DETECTING BURSA MALAYSIA'S LONG-TERM DEPENDENCE OF RETURNS USING RESCALED RANGE ANALYSIS: (1982-2001)

OOI KOK HWA

UNIVERSITI SAINS MALAYSIA
2006

DETECTING BURSA MALAYSIA'S LONG-TERM DEPENDENCE OF RETURNS USING RESCALED RANGE ANALYSIS: (1982-2001)

By

OOI KOK HWA

Thesis submitted in fulfillment of the requirements for the degree of Master of Arts (Finance)

June 2006

ACKNOWLEDGEMENTS

I wish to express my gratitude to my supervisors, Dr. Zamri Ahmad and Assoc. Prof Ruhani Ali for spending their invaluable time in discussion and supervision during the course of this study. Their assistance and guidance have contributed to the preparation of this thesis.

I would also like to take this opportunity to thank Professor Muhamad Jantan, Assoc. Prof. Zainal Ariffin Ahmad, Assoc. Prof. Yuserrie Zainuddin and Assoc. Prof. Fauziah Md. Taib for their most valuable advice.

Last but not least, to my mom, thanks for being understanding. All would not have been possible if not for the kind understanding and continuous moral support from my wife, Chooi Theng. To her, I dedicate this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

		Pages
P	Stock price	18
φ_t	All available information at time t	42
R	The expected stock returns	42
$e_{j,t+1}$	Random variable with zero mean	43
H	H Statistic	45
N	The logarithmic return on the index at time i	57
M	A time series of length N=M-1	57
Q_T	The modified R/S statistic, denoted by, is given by the range of cumulative sums of deviations of the time series from its mean, rescaled by a consistent estimate of its standard deviation:	61
R	The range of cumulative sums of deviations from the sample mean	61
q	A truncation lag	62

MENGESAN KEBERGANTUNGAN PULANGAN-PULANGAN JANGKA PANJANG BURSA MALAYSIA DENGAN MENGGUNAKAN KAEDAH ANALISIS JULAT YANG DISKEL SEMULA: (1982-2001)

ABSTRAK

Tesis ini mengkaji gelagat ingatan jangka panjang bagi harga-harga saham di Bursa Malaysia menggunakan Kaedah *Rescaled Range Analysis* (RSA). Kaedah yang sedia ada berguna dalam meramal pergerakan pasaran hanya sesuai dalam keadaan yang norma di bawah andaian bahawa data adalah bebas dan bertaburan serupa dan perhubungan antara pembolehubah-pembolehubah bersandar dan tak bersandar adalah linear. Bagaimanapun, kaedah-kaedah konvensional ini cuma berupaya mengenal pasti kitaran berjangka yang tetap. Mereka gagal mengesan kitaran bila data adalah tidak bebas, kitaran adalah tidak berjangka dan sistem adalah dinamik tak linear. Tesis ini mengaplikasi *Choas Theory* dan *Fractal* untuk menjelaskan fenomena pasaran kewangan di mana hipotesis pasaran cekap (EMH) tidak berupaya menjelaskan fenomena secara efektif. RSA adalah teknik *non-parametric* yang boleh membezakan tempoh purata kitaran bagi putaran yang tidak tetap dalam sistem yang dinamik tak linear. Kajian ini juga memperkenalkan satu kaedah baru dengan menggunakan aspek keberubahan RSA. Kaedah in dinamakan *Moving RSA* (MRSA). MRSA menggunakan tempoh rujukan yang bergerak, di mana tempoh rujukan yang berlainan bagi permulaan dan akhirannya. Ia akan mengkaji sama ada terdapat perubahan ingatan jangka panjang pulangan saham sebelum dan selepas kejatuhan yang besar dan mendadak (*large and abrupt drop* (LAD) ataupun kejatuhan pasaran). Secara umumnya, kajian mendapati bahawa terdapat ingatan jangka panjang bagi pulangan saham di pasaran Malaysia. Walau bagaimanapun, ingatan jangka panjang ini adalah

lemah bagi kebanyakan tempoh, kecuali ketika kejatuhan pasaran yang menyaksikan kemeruapan yang tinggi. Ini adalah bertentangan dengan teori *random walk*. Kajian ini berjaya mengesan corak-corak yang boleh dijangkakan sebelum kejatuhan pasaran. Bagaimanapun, kajian ini tidak boleh menjangka secara tepat bila kejatuhan pasaran berikutnya akan berlaku. Oleh itu, Bursa Malaysia masih cekap bentuk lemah. Akhir sekali, perubahan pada volum dagangan tidak menunjukkan ingatan jangka panjang. Ia nya hanya menunjukkan ingatan jangka pendek.

