



Final Examination  
2017/2018 Academic Session

May/June 2018

**JIM420 – Design and Analysis of Experiments**  
**[Rekabentuk dan Analisis Ujikaji]**

Duration : 3 hours  
[Masa: 3 jam]

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Please ensure that this examination paper contains **EIGHTEEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN BELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

*Baca arahan dengan teliti sebelum anda menjawab soalan.*

*Setiap soalan diperuntukkan 100 markah.*

*Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan.*

- 2 -

1. (a). A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one supplier to another, the chemist applies all four chemicals in random order to the cloth from each of the 5 suppliers. The resulting tensile strengths are as follow.

Chemical	Supplier				
	1	2	3	4	5
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

- (i). What experimental design is the chemist using?  
(ii). Write the effects model for this experiment and their assumptions.  
(iii). Analyze the data and draw conclusions. Use  $\alpha = 0.05$

(55 marks)

- (b). Suppose that the observation for chemical type 2 and supplier 3 is missing. Analyze the problem by estimating the missing value. Perform the same analysis as in (a). (iii). and compare the results.

(45 marks)

- 3 -

2. A factorial experiment is carried out to study the effect of four factors (A, B, C & D) on the filtration rate of a product. Each factor has two levels (L and H). The design matrix and the response data from a single replicate of the  $2^4$  experiment are shown below.

Run Number	Factor				Run Label	Filtration Rate (gal/h)
	A	B	C	D		
1	—	—	—	—	(1)	45
2	+	—	—	—	a	71
3	—	+	—	—	b	48
4	+	+	—	—	ab	65
5	—	—	+	—	c	68
6	+	—	+	—	ac	60
7	—	+	+	—	bc	80
8	+	+	+	—	abc	65
9	—	—	—	+	d	43
10	+	—	—	+	ad	100
11	—	+	—	+	bd	45
12	+	+	—	+	abd	104
13	—	—	+	+	cd	75
14	+	—	+	+	acd	86
15	—	+	+	+	bcd	70
16	+	+	+	+	abcd	96

- (a). Can you run an ANOVA if you fit the full model? Explain.

(10 marks)

- 4 -

- (b). Verify the effect estimates and the sum of squares for factors A, B and C given in the table below.

Model Term	Effect Estimate	Sum of Squares	Percent Contribution
A	21.625	1870.56	32.6397
B	3.125	39.0625	0.681608
C	9.875	390.062	6.80626
D	14.625	855.563	14.9288
AB	0.125	0.0625	0.00109057
AC	-18.125	1314.06	22.9293
AD	16.625	1105.56	19.2911
BC	2.375	22.5625	0.393696
BD	-0.375	0.5625	0.00981515
CD	-1.125	5.0625	0.0883363
ABC	1.875	14.0625	0.245379
ABD	4.125	68.0625	1.18763
ACD	-1.625	10.5625	0.184307
BCD	-2.625	27.5625	0.480942
ABCD	1.375	7.5625	0.131959

(30 marks)

- (c). Given the table in (b)., use the normal probability plot analysis to find significant effects.

(30 marks)

- (d). Use the main effects plots and interaction plots to determine the factor levels production should be run in order to maximize filtration rates.

(30 marks)

- 5 -

3. (a). (i). Construct a  $2^3$  design with  $ABC$  confounded in the first two replicates and  $BC$  confounded in the third.
- (ii). Determine each source of variation and its degrees of freedom for the ANOVA in part (a). (i).

(40 marks)

- (b). An experiment is conducted to investigate the effect of factors A, B, C, and D on the response variable. To conserve time, she decides to investigate these factors using a  $2_{III}^{5-2}$  design with  $I = ABD$  and  $I = BCE$ . The results obtained are  $de = 95$ ,  $ae = 134$ ,  $b = 158$ ,  $abd = 190$ ,  $cd = 92$ ,  $ac = 187$ ,  $bce = 155$ , and  $abcde = 185$ .
- (i). Verify that the treatment combinations given are correct.
- (ii). Assume the three-factor and higher interactions effects are negligible and the effect estimate, SS and % contribution of the factors are as below. Identify factors that are important using the half normal plot.

Term	Effect	SumSqr	%Contribution
A	49	4802	43.9502
B	45	4050	37.0675
C	10.5	220.5	2.01812
D	-18	648	5.93081
E	-14.5	420.5	3.84862
AC	13.5	364.5	3.33608
AE	-14.5	420.5	3.84862

- (iii). Suppose that a second fraction is added to the first, for example  $ade = 136$ ,  $e = 93$ ,  $ab = 187$ ,  $bd = 153$ ,  $acd = 139$ ,  $c = 99$ ,  $abce = 191$ , and  $bcde = 150$ . How was this second fraction obtained?

(60 marks)

- 6 -

4. (a). Consider a company that buys raw material in batches (B) from 3 different suppliers (A). The purity of this raw material varies considerably, which causes problems in manufacturing the finished product. Four batches of raw material are selected at random from each supplier, and three determinations of purity are made on each batch.

- (i) What experimental design is being used here?
- (ii) Given  $SS_T=148$ ,  $SS_A= 15.06$ ,  $SS_{B(A)}=69.92$ , is the variability in purity attributable to differences between or within suppliers? Use  $\alpha = 0.05$ .

(40 marks)

(b). A textile mill has a large number of looms. Each loom is supposed to provide the same output of cloth per minute. To investigate this assumption, five looms are chosen at random and their output is noted at different times. The following data are obtained:

Loom	Output (lb/min)				
1	14.0	14.1	14.2	14.0	14.1
2	13.9	13.8	13.9	14.0	14.0
3	14.1	14.2	14.1	14.0	13.9
4	13.6	13.5	14.0	13.9	13.7
5	13.8	13.6	13.9	13.8	14.0

- (i). Explain why this is a random effects experiment.
- