Final Examination
2018/2019 Academic Session

June 2019

JIB323 – Biostatistics
(Biostatistik)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **TWENTY (20)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH (20)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **FIVE (5)** questions. Mark for each sub question is given at the end of that sub question. You may answer **either** in Bahasa Malaysia or English.

*[**Arahan** : Jawab **LIMA (5)** soalan. Markah untuk setiap subsoalan diperlihatkan di penghujung subsoalan itu. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Answer FIVE (5) questions.
Jawab LIMA (5) soalan.

1. A researcher conducted a study on the distribution of oxygen in surface sediments from Central Sagami Bay, Japan. Table 1 shows the measurements performed at 16 sites in millimoles per square metre per day (mmol $m^{-2} d^{-1}$).

Table 1: Measurement of oxygen distributions (mmol $m^{-2} d^{-1}$)							
1.8	2.0	1.8	2.3	3.8	3.4	2.7	1.1
3.3	1.2	3.6	1.9	7.6	2.0	1.5	1.1

Find,

(a). mean

(3 marks)

(b). median

(2 marks)

(c). mode

(3 marks)

(d). midrange

(3 marks)

(e). range

(3 marks)

(f). standard deviation

(6 marks)

Seorang penyelidik menjalankan kajian tentang taburan oksigen dalam permukaan sedimen di Teluk Tengah Sagami, Jepun. Jadual 1 menunjukkan ukuran yang diambil pada 16 tapak dalam milimol per meter persegi sehari (mmol m^{-2} d^{-1}).

Jadual 1: Ukuran taburan oksigen (mmol m^{-2} d^{-1})							
1.8	*2.0*	*1.8*	*2.3*	*3.8*	*3.4*	*2.7*	*1.1*
3.3	*1.2*	*3.6*	*1.9*	*7.6*	*2.0*	*1.5*	*1.1*

Cari,

(a). min

(3 markah)

(b). median

(2 markah)

(c). mod

(3 markah)

(d). julat tengah

(3 markah)

(e). julat

(3 markah)

(f). sisihan piawai

(6 markah)

2. A university committee proposed that all students must take a course in ethics as a requirement for graduation. A total of 300 faculty members and students from this university were asked about their opinion on this issue. Table 2 shows the responses of these faculty members and students.

Table 2: Responses from faculty members and students

	Favour	Oppose	Neutral
Faculty	45	15	10
Student	90	110	30

(a). What is the probability that a randomly selected person will favour the ethics course?

(4 marks)

(b). What is the probability that a randomly selected person will oppose the ethics course or is a student?

(4 marks)

(c). What is the probability that a randomly selected person will favour the ethics course or is a faculty member?

(4 marks)

(d). What is the probability of a person being neutral, given that they are students?

(4 marks)

(e). What is the probability of a person is a student, given that they are in favour of the ethics course?

(4 marks)

Jawatankuasa universiti mencadangkan agar semua pelajar mengambil kursus dalam bidang etika sebagai syarat untuk tamat pengajian. Seramai 300 ahli fakulti dan pelajar dari universiti ini telah ditanya pendapat mereka mengenai isu tersebut. Jadual 2 menunjukkan jawapan ahli fakulti dan pelajar.

Jadual 2: Jawapan daripada ahli fakulti dan pelajar

	Setuju	*Bangkang*	*Neutral*
Fakulti	45	15	10
Pelajar	90	110	30

(a). Apakah kebarangkalian seseorang dipilih secara rawak akan bersetuju dengan kursus etika?

(4 markah)

(b). Apakah kebarangkalian seseorang dipilih secara rawak akan membangkang kursus etika atau ialah seorang pelajar?

(4 markah)

(c). Apakah kebarangkalian seseorang dipilih secara rawak akan bersetuju dengan kursus etika atau ialah seorang ahli fakulti?

(4 markah)

(d). Apakah kebarangkalian seseorang itu neutral, sekiranya mereka pelajar?

(4 markah)

(e). Apakah kebarangkalian seseorang itu pelajar, sekiranya mereka bersetuju dengan kursus etika?

(4 markah)

…6/-

3. Baier's Electronics manufactures computer parts that are supplied to many computer companies. Although there are two quality control inspectors checking every part before shipment, a few defective parts do pass through these inspections undetected. Table 3 shows the probability distribution of defective computer parts in a shipment of 400.

Table 3: Probability distribution of defective computer parts in a shipment of 400.

x	*P*(x)
0	0.02
1	0.20
2	0.30
3	0.30
4	0.10
5	0.08

Based on Table 3,

(a). find the mean of the probability distribution.

(5 marks)

(b). find the variance of the probability distribution.

(4 marks)

(c). find the standard deviation of the probability distribution.

(3 marks)

(d). find the minimum and maximum values.

(6 marks)

(e). what do the minimum and maximum values interpret?

(2 marks)

Baier's Electronics menghasilkan bahagian komputer yang merupakan pembekal kepada banyak syarikat komputer. Walaupun terdapat dua pemeriksa kawalan kualiti memeriksa setiap bahagian sebelum penghantaran, beberapa bahagian yang defektif melepasi pemeriksaan tanpa dikesan. Jadual 3 menunjukkan taburan kebarangkalian bahagian komputer yang defektif dalam penghantaran 400.

Jadual 3: *Taburan kebarangkalian bahagian komputer yang defektif dalam penghantaran 400*.

x	*P*(x)
0	0.02
1	0.20
2	0.30
3	0.30
4	0.10
5	0.08

Berdasarkan Jadual 3,

(a). cari min taburan kebarangkalian.

(5 markah)

(b). cari varians taburan kebarangkalian.

(4 markah)

(c). cari sisihan piawai taburan kebarangkalian.

(3 markah)

(d). cari nilai minimum dan maksimum.

(6 markah)

(e). apakah maksud nilai minimum dan maksimum?

(2 markah)

4. A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Table 4 shows the values before and after hypnosis. Measurements are in centimetres on a pain scale.

Table 4: Measurements of pain (cm).

Subject	A	B	C	D	E	F	G	H
Before	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

(a). Construct a 95% confidence interval for the mean of the before-after differences.

(11 marks)

(b). Use a 0.05 significance level to test the claim that the sensory measurements are lower after hypnotism.

(7 marks)

(c). Does hypnotism appear to be effective in reducing pain?

(2 marks)

Satu kajian telah dijalankan untuk mengkaji keberkesanan hipnotisme dalam mengurangkan kesakitan. Jadual 4 menunjukkan nilai sebelum dan selepas hipnosis. Pengukuran adalah dalam sentimeter pada skala kesakitan.

Jadual 4: Ukuran kesakitan (cm).

Subjek	A	B	C	D	E	F	G	H
Sebelum	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
Selepas	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

(a). Hasilkan selang keyakinan 95% untuk min perbezaan sebelum-selepas.

(11 markah)

(b). Guna aras keertian 0.05 untuk menguji tuntutan bahawa pengukuran kesakitan lebih rendah selepas hipnotisme.

(7 markah)

(c). Adakah hipnotisme berkesan dalam mengurangkan kesakitan?

(2 markah)

…9/-

5. Table 5 shows the times of birth recorded for 74 babies born in Penang. If birth times are uniformly distributed throughout the day, the relative frequencies in the table should be 0.250, 0.583 and 0.167. Use a 0.05 significance level to test the claim that the births are uniformly distributed throughout the day.

Jadual 5 menunjukkan masa kelahiran yang dicatatkan untuk 74 bayi yang lahir di Pulau Pinang. Sekiranya masa kelahiran diagihkan secara seragam sepanjang hari, frekuensi relatif dalam jadual sepatutnya 0.250, 0.583 dan 0.167. Gunakan aras keertian 0.05 untuk menguji tuntutan bahawa kelahiran teragih secara seragam sepanjang hari.