DETECTING BURSA MALAYSIA'S LONG-TERM DEPENDENCE OF RETURNS USING RESCALED RANGE ANALYSIS: (1982-2001)

ABSTRACT

This study examines the long memory behavior in Malaysian stock prices by using a method called Rescaled Range Analysis (RSA). The standard conventional statistical methods are useful in predicting market movements under the assumption that the data are independent and identically distributed as well as the relationship between dependent and independent variables are linear. However, these conventional methods are only able to identify regular periodic cycles. They fail to detect any cycles when the data are not independent, the cycles are not periodic and the system is nonlinear dynamic. This thesis applies Chaos Theory and fractals to explain the financial market phenomenon where the conventional Efficient Market Hypothesis (EMH) is unable to explain them effectively. RSA is a nonparametric technique that can distinguish the average cycle length of irregular cycles in a nonlinear dynamic deterministic chaotic system. We also introduce a new approach by introducing a moving aspect of RSA. We name this Moving RSA as MRSA. MRSA uses a moving reference period, a moving starting and ending period instead of a static reference period. It will examine whether there are changing long-term memory in stock returns before and after large and abrupt drop (LAD) (or market crash). In general, we have found long-term memory in stock returns for Bursa Malaysia. However, the long-term memories are weak in most periods except during the LAD period where stocks exhibited high volatility in returns. This is against the Random Walk Theory. We are able to detect some predictable patterns before the market crash. However, we are unable to forecast the exact date for the next market crash. Hence, Bursa Malaysia is still weak-form efficient. Lastly, the changes in trading volume do not show any long-term memories. They have only short-term memories.

CHAPTER ONE

INTRODUCTION

1.1 Background

Studies on the behaviour of stock prices have been going on for more than 50 years, and it seems that there is still a huge interest in the area. The basic question being asked by earlier researchers is whether stock prices are predictable. Fama (1970, 1991) reviews related studies published in the 1950s and 1960s, and concludes that the movement of stock prices are best characterized by a random walk process, and from there on, he formulates what is widely known today as the Efficient Market Hypothesis (EMH). EMH postulates that it will be a futile effort to predict stock prices as all information has already been incorporated into prices.

Though hailed as one of the most remarkable idea in economics, the hypothesis started to show its weaknesses as more and more studies starting in the late 70s revealed some pricing anomalies in the capital markets, which could not be explained by the EMH. Evidence show that there exists firm size effect (Banz, 1981), price-to-earnings effect (Basu, 1977), overreaction effect (DeBondt & Thaler, 1985) and other numerous anomalies which imply that prices are predictable. Fama himself documented that the autocorrelation in returns was stronger in the long-term period than the short-term period which implies that there is a long-term dependence in return (see Fama & French, 1988).

The long-range dependence of returns implies the current returns on a security can be determined by its historical long-period returns. The importance of long-range dependence or long-term memories of returns in financial markets was first considered by Mandelbrot (1971). The findings uncovered by Fama and French (1988), Lo and

MacKinlay (1988), and Poterba and Summers (1988) can be considered as early evidence of the presence of a long-range dependent component in stock market prices.

1.2 Statement of the Problem

Though EMH has some answers in most of stock market anomalies, many questions still remain. The absence of any significant news during the stock market crash of October 1987 further increases the vulnerability of the EMH. The efficient market hypothesis (EMH) and the capital asset pricing model (CAPM) originally developed by Sharpe (1964), Linter (1965), and Mossin (1966), were very successful in making the mathematical environment easier for explaining stock returns, but unfortunately are not justified by the real data. Mandelbrot (1961) asserts that historical research is too dependent on the assumption of normality in price changes and has neglected certain observations that depart from normality. He proposes a new approach called stable Paretian hypothesis. Stable Paretian distributions have high peaks at the mean and fat tails. Fat tails mean that the probability of having high abrupt and discontinuous changes is higher than a Gaussian distribution (normal distribution).

There are two concepts, namely fractal and chaos theory, that can be used to explain stock returns without the assumption of normality. Fractal and chaos theory (see Gleick, 1987; Mandelbrot, 1972; Peters, 1989, 1991, 1992, 1994, 1996; Mirowski, 1990; Weron and Weron, 2000) originated from mathematics. Chaos refers to disorder or irregularity whereas fractal refers to self-similar structure. These two concepts are used to explain the hidden self-similar structure (fractal) of the large and abrupt drop (LAD) in stock returns for a disorderly (chaotic) market situation, especially during a stock market crash. The advantage of using fractal and chaos theory is that they do not require the assumption of rationality, order and optimization as assumed by the EMH (Lo and MacKinlay, 1988; Fama, 1963; Fama and French, 1988; Scheinkman and LeBaron, 1989; Hsieh, 1991). In this thesis, we explore the predictability of these long-term dependences of LAD in stock returns. We try to detect and later predict a self-similar structure of LAD in stock returns for the period of market crashes in 1987, 1994, 1998 and 2000. Any predictable pattern of returns during these periods may imply that we are able to predict when the next market crash will happen. If series of past stock returns reveal recognizable pattern, then the market will have long memory (persistency). This would be against the random walk hypothesis which claims that stock returns are random and they have equal chances of moving up as well as moving down.

The methodology that will be used in this study is based on rescaled range analysis (RSA) (Peters (1994) calls it as fractal market analysis) and modified rescaled range analysis (MRS) as modified by Lo (1991). For MRS, Lo (1991) has modified the classical RSA for long memory in a time series to account for short-range dependence under the null hypothesis. The main use of RSA is to detect the variety of long-term dependence of returns. Long-term dependence (persistence) occurs when there is persistent dependence between observations far apart in time. Cheung and Lai (1993), Fung and Lo (1993) and So (2000) uses the MRS whereas Aydogan and Booth (1982), Freund, Larrain and Pagano (1997) and McKenzie (2001) use RSA to test the long memory. Fung and Lo (1993), Blasco and Santamaria (1996), Howe, Martin and Wood (1999) and Mulligan (2000) used both MRS and RSA. Except for the results from Lo (1991), Cheung and Lai (1993), Fung and Lo (1993), Berg and Lyhagen (1998), Fung, Lo and Peterson (1994), Blasco and Santamaria (1996) and Howe et al (1999), all other researches found the existence of long memory of returns.

It should also be noted that except for Cheung and Lai (1993) and Freund et al. (1997), all the above works were conducted on a static reference period. For example, McKenzie (2001) applied RSA on monthly Australian stock market returns over the static period from April 1876 to March 1996. Mulligan (2000) used the monthly average dollar exchange rates for a selection of countries covering the fixed period of January 1973 to December 1997. Both of them tested on a fixed period, meaning that the tests were carried out based on a fixed initial and ending condition. For Cheung and Lai (1993) and Freund et al. (1997), their studies examined data based on a rolling reference periods with different initial and ending conditions. As a result, they noticed some inconsistency in results when the data covered a period with high market volatility in stock prices. This discovery is in line with one of the characteristic of a nonlinear dynamic system. According to a nonlinear dynamic system, the predictability of a nonlinear system decline with time and is very sensitive to its initial condition. Any slight changes to the initial condition will affect the predictability of a nonlinear dynamic system.

Cheung and Lai (1993) applied MRS on the returns on gold prices. In general, they found no long memory in gold returns if they excluded the period of LAD. However, they discovered long memories in gold price returns if they included the event of large volatility in their time period. The sensitivity of MRS method is dependent on the volatility of returns. MRS will show long memory when the data covers the period of high volatility in returns. Cheung and Lai (1993) concluded that future research should remove data in those LAD periods. They argued that gold price returns do not show long memory in most periods except for the LAD period.

However, according to Peters (1994) and Mouck (1998), any trimming in raw data may result in misrepresentation of the real market situation. As a result, we intend to explore further the characteristic of long-term memories in returns, especially during the LAD period. By using the full-data (without any trimming activities for exceptional high volatilities), we attempt to detect the self-similar structure of LAD. This thesis explores the possibility of getting a self-similar changing pattern of long memories in stock returns before and after the LAD period. Any predictable patterns may imply that we are able to know when the next market crash will be.

In addition, there are many evidences that increased trading activities and stock return volatilities can occur together (Schwert, 1990). It is difficult to determine what causes this association. There are many studies on the relationship between trading volume and price (see Ying, 1966; Tauchen & Pitts, 1983; Karpoff, 1987; Ali & Sanda, 2000). For example, Ying (1996) and Tauchen and Pitts (1983) show that absolute price changes and trading volume are positively correlated. However, none of the above studies try to examine the characteristic of long memory in trading volume. Thus, this thesis will also explore the pattern of long memory in trading volume.

1.3 Research Questions

Given the focus earlier and based on the discussion thus far, the study would like to answer the following questions:

1. By comparing three main indices: Kuala Lumpur Composite Index (KLCI), Kuala Lumpur Emas Index (KLEI) and Second Board Index (SBI), are there any long-term memories in Bursa Malaysia? Does KLCI, which represent liquid stocks, has a different pattern as compared to KLEI and SBI? Market liquidity is a crucial factor in determining whether a market is efficient.
2. Are there any detectable changing patterns in long-term memories during LAD (market crash) in 1987, 1994, 1998 and 2000?
3. Are there any long-term memories in trading volume? Do they exhibit any similar pattern like prices?

1.4 Objectives and Scope of the Study

The overall objective of this study is to examine the long-term dependence of returns of Malaysian indices. The first objective is to detect whether there is any long-term dependence of returns for Malaysian stock market. We will test the long-term memories of returns in the Bursa Malaysia by looking into three main indices: Kuala Lumpur Composite Index (KLCI), Kuala Lumpur Emas Index (KLEI) and Second Board Index (SBI). Any significant long persistency in returns from these three indices indicates current returns have relationship with past returns. This would show evidence against random walk hypothesis that past returns have no influence in the present returns. Three indices are used to test the issue of liquidity as selected companies in KLCI are generally more liquid than companies in SBI. The results from KLEI and SBI will examine whether the issue of market capitalization can influence the long memory of stock returns.

The second objective is to test the predictability of LAD in Malaysian stock market. We introduce a new approach by introducing a moving aspect of RSA. We name this Moving RSA as MRSA as compared to the conventional RSA used by Peters (1994). "Moving" aspect of MRS is not explored in this study as we are examining the changing behavior patterns of H-statistic. H-statistic is a measure of the bias in fractional Brownian motion. It can be used to examine whether a time series is short-term or long-term dependence. The behavior patterns of H statistic can only be analyzed by using the RSA that was modified by Peters (1994), thus the use of moving RSA (MRSA).

MRSA uses a moving reference period, a moving starting and ending period instead of a static reference period. It will examine whether there are changing long memory pattern in returns before and after LAD (as opposed to the present RSA which focus only on finding a static long memory). The results will show not only persistency in returns (against the random walk hypothesis), but more importantly would indicate if future

LAD can be predicted. Any predictability of future LAD implies the market is not weak-form efficient. The study will cover the period from 1982 to 2001, but special attention will be placed on the LAD during 1987, 1994, 1998 and 2000. The main reason for choosing these four special periods is because the market shows a significant large percentage drop from its peak during the year. The maximum one-day drop for 1987, 1994, 1998 and 2000 were 15.7%, 6.4%, 21.5% and 6.0% respectively. This was to compare with the average absolute one-day change of 1% for the period 1982-2001.

