



Final Examination
2018/2019 Academic Session

June 2019

**JIF315 – Mathematical Methods
(Kaedah Matematik)**

Time: 3 hours
(Masa: 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Arahan : Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris].

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Table of Laplace Transform
[Jadual transformasi Laplace]

| $f(t)$ | $L\{f(t)\}=F(s)$ |
|-------------------|----------------------------------|
| a | $\frac{a}{s}$ |
| $t^n, n=1,2,3,*$ | $\frac{n!}{s^{n+1}}$ |
| e^{at} | $\frac{1}{s-a}$ |
| $\sin(at)$ | $\frac{a}{s^2+a^2}$ |
| $\cos(at)$ | $\frac{s}{s^2+a^2}$ |
| $\sinh(at)$ | $\frac{a}{s^2-a^2}$ |
| $\cosh(at)$ | $\frac{s}{s^2-a^2}$ |
| $e^{at} \sin(bt)$ | $\frac{b}{(s-a)^2+b^2}$ |
| $e^{at} \cos(bt)$ | $\frac{s-a}{(s-a)^2+b^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ |
| $t^n f(t)$ | $(-1)^n \frac{d^n}{ds^n} [F(s)]$ |
| $e^{at} f(t)$ | $F(s-a)$ |

Legendra Polynomial Function*[Jadual fungsi Legendra Polynomial]*

$$P_0(x) = 1,$$

$$P_1(x) = x,$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1),$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x),$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3),$$

$$P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

1. (a). The Laplace transform of a function, $f(t)$, is given by
Jelmaan Laplace fungsi, $f(t)$, diberikan oleh

$$F(s) = \frac{2s+1}{s(s+1)}$$

Determine the Laplace transform of
Tentukan jelmaan Laplace

(i). $e^{-2t}f(t)$

(ii). $e^{3t}f(t)$

(30 marks/markah)

- (b). Determine the inverse Laplace transform of
Tentukan jelmaan Laplace songsang

$$\frac{6s+8}{s^2+3s+2}$$

by expressing as its partial fractions.
dengan mengungkap pecahan separa.

(30 marks/markah)

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- (c). Consider the circuit in Figure 1 when the switch is closed at $t = 0$, $V_c(0) = 1.0 \text{ V}$. Solve for the current, $i(t)$ in the circuit by using Laplace transform.

*Pertimbangkan litar pada Rajah 1 apabila suis ditutup $t = 0$, $V_c(0) = 1.0 \text{ V}$
Selesaikan arus $i(t)$ dalam litar dengan menggunakan jelmaan Laplace.*

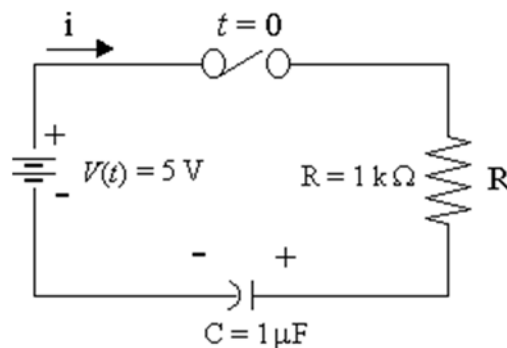


Figure 1

Rajah 1

(40 marks/markah)

2. (a). Determine the general solution of the following equations in terms of Bessel functions.

Tentukan penyelesaian umum persamaan berikut dalam sebutan fungsi Bessel.

- (i). $y'' + 9xy = 0$
(ii). $4x^2y'' + 8xy' + (x^4 - 3)y = 0$

(50 marks/markah)

...6/-

(b). The Legendre's equation is given as follows

Persamaan Legendre diberi seperti berikut

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

where n is a constant. Express the following equations in terms of the Legendre's Polynomial:

dengan n adalah pemalar. Ungkapkan persamaan-persamaan berikut dalam sebutan polinomial Legendre:

(i). $2x^3 - 5x^2 + 6x + 1$

(ii). $-x^3 - 4x^2 - 5x + 5$

(50 marks/markah)

3. Determine all the eigenvalues and associated eigenfunctions of the following Sturm-Liouville problem. Consider all **THREE (3)** cases of λ .

