Final Examination
2018/2019 Academic Session

June 2019

JIF424 – Quantum Mechanics
(Mekanik Kuantum)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **THIRTEEN (13)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TIGA BELAS (13)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

*[**Arahan** : Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

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Answer **ALL** questions.
*Jawab **SEMUA** soalan.*

1. (a). Explain how Max Planck managed to describe the spectrum of the black-body radiation.
Jelaskan bagaimana Max Planck berjaya memerihalkan spektrum sinaran jasad hitam.

(8 marks/*markah*)

(b). Describe the photoelectric effect experiment in detail. Explain how the concept of photon was able to interpret the results of this experiment.
Perihalkan secara terperinci ujikaji kesan fotoelektrik. Jelaskan bagaimana konsep foton mampu mentafsir keputusan ujikaji ini.

(12 marks/*markah*)

2. (a). (i). Define the Hermitian operator.
Takrifkan operator Hermitian.

(ii). Show that the eigenvalue of a Hermitian operator must be a real number.
Tunjukkan bahawa nilai-eigen suatu operator Hermitian mestilah suatu nombor hakiki.

(iii). Show that two eigenfunctions of a Hermitian operator is orthogonal if their corresponding eigenvalues are not the same.
Tunjukkan bahawa dua fungsi-eigen suatu operator Hermitian adalah berortogon jika nilai eigen sepadannya tidak sama.

(8 marks/*markah*)

(b). (i). If $\hat{A}$ and $\hat{B}$ are two operators in quantum mechanics, give the definition of the commutator of the two operators.

Jika $\hat{A}$ dan $\hat{B}$ adalah dua operator dalam mekanik kuantum, berikan takrifan komutator dua operator tersebut.

(ii). When will we be able to say that the two operators as commutative or non-commutative?

Bilakah kita boleh mengatakan dua operator itu berkomutatif atau tak berkomutatif?

(iii). Are the operator for the position of a particle, $\hat{X}$, and its momentum operator along the x-axis, $\hat{P}_x$, commutative? Prove it.

Adakah operator bagi kedudukan suatu zarah, $\hat{X}$, dan operator momentumnya di sepanjang paksi x, $\hat{P}_x$, berkomutatif? Buktikannya.

(iv). What is the interpretation for each case if two operators, $\hat{A}$ and $\hat{B}$ are commutative or non-commutative? Give an example of two commutative operators.

Apakah tafsiran bagi setiap kes jika dua operator, $\hat{A}$ dan $\hat{B}$ berkomutatif atau tak berkomutatif? Beri satu contoh dua operator berkomutatif.

(12 marks/*markah*)

3. (a). A particle of mass *m* is moving along a line and has a wave function
Suatu zarah berjisim m bergerak di sepanjang suatu garisan dan mempunyai fungsi gelombang

$$\psi = C\exp\left(-\alpha^2 x^2/2\right).$$

where C and α are constants.
dengan C dan α adalah pemalar.

(i). Calculate *C* in terms of α.
Hitung C dalam sebutan α.

(ii). Determine the expression for the potential energy at a distance x from the origin if the total energy of the particle is $\left(\hbar^2\alpha^2\right)/\left(8\pi^2 m\right)$.
Tentukan ungkapan bagi tenaga keupayaan pada suatu jarak x dari asalan jika jumlah tenaga zarah ialah $\left(\hbar^2\alpha^2\right)/\left(8\pi^2 m\right)$.

(iii). Write down an integral expression for the probability of finding the particle in between point x = 2 and x = 3.
Tuliskan suatu ungkapan kamiran bagi kebarangkalian menemui zarah itu di antara titik x = 2 dan x = 3.

Hint: You may assume
Petunjuk: Anda boleh menganggapnya

$$\int_{-\infty}^{\infty} \exp\left(-y^2\right)dy = \sqrt{\pi}\ .$$

(12 marks/*markah*)

(b). The time-independent Schrodinger equation (TISE) for a 1-dimensional harmonic oscillator is

Persamaan Schrodinger tak bersandar masa bagi suatu pengayun harmonik 1-dimensi ialah

$$\left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}+\frac{1}{2}kx^2\right]\psi(x)=E\,\psi(x)$$

Determine

Tentukan

(i). $\psi(x)$,

(ii). *E*.

(8 marks/*markah*)

4. (a). Define the transmission coefficient of a particle moving across a potential boundary.

Takrifkan pekali transmissi suatu zarah yang bergerak merentasi suatu sempadan keupayaan.

(6 marks/*markah*)

(b). Consider a square-well potential of depth V_0 and width a as shown in Figure 1. A particle with energy E (> V_0) enters the regions from left to right; i.e. from $-\infty$ towards $+\infty$.
Pertimbangkan suatu keupayaan telaga bersegi dengan kedalaman V_0 dan lebar a seperti pada Rajah 1. Suatu zarah dengan tenaga E (> V_0) memasuki rantau-rantau itu dari kiri ke kanan; iaitu, dari $-\infty$ menghala ke $+\infty$.

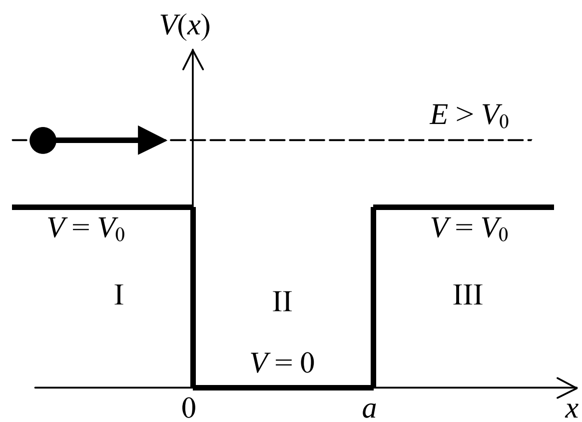

Figure 1
Rajah 1

(i). Solve the Schrodinger equation in order to obtain the solutions for the three regions I, II and III.
Selesaikan persamaan Schrodinger untuk mendapatkan penyelesaian bagi ketiga-tiga rantau I, II dan III.

(ii). State the coefficient of the wave numbers for the three regions.
Nyatakan pekali nombor gelombang bagi ketiga-tiga rantau.

(iii). State the similarities as well as the differences between the results obtained with those predicted by classical physics.
Nyatakan keserupaan dan juga perbezaan antara keputusan-keputusan yang diperolehi dengan keputusan-keputusan yang diramalkan oleh fizik klasik.

(14 marks/*markah*)

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5. (a). What is degeneracy? What are the conditions needed for degeneracy to occur?

Apakah kedegeneratan? Apakah syarat-syarat yang diperlukan untuk kedegeneratan berlaku?

(6 marks/*markah*)

(b). The angular momentum $\vec{L}$ in the classical mechanics is defined as $\vec{L} = \vec{r} \times \vec{p}$ where $\vec{r}$ is the location vector of the particle with respect to the origin 0 and $\vec{p}$ is the linear momentum of the particle.

Momentum sudut $\vec{L}$ dalam mekanik klasik ditakrifkan sebagai $\vec{L} = \vec{r} \times \vec{p}$ di mana $\vec{r}$ ialah vektor kedudukan zarah terhadap asalan 0 dan $\vec{p}$ ialah momentum linear zarah itu.

(i). Obtain the components of the angular momentum, i.e. L_x, L_y and L_z, according to classical mechanics.

Dapatkan komponen-komponen momentum sudut, iaitu L_x, L_y dan L_z berdasarkan mekanik klasik.

(ii). Express these components according to quantum mechanics.

Ungkapkan komponen-komponen ini menurut mekanik kuantum.

(iii). Write down the orbital angular momentum $\vec{L}$ as a 3-dimensional vector operator in quantum mechanics.

Tuliskan momentum sudut orbital $\vec{L}$ sebagai suatu operator vektor 3-dimensi dalam mekanik kuantum.