Finally, the third objective is to detect whether there is any long-term memories for trading volume by using RSA. In general, changes in trading volume are more volatile than changes in prices. This test will provide not only the characteristic of long memory in trading volume but more importantly the pattern of RSA under high volatility in returns.

The examination of first objective will determine whether the returns on Bursa Malaysia show any long-term dependence. The issues of selected high liquidity companies (via KLCI), all main board companies (via KLEI) and small capital companies (via SBI) are explored in this test. The second objective will examine the behavior of long memory pattern of returns. It will detect whether there is any predictable pattern of long-memory during the period of LADs. Final objective is to test whether there are any long-memories for the daily changing pattern of trading volume.

The objectives of the thesis are as follows:

1. To determine the long-term memories of KLCI, KLEI and SBI using RSA and MRS.
2. To determine the changing long memory pattern during LAD. To detect the self-similarity (or fractal structure) in long-term memories before or after LAD (Using MRSA).
3. To detect the long memory of returns in trading volume using RSA.

1.5 Significance of This Study

This is the first attempt to use a nonlinear dynamic technique: rescaled range analysis to test the efficiency of the Bursa Malaysia. We introduce a new "dynamic" approach for nonlinear dynamic techniques. Even though RSA is nonlinear dynamic technique, its "dynamic" are confined to a fixed reference period. Nonlinear dynamic system says that any nonlinear dynamic model is sensitive to its initial conditions. Hence, we provide a "dynamic approach" for this "dynamic" technique using MRSA which will look at moving reference period before and after the LAD. It will be used to test the market efficiency by looking into the changing pattern of long-term memories. There is a missing link between the conventional RSA and market efficiency. The conventional RSA is able to show persistency (against random walk hypothesis) but it is unable to tell whether the market is not weak-form efficient (they are unable to tell whether future LAD can be predictable). If there is any consistent changing long-term memories pattern (by using MRSA), we are able to predict when will the next LAD be. This would be against weak-form market efficiency. Lastly, this thesis also applies the RSA on testing any long memory for trading volume. Past evidence suggest that changes in trading volume are more volatile than changes in prices. By carrying out this final test, we will have a better picture of the long memory in trading volume.

1.6 Definition of Terms

Autoregressive conditional heteroskedasticity (ARCH) process	A nonlinear stochastic process, where the variance is related to the past variance (Engle, 1982)

Chaos	A system is classified as chaotic when it exhibits orders in a disorder situation. It may appear to be random or discontinued from the previous movement.
Fractal	It is a shape made of parts similar to the whole in some way. They may be different between each other but they have the same underlying patterns. Sometimes, it is also called self-similar structure (Peters, 1994).
Gaussian	A system whose probabilities are described by a normal distribution or bell-shaped curve.
Generalized ARCH (GARCH) process	The ARCH model was modified to make the *s* variable dependent on the past (Bollerslev, 1986, 1992).
H-Statistic	A measure of the bias in fractional Brownian motion. $H=0.50$ for Brownian motion; $0.5<H\leq1.0$ for persistent or trend-reinforcing series; $0\leq H<0.50$ for an antipersistent or mean-reverting system. (Peters, 1994)
Joseph effect	The tendency for persistent time series ($0.50<H\leq1.00$) to have long-term trends and non-periodic cycles (Mandelbrot, 1966b).
Large and Abrupt drop (LAD)	A sudden huge drop in prices or market crash

Leptokurtosis	The condition of a probability density curve that has fatter tails and a higher peak at the mean than at the normal distribution (Peters, 1994).
Long-term memory	Long-term dependence of returns, meaning the current returns is related to the previous long period of returns.
Modified Rescaled Range Analysis (MRS)	A modified version of RSA used by Lo (1991). Lo replaced the denominator of the R/S with the square root of the sum of the sample variance and weighted covariance terms.
Moving RSA (MRSA)	It is a dynamic approach on RSA which uses different initial conditions on the same time series period.
Noah effect	The observed instances of large "discontinues" jumps in stock prices (Mandelbrot, 1966b).
Non-periodic cycle	The cyclical patterns can be discerned from afar, but cannot be predicted when they will begin, when they will end, or how severe they will be. (Peters, 1994)
Nonlinear dynamic system	It has sample path trajectories which appear random but are in fact deterministic (Peters, 1994).
Rescaled Range Analysis (RSA)	The method developed by H.E. Hurst to determine long-memory effects and fractional Brownian motion. A measurement of how the distance covered by a particle increases over longer and

	longer time scales. For Brownian motion, the distance covered increases with the square root of time. A series that increases at a different rate is not random (Peters, 1994).
V-Statistic	A method to estimate the cycle length.

1.7 Organisation of the Thesis

Chapter 1 introduces the subject matter, explains the research problem and states the objectives of the study. Chapter 2 highlights the previous studies, concepts on fractal and chaos theory as well as long-term dependence on returns. Chapter 3 describes the methodologies used for the analyses. Chapter 4 discusses and presents the results of the study. Chapter 5 concludes the study, discuss some limitations and implications and give some suggestions for future studies.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

Before 1987, most of the empirical studies are centered on the linear equilibrium system. As a result of the stock market crash in 1987, the traditional assumption on normal distribution appears to be inadequate to explain the irregular and large movements on the stock prices. Thus, chaos theory and fractals are getting more attention after 1987.

