselves based on their own understanding.