(iv). Show that

Tunjukkan bahawa

$$\left[L_x, L_y\right] = i\hbar L_z$$

(14 marks/*markah*)

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Useful Information:

Speed of light $c = 3.0 \times 10^{8}$ m s^{-1}

Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

Planck constant $h = 6.63 \times 10^{-34}$ J s

Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}

Permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}

Permeability of free space $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

Basic charge $e = 1.6 \times 10^{-19}$ C

Electron rest-mass $m_e = 9.1 \times 10^{-31}$ kg

Proton rest-mass $m_p = 1.6725 \times 10^{-27}$ kg $\equiv$ 1.0072766 u

Neutron rest-mass $m_n = 1.6748 \times 10^{-27}$ kg $\equiv$ 1.0086654 u

Bohr's radius $a = 5.3 \times 10^{-11}$ m

1 eV = 1.6×10^{-19} J

1 u $\equiv$ 931 MeV c^{-2}

1 barn = 10^{-28} m^{2}

1 fm = 10^{-15} m

1 Ci = 3.7×10^{10} s^{-1}

USEFUL MATHEMATICS IN QUANTUM MECHANICS

Exponential series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

Trigonometric series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Differentiation and integration (Standard forms)

Differentiation	Integration
$\frac{d}{dx} x^n = nx^{n-1}$ $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
$\frac{d}{dx}\log x = \frac{1}{x}$ $\frac{d}{dx}\log(ax+b) = \frac{a}{ax+b}$	$\int \frac{dx}{x} = \log x + c$ $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b) + c$
$\frac{d}{dx} e^x = e^x$ $\frac{d}{dx} e^{mx} = me^{mx}$	$\int e^x dx = e^x + c$ $\int e^{mx} dx = \frac{e^{mx}}{m} + c$
$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\sin mx = m\cos mx$	$\int \cos x\, dx = \sin x + c$ $\int \cos mx\, dx = \frac{\sin mx}{m} + c$
$\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\cos mx = -m\sin mx$	$\int \sin x\, dx = -\cos x + c$ $\int \sin mx\, dx = -\frac{\cos mx}{m} + c$
$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\tan mx = m\sec^2 mx$	$\int \sec^2 x\, dx = \tan x + c$ $\int \sec^2 mx\, dx = \frac{\tan mx}{m} + c$
$\frac{d}{dx}\cot x = -\text{cosec}^2 x$ $\frac{d}{dx}\cot mx = -m\,\text{cosec}^2 mx$	$\int \text{cosec}^2 x\, dx = -\cot x + c$ $\int \text{cosec}^2 mx\, dx = -\frac{\cot mx}{m} + c$
$\frac{d}{dx}\sinh x = \cosh x$	$\int \cosh x\, dx = \sinh x + c$
$\frac{d}{dx}\cosh x = \sinh x$	$\int \sinh x\, dx = \cosh x + c$

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Integration by parts

$$\int u\upsilon\, dx = u\int \upsilon\, dx - \int\left\{\int \upsilon\, dx\right\}\frac{du}{dx}\, dx$$

Integration common in Quantum Mechanics

$$f(x) = \int_0^\infty x^n e^{-ax^2}\, dx$$

n	$f(n)$	*n*	$f(n)$
0	$\frac{1}{2}\sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4}\sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8}\sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16}\sqrt{\frac{\pi}{a^7}}$	7	$\frac{3}{a^4}$

If *n* is even, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\, dx = 2\, f(x)$

If *n* is odd, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\, dx = 0$

Other standard integrals

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

$$\int_0^\infty \frac{x}{(e^x - 1)}\, dx = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^3}{(e^x - 1)}\, dx = \frac{\pi^4}{15}$$

Pythagorean identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

Sum & difference formulas

$$\sin(u \pm \upsilon) = \sin u \cos \upsilon \pm \cos u \sin \upsilon$$
$$\cos(u \pm \upsilon) = \cos u \cos \upsilon \mp \sin u \sin \upsilon$$
$$\tan(u \pm \upsilon) = \frac{\tan u \pm \tan \upsilon}{1 \mp \tan u \tan \upsilon}$$

Double angle formulas

$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

Power reducing/half angle formulas

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$
$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$
$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Sum-to-product formulas

$$\sin u + \sin \upsilon = 2\sin\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\sin u - \sin \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u + \cos \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u - \cos \upsilon = -2\sin\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$

Product-to-sum formulas

$$\sin u \sin \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) - \cos(u+\upsilon)\right]$$
$$\cos u \cos \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) + \cos(u+\upsilon)\right]$$
$$\sin u \cos \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) + \sin(u-\upsilon)\right]$$
$$\cos u \sin \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) - \sin(u-\upsilon)\right]$$

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