This chapter is divided into four main sections. Section 2.1 will discuss the concept and literature review on chaos theory and its other related concepts. Other relevant terms, like nonlinear dynamic system, fractals, large and abrupt drop (LAD) will be explored in this section. Under section 2.2, the application of all these theories and concepts into financial markets will be reviewed. The theoretical framework on efficient market hypothesis, random walk theory and weak-form market efficiency will also be explained in this section. Next, section 2.3 will review the evidence of long-term memory in the financial markets. Finally, section 2.4 will summarize the chapter.

2.1 Chaos theory and its related concepts

This section will explain in details the chaos theory and its relation to nonlinear dynamic system. Besides, chaos theory and fractals will be compared with other nonlinear method like autoregression, ARCH and GARCH.

2.1.1 The complex system and definition of chaos theory

One of the famous books on chaos titled "Chaos, making a new science", written by James Gleick in 1987, states that:

"Where chaos begins, classical science stops. For as long as the world has had physicists inquiring into the laws of nature, it has suffered a special ignorance about disorder in the atmosphere, in the turbulent sea, in the fluctuations of wildlife populations, in the oscillations of the heart and the brain. The irregular side of nature, the discontinuous and erratic side – these have been puzzles to science, or worse, monstrosities" (Gleick, 1987, p.3)

Nature is a complex system. It is deemed to be unstable and unpredictable. However, some scientists formulate the nature into a simple manner where it can be easily understood by using some deterministic laws. As a result, some prediction models are created to predict the short and long-term behavior of the nature. However, these models may not provide the correct representation of the true nature. A complex system does not imply that it is out of control. It could be influenced by random external factors as well as multitude of independent aspects.

A complex system can be explained by complexity theory. According to Chorafas (1994), there are several topics classified under complexity theory, namely nonlinearities, bifurcations, chaos theory, attractors, fractals, entropy, genetic algorithms, predictors, adaptive agents, swarms, fuzzy engineering, Monte Carlo, and patterning. By using algorithms, heuristics and the associated information, the studies on complexity theory helps to explain the dynamic nature of real-world problems. In financial market, he postulates that a well-defined and effective algorithms and heuristics approach can provide some fundamental empirical features of price dynamics in financial transactions, decompose global portfolio risks into a definable and controllable way and set future risks into a certain ranges.

Chaos theory is one of the topics classified under complexity theory. Since 1970s, there have been a growing number of studies on chaos theory. Chaos theory is a branch of mathematics. The origin of chaos theory was referred far back to Henri Poincare (1854-

1912) where he discovered that if a system consisted of a few parts that interacted strongly, it could exhibit unpredictable behavior. He was the first to understand the possibility of chaos. The study of chaos theory is to study the orders in a chaotic system. A chaotic system must have a fractal (or self-similar) structure and exhibit sensitive dependence on initial conditions. The concept of fractal structure will be explained in details in 2.1.7.

2.1.2 Characteristics of chaos theory

2.1.2.a Disorder, irregularity and fractal structure

A system is classified as chaotic when it exhibits orders in a disorder situation. It may appear to be random or discontinued from the previous movement. However, it is a logical consequence of preceding events. Thus, it rejects the concept of discontinuity. It has some hidden orders in a very complex system. The hidden order is also referred to as fractal structure (or self-similar structure) which is generated by a deterministic model. However, the fractal structure may not be explained by some simple linear models. Nevertheless, it can be determined by using some non-parametric techniques like rescaled range analysis which consolidates the data into detectable patterns.

The significance of a chaos time series is that the accuracy of its prediction falls off with the increasing passage of time. Even though its patterns show irregularity and are unpredictable in a short period of time (locally unpredictable), it can be determined over a long period of time (globally stable). As a result, Peters (1994) named this phenomenon as "local randomness but global deterministic". Section 2.1.7 provides further details on this concept.

2.1.2.b Non-periodic cycle and trend-reinforcement

Chaotic time series often display non-periodic cycles and trend-reinforcing behavior. In other words, we may discern cyclical patterns from afar, but we cannot predict when they will begin, when they will end, or how severe they will be. Hsieh (1991) provided some key features on deterministic chaotic systems and used correlation dimension to detect deterministic chaos. Chaotic behavior appeared to be able to explaining the volatility in the stock market. However, he claimed that chaos theory was limited to detect low complexity chaotic behavior. If a market showed a highly complex chaotic behavior, it would be similar to random movement. A short-term prediction on stock prices was possible with the condition that the market exhibited a not-too-complex chaotic process.

2.1.3 Nonlinear Dynamic systems and Nonlinearity

Chaos is a subset of nonlinear analysis. Chaos theory is sometimes referred to as nonlinear dynamic analysis, but these two concepts are different. A nonlinear dynamic system also has sample path trajectories which appear random but are in fact deterministic. A system may be nonlinear, but it may not be chaotic. According to Larrain (1991), nonlinear equations could exhibit volatile behavior without necessarily being chaotic. When a nonlinear system turns into a more complex situation then it will be called a chaotic system. Fractal geometry is related to chaos theory. It is the pictorial representation of some of the equations of chaos.