Table 5/*Jadual 5*: Times of birth for babies born in Penang / *Masa kelahiran bayi di Pulau Pinang.*		
12.00 am – 5.59 am	6.00 am – 7.59 pm	8.00 pm – 11.59 pm
16	45	13

(20 marks/*markah*)

6. A consumer agency wanted to find out whether each of the three brands of medicines in providing relief from a headache has the same mean. Table 6 shows the time (minutes) taken by each patient to get a relief from a headache after taking the medicine. At a 0.05 significance level, can it be concluded that the mean time taken to provide relief from a headache is the same for each of the three drugs?

Agensi pengguna ingin mengetahui sama ada setiap tiga jenama ubat dapat memberikan kelegaan daripada sakit kepala mempunyai min yang sama. Jadual 6 menunjukkan masa (minit) yang diambil oleh setiap pesakit untuk lega daripada sakit kepala selepas mengambil ubat. Pada aras keertian 0.05, adakah boleh disimpulkan masa yang diambil untuk memberikan kelegaan daripada sakit kepala mempunyai min yang sama untuk setiap tiga ubat?

Table 6/*Jadual 6*: Time taken by each patient to get a relief from a headache (minutes) / *Masa yang diambil untuk setiap pesakit mendapat kelegaan selepas sakit kepala (minit)*

Drug I/*Ubat I*	Drug II/ *Ubat II*	Drug III/ *Ubat III*
25	15	44
38	21	39
42	19	64
65	28	58
41	18	73

(20 marks/*markah*)

APPENDIX

Descriptive statistics

$\bar{x} = \frac{\Sigma x}{n}$ Mean

$\bar{x} = \frac{\Sigma f \cdot x}{\Sigma f}$ Mean (frequency table)

$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$ Standard deviation

$s = \sqrt{\frac{n(\Sigma x^2) - (\Sigma x)^2}{n(n - 1)}}$ Standard deviation (shortcut)

$s = \sqrt{\frac{n[\Sigma(f \cdot x^2)] - [\Sigma(f \cdot x)]^2}{n(n - 1)}}$ Standard deviation (frequency table)

variance $= s^2$

variance $s^2 = \frac{n(\Sigma x^2) - (\Sigma x)^2}{n(n-1)}$

Coefficient of variation $CV = \frac{s}{\bar{x}} \times 100$

Probability Distributions

$\mu = \Sigma x \cdot P(x)$ Mean (prob. dist.)

$\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$ variance (prob. dist.)

$\sigma^2 = \Sigma[x^2 \cdot P(x)] - \mu^2$ variance (prob. dist.)

$\sigma = \sqrt{[\Sigma x^2 \cdot P(x)] - \mu^2}$ Standard deviation (prob. dist.)

$P(x) = \frac{n!}{(n - x)!\, x!} \cdot p^x \cdot q^{n-x}$ Binomial probability

$\mu = n \cdot p$ Mean (binomial)

$\sigma^2 = n \cdot p \cdot q$ Variance (binomial)

$\sigma = \sqrt{n \cdot p \cdot q}$ Standard deviation (binomial)

$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$ Poisson Distribution where $e \approx 2.71828$

Probability

$P(A \text{ or } B) = P(A) + P(B)$ if A, B are mutually exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ if A, B are not mutually exclusive

$P(A \text{ and } B) = P(A) \cdot P(B)$ if A, B are independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if A, B are dependent

$P(\bar{A}) = 1 - P(A)$ Rule of complements

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Conditional probability

Confidence Interval (one population)

$$\hat{p} - E < p < \hat{p} + E \quad \text{Proportion}$$
$$\text{where } E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E \quad \text{Mean}$$
$$\text{where } E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad (\sigma \text{ known})$$
$$\text{or } E = t_{\alpha/2}\frac{s}{\sqrt{n}} \quad (\sigma \text{ unknown})$$

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2} \quad \text{Variance}$$

Test statistics (one population)

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \quad \text{Proportion—one population}$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \text{Mean—one population } (\sigma \text{ known})$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{Mean—one population } (\sigma \text{ unknown})$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{Standard deviation or variance—one population}$$

Confidence Interval (two populations)

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

where $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

$(\overline{x}_1 - \overline{x}_2) - E < (\mu_1 - \mu_2) < (\overline{x}_1 - \overline{x}_2) + E$ (Indep.)