*Tentukan semua nilai eigen dan eigenfunctions berkaitan masalah Sturm-Liouville yang berikut. Pertimbangkan semua **TIGA (3)** kes λ .*

$$y'' + \lambda y = 0, \quad (0 < x < L); \quad y'(0) = 0, y(L) = 0$$

(a). $\lambda < 0$

(b). $\lambda = 0$

(c). $\lambda > 0$

(100 marks/markah)

4. (a). Consider the following function

Pertimbangkan fungsi berikut

$$f(t) = \begin{cases} 1 & 0 \leq t \leq 0.01 \\ 0 & 0.01 \leq t \leq 0.02 \end{cases}$$

- (i) Determine the Fourier series for $f(t)$ with period, $T = \frac{1}{50}$.
Tentukan siri Fourier bagi $f(t)$ dengan diberikan tempoh, $T = \frac{1}{50}$.
- (ii). Sketch the graph of the function $f(t)$.
Lakarkan graf bagi fungsi $f(t)$

(60 marks/markah)

- (b). Determine the Fourier transform of
Tentukan jelmaan Fourier bagi

$$f(t) = \begin{cases} e^{-3t} & t \geq 0 \\ e^{3t} & t < 0 \end{cases}$$

(40 marks/markah)

5. (a). Suppose the length of rod, $L = 50$ cm is immersed in steam until its temperature is throughout $u_0 = 100$ °C. At time $t = 0$, its lateral surface is insulated and its two ends are imbedded in ice at 0 °C. Calculate the rod's temperature at its midpoint after half an hour if it is made of iron ($k_{iron} = 0.15$)

Sekiranya panjang rod, $L = 50$ cm ditenggelamkan dalam wap sehingga seluruh suhunya, $u_0 = 100$ °C. Pada masa $t = 0$, permukaan sisi ditebatkan dan hujung kedua-duanya di tanam dalam ais pada 0 °C. Hitung suhu rod pada titik tengah selepas setengah jam jika ia diperbuat daripada besi ($k_{besi} = 0.15$)

(50 marks/markah)

...8/-

- (b). Figure 2 shows the initial position function $f(x)$ for a stretched string length, L that is set in motion by moving its midpoint $x = \frac{L}{2}$ aside the distance $\frac{1}{2}bL$ and then releasing it from rest at time $t = 0$. The corresponding boundary value problem is

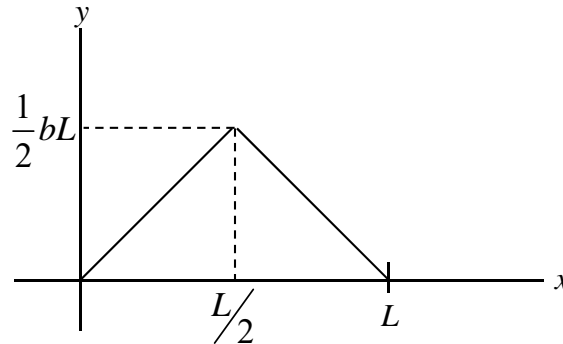


Figure 2
Rajah 2

Rajah 2 menunjukkan fungsi kedudukan awal $f(x)$ untuk tali diregangkan dengan panjang, L yang digerakkan dengan memindahkan titik tengah $x = \frac{L}{2}$ selain jarak $\frac{1}{2}bL$ dan kemudian melepaskannya daripada rehat pada masa $t = 0$. Masalah nilai sempadan sepadan ialah

$$y_{tt} = a^2 y_{xx} \quad 0 < x < L, t > 0;$$

$$y(0, t) = y(L, t) = 0,$$

$$y(x, 0) = f(x)$$

$$y_t(x, 0) = 0,$$

Given: $f(x) = bx$ for $0 \leq x \leq \frac{L}{2}$ and $f(x) = b(L - x)$ for $\frac{L}{2} \leq x \leq L$.

Determine $y(x, t)$.

Diberi: $f(x) = bx$ untuk $0 \leq x \leq \frac{L}{2}$ dan $f(x) = b(L - x)$ untuk $\frac{L}{2} \leq x \leq L$.

Tentukan $y(x, t)$.

(50 marks/markah)

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