Chaos theory suggests that we should not ignore errors that happened in forecasts of time series. An example of these errors is the happening of extraordinary volatility in stock prices. It can be a super bull or a sudden drop in stock prices. However, these errors may be explained by nonlinear equations. A nonlinear relationship between two variables shows the disproportionate change in one variable due to a small change in the other. Any external shocks to such an equilibrium system may result in extremely turbulent behavior. It can be further explained by the Butterfly Effect. The Butterfly Effect is a state where it is extremely sensitive to its initial conditions. Section 2.1.3.a. provides further details on this concept.

2.1.3.a Characteristic of Nonlinear Dynamic System

The characteristics of nonlinear dynamic systems can be broadly classified into two characteristics. First, they are feedback systems. Things that happened in the past can affect what will happen today. P(t+1) is a product of P(t). Second, they have critical levels, meaning that more than a single equilibrium exists.

According to Peters (1994), a nonlinear dynamic system has the following characteristics:-

i). It has long-term correlations and trends as a result of feedback effect;

ii). It has critical levels under certain conditions and at certain times;

iii). It has fractal structure characterized by local randomness but global determinism;

iv). Due to the sensitive dependence on initial conditions, it may be less reliable forecasts on long-term.

The sensitivity of dependence on initial conditions was tested by Lorenz in the year 1961. The following chart (Figure 1) was part of Lorenz's 1961 printout showing the effect of the sensitive dependence on initial conditions.

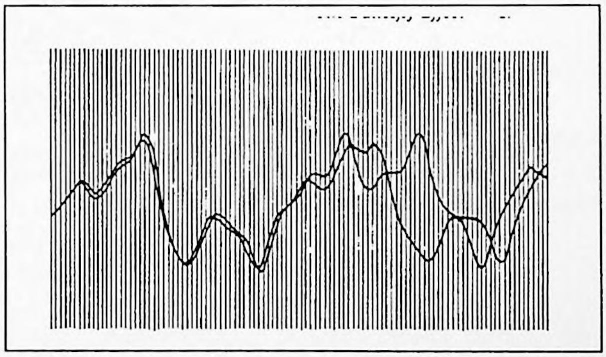

Figure 2.1. Lorenz's 1961 printouts
Source: "Chaos: Making A New Science"(Gleick, 1987, pp. 17)

He conducted computer experiments on forecasting long-range weather patterns in the early 1960s. The two lines indicated the two different starting points on its forecasting model. He encountered the problem of sensitive dependence on initial conditions, in which small changes can lead to major changes later on. The above chart shows how two weather patterns diverge. From nearly the same starting point, the patterns grew farther and farther apart until the same structure disappeared.

2.1.4 Other nonlinear method- Autoregression, ARCH and GARCH

Fat-tailed distributions (this issue will be explored further in 2.2.1.a) are one of the symptoms of nonlinear dynamic system. Roberts (1959) proposes using autoregression method if a time series exhibit a strong dependence. This nonlinear process can be caused by time-varying variance (ARCH), or a long-memory process called Pareto-Levy. ARCH (Autoregressive Conditional Heteroskedasticity) is a nonlinear stochastic process, where the variance is time-varying and conditional upon the past variance. ARCH has frequency distributions that are quite similar to stable Paretian distribution. Its generalized version is called GARCH (Generalized Autoregressive Conditional Heteroskedasticity). GARCH was introduced by Bollerslev (1986). Sewell, Stansell, Lee and Below (1996) assert that once the linear dependencies and conditional heteroskedasticity have been filtered out with ARMA (Autoregressive moving average) and GARCH procedures, some of the stock market indices and exchange rate series will be stochastic. There is evidence in some of the stock market and exchange rate series of nonlinear dependencies. Gilmore (2000) asserts that GARCH may be able to explain certain parts of the nonlinear dependence in exchange rate series. He proposes the need to develop a more complex model to capture the remaining nonlinearity that present in those series.

According to Peters (1994), ARCH and GARCH are less superior as compared to fractals. ARCH is a local process which relates past volatility to future volatility for a particular investment horizon, meaning recent volatility in daily prices may be related to future daily volatility, but not to future volatility of weekly or monthly price movements. As compared to fractals, he argues that ARCH is unable to deal with all investment horizons simultaneously. Despite local randomness, fractal process can deal with global structures with all investment horizons simultaneously. Besides, ARCH models do not correspond to the persistence (or long-memory) effects from Mandelbrot's "Joseph effect". The "Joseph effect" refers to the apparent tendency toward long-term trends and non-periodic cycles.

Findings from Fama and French (1988) provide further weakness in autocorrelation methods. They assert that autocorrelation may reflect market inefficiency or time-varying equilibrium expected returns generated by rational investor behavior. However, the patterns of autocorrelation may not be stable for a sample period as long as 60 years. They found out that there are large negative autocorrelations for return horizons beyond a year. This may imply that predictable price variation due to mean reversion accounts for large fractions of 3-5 year return variances (40 percent for portfolios of small firms and 25 percent for portfolios of large firms).