where $E = t_{\alpha/2}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ (df = smaller of $n_1 - 1, n_2 - 1$)

(σ_1 and σ_2 unknown and not assumed equal)

$E = t_{\alpha/2}\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$ (df = $n_1 + n_2 - 2$)

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

(σ_1 and σ_2 unknown but assumed equal)

$E = z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

(σ_1, σ_2 known)

$\overline{d} - E < \mu_d < \overline{d} + E$ (Matched Pairs)

where $E = t_{\alpha/2}\dfrac{s_d}{\sqrt{n}}$ (df = $n - 1$)

$s_d = \sqrt{\dfrac{\Sigma(d-\bar{d})^2}{n-1}}$ Standard deviation (matched pairs)

Test statistics (two populations)

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p}\,\overline{q}}{n_1} + \frac{\overline{p}\,\overline{q}}{n_2}}}$$ Two proportions

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ df = smaller of $n_1 - 1, n_2 - 1$

Two means—independent; σ_1 and σ_2 unknown, and not assumed equal.

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$ $(\text{df} = n_1 + n_2 - 2)$

where $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Two means—independent; σ_1 and σ_2 unknown, but assumed equal.

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$ Two means—independent; σ_1, σ_2 known.

$$t = \frac{\overline{d} - \mu_d}{s_d/\sqrt{n}}$$ Two means—matched pairs $(\text{df} = n - 1)$

$$s_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$$ Standard deviation (matched pairs)

$$F = \frac{s_1^2}{s_2^2}$$ Standard deviation or variance—two populations (where $s_1^2 \geq s_2^2$)

Multinomial and Contingency Tables

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Multinomial $(\text{df} = k - 1)$

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Contingency table $[\text{df} = (r - 1)(c - 1)]$

where $E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$

One-Way Analysis of Variance

$$df_{between} = k - 1$$
$$df_{within} = N - k$$
$$df_{total} = N - 1$$

$\bar{\bar{x}} = \frac{\Sigma x}{N}$ mean of all values combined

$F = \frac{variance\ between\ samples}{variance\ within\ samples}$ or $F = \frac{MS_{between}}{MS_{within}}$

$MS_{between} = \frac{SS_{between}}{k-1}$ $\quad MS_{within} = \frac{SS_{within}}{N-k}$ $\quad MS_{total} = \frac{SS_{total}}{N-1}$

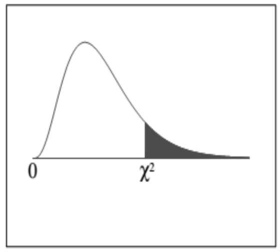
$$SS_{between} = n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + \dots + n_k(\bar{x}_k - \bar{\bar{x}})^2$$

$$SS_{within} = \sum_1 (x_{i1} - \bar{x}_1)^2 + \sum_2 (x_{i2} - \bar{x}_2)^2 + \dots + \sum_k (x_{ik} - \bar{x}_k)^2$$

$$SS_{total} = \sum_1 (x_{i1} - \bar{\bar{x}}_1)^2 + \sum_2 (x_{i2} - \bar{\bar{x}}_2)^2 + \dots + \sum_k (x_{ik} - \bar{\bar{x}}_k)^2$$
$$= \Sigma (x - \bar{\bar{x}})^2$$

$$SS_{total} = SS_{between} + SS_{within}$$

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

F Values for $\alpha = 0.05$

d_2 \ d_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

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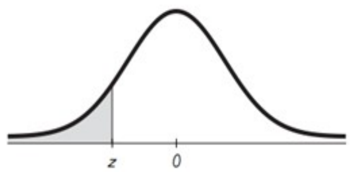
TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
Large	2.576	2.326	1.960	1.645	1.282



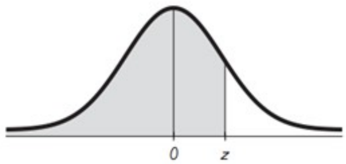
NEGATIVE z Scores

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below −3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z score	Area
−1.645	0.0500
−2.575	0.0050



POSITIVE *z* Scores

TABLE A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949 *	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

NOTE: For values of z above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575

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