2.1.5 Simple financial model for a chaotic system

Peter (1994) applies the above model on a penny stock (i.e. stock price of less than US$1.00).

$$P_{t+1} = aP_t - aP^2_t$$

Or $$P_{t+1} = aP_t(1 - P_t) \tag{2.1}$$

P is a stock price which can only take values ranging between zero and one. "a" represents the rate at which the price would rise as a result of buying pressure alone, and "aP^2_t" represents the price reduction that would result from selling pressure. Assume that a=2.0 and we begin with a price (P_0) of US$0.20 per share. Based on a series of iterative calculations for P_1, P_2, P_3, P_4, P_5 and subsequent prices, those prices will converge to US$0.50 per share. This is the fair value for the stock under the buying pressure of a=2 and at an initial price of P_0=$0.2. At this point, the model is at a stable steady state system and the value US$0.50 is said to be fixed point attractor with a=2.0. However, if we use different values of *"a"* from one to three, the model can produce different stable steady state system. If we use *"a"*=3.4, then we will have two repeated long-term stable price of $0.452 and $0.842 instead of one single point. The two prices show the buyer and seller are unable to agree with a single price. If we use a slightly higher value of *"a"*=3.4, we will get four alternate regular different values in the long-run. A slight increase in "a" will result in a periodic system of period 8 and so forth.

2.1.5.a Bifurcation Diagram

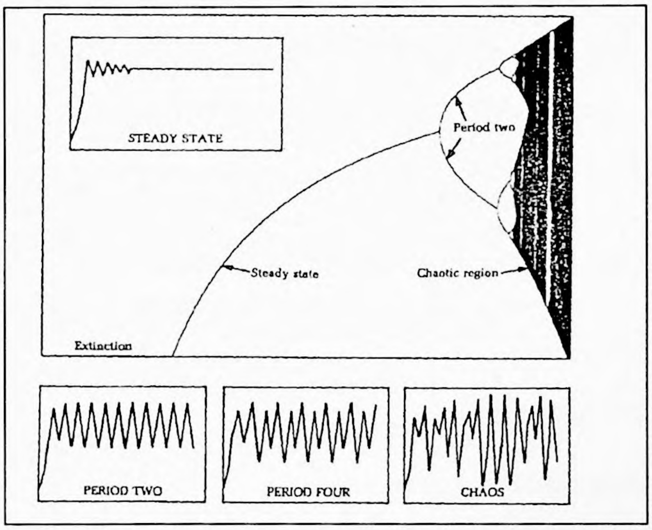

Figure 2.2. Bifurcation Diagram
Source: This chart is extracted from the book titled "Chaos: Making A New Science" (Gleick, 1987, pp.71)

The relationship between *"a"* and P_n can be represented by a graph, namely "bifurcation diagram" as shown in Figure 2.2. "Bifurcation diagram" shows the critical values of different *"a"* and the number of corresponding fair values. It is a diagram showing all possible solutions to a nonlinear dynamic system. It exhibits how an equation evolves from yielding first one solution, then two solutions, then four, etc, until the system reaches multiple solutions and then chaos (Gleick, 1987). According to Peters (1994), he identifies this self-similar property as a characteristic of nonlinear dynamic systems and is part of the nonlinear feedback process. However, this system will occur when the system is not in equilibrium. This is in contrast to the EMH.

2.1.6 Other Terms and Common Concepts in Chaos

2.1.6.a Butterfly Effect

The "butterfly effect" was first described by Edward Lorenz in the early 1960s. The butterfly effect states that a butterfly flapping its wings in the Brazilian rain forest may result a hurricane in the Atlantic Ocean a few months later. The butterfly effect suggests that assuming away as insignificant any variable or interdependence of variables in a model could result in a model that may not reflect the reality. In financial market, a small piece of news at a certain time may have a great impact to the stock market. This is one of the characteristics in a chaotic system.

2.1.6.b Noah Effect and Joseph Effect

Mandelbrot (1963c, 1966b) points out two characteristics of stock price behavior, namely the "Noah effect" and "Joseph effect". The "Noah effect" refers to the observed instances of large "discontinuous" jumps in stock prices. As explained earlier, the large jumps do not imply they are discontinued from the previous movements. Normally, we refer this phenomenon as "fat tails" or extreme tails in the normal distribution. The "Joseph effect" refers to the apparent tendency toward long-term trends and non-periodic cycles. It challenges the validity of IID assumption regarding the independence of error terms from the random walk model. Hence, it challenges the validity of the EMH as well as the conventional capital market research (CMR), like CAPM and APT. Even though these challenges have not deterred the empirical studies on the usefulness of CAPM and APT to investors, it has however created interest in market research on chaotic dynamics.

2.1.7 Fractal structure in a chaotic system

Since Mandelbrot (1963c) shifted his attention from income distribution to speculative prices, there were a growing number of researches on the application of fractal geometry and chaos theory to economics. The earlier studies were still focused on the searching of long memory or fractal structure in capital markets. By using fractals on financial markets, the earlier studies from Mandelbrot (1963a, 1963b, 1966a, 1969, 1971, 1972) challenged the traditional approach of using linear equilibrium model. Mouck (1998) viewed Mandelbrot's studies posed serious threat to the conventional CMR paradigm. During 1950s and 1960s, the CMR paradigms were focused on linear equilibrium model which were centered on order and rationality. Mandelbrot (1972) contended that the conventional method of using covariance analysis testing was unable to detect the long-term persistence and non-periodic cycles. As a result, investors are always interested in the issue of persistency because any predictable returns should be readily exploitable by an appropriate trading strategy.

A fractal is a shape made of parts similar to the whole in some way. They may be different between each other but they have the same underlying patterns. Sometimes, it is also called self-similar structure. For example, branches for a tree. Each branch and the following branches are different, but they have the same qualities similar to the structure of the whole tree. In a financial market, we may not be able to predict short-term movement of stock prices, but we may be able to detect its long-term trends. The father of fractal geometry, Benoit Mandelbrot, is the person most closely linked to the development of chaos theory and fractals. The word fractal is derived from Latin roots meaning "breaking". Benoit Mandelbrot experimented with fractional Brownian motion and published his groundbreaking work, *The Fractal Geometry of Nature*, in 1977.

The main contribution from Mandelbrot (1982) is "breaking" nature into patterns compatible with ready-made mathematical models. He claims that many patterns of nature are very irregular as compared with Euclid, a term to denote all of classical geometry. In his book, titled *The Fractal Geometry of Nature*, Mandelbrot explains in the introduction that:

> Why is geometry often described as "cold" and "dry"? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lighting travel in a straight line (Mandelbrot, 1982, p.1).

By developing a new geometry of nature, Mandelbrot explains the irregular and fragmented patterns of nature. He identifies a family of shapes and names them as fractals. Euclidean geometry is referred to plane geometry which is based on ideal, smooth and symmetric shapes. However, fractal geometry develops on roughness and asymmetry. Peters (1994) asserts that it is hard to describe precisely the meaning of fractals. But fractals do have certain characteristics that are measurable, and properties that are desirable for modeling purposes.

2.1.7.a Local Randomness and Global Determinism

Peter (1994) argues that chaos and order are coexisted. For example, a coconut tree has global structure and local randomness. Local randomness means that we are unable to determine in specific how an individual shape of each branch. However, global determinism means we are able to know in general what a coconut tree looks like and we can predict the general structure of any coconut trees. Thus, each branch has its own characteristic that we are unable to predict, they have certain global properties that we can determine from afar.

Local randomness does not have equal probability for all possible solutions. It is random but not independence. Its next movement is dependent on the present point or situation. In financial market, many empirical studies have shown that it is difficult to profit from short-term market movement. Filter rules from Alexander (1961) are unable to generate trading profits in the short-term. However, the ability to predict is seemed to improve over the longer-term. Hence, each market cycle may have different characteristic but it has the same global characteristics. Global characteristics when referred to bull and bear market consist of rising and falling prices, during rising and falling business cycles. However, the causes or circumstances around each cycle are not the same – local randomness. Thus, they are locally random and globally deterministic.

2.1.7.b Fractal Market Hypothesis (FMH)

EMH is only valid when a market is stable and has enough liquidity. Peters (1994) proposes Fractal Market Hypothesis (FMH) to explain the impact of liquidity and investors' investment horizons on stock market behavior. According to FMH, liquidity is one of the main factors that contribute to the stability of a market. A market will remain stable when there are a lot of investors who participate with different investment horizons. This is in contrast to EMH which says that all investors have homogenous expectation. The investment horizons can be divided into short-term (used by technical analysts) and long-term horizon (used by fundamental analysts). A market will be stable when these two groups hold different views. As FMH is based on the fractal concept, the market will remain stable as long as there is no break down on the fractal structure. A break down will occur when investors with long investment horizons either stop participating in the market or become short-term investors themselves. Investment horizons are shortened when investors feel that longer-term fundamental information is no longer important. Hence, traders with short investment horizons dominate a market crash. The market will remain unstable until long-term investors step in to buy. However, this type of instability is not the same as bear markets. Bear market is based on declining fundamental valuation. Fundamental information remains important but the market declines on a gradual basis.

The instability is referred to high levels of short-term volatility as a result of sharp jump or drop within a short period of time. When investors with different investment horizons are participating in a stock market, any panic buying or selling will be absorbed by the other investment horizons. However, if all investors buy or sell together, the lack of liquidity will cause the market to become unstable (a break down on fractal structure). A panic buying or selling will occur and cause discontinuity in price movements. This occurrence is named as "Noah Effect" by Mandelbrot and fat tails will appear in the frequency distribution of returns (Peters, 1994). Peters (1994) calls for the replacement of the EMH by FMH. Rachev, Weron and Weron (1999) concur with Peter's view that EMH fails to explain the relationship of information and investors' investment horizons.

There are five assumptions of FMH. They are as follows:-

1. **When there are a lot of investors with different investment horizons, the market will be stable due to the large liquidity in the market.** At any point in time, stock prices will be traded in a smaller band when there always have buyers and sellers who are willing to buy and sell.
2. **Information has a different impact on different investment horizons.** For shorter-term investment horizon, information that is related to market sentiment and technical factors will dominate. For longer-term investment horizon, fundamental information will be more useful and applicable. This is against the assumption used by EMH that information has the same impact on all investors.
3. **The liquidity, determined by the balance between supply and demand, can affect the stability of a market. Many investors with many different investment horizons provide the market liquidity.** Whenever there are any changes in fundamental factors, long-term investors will stop taking long-term position in the market. They will instead focus on short-term information. If the fundamental information has negative impact, long-term investors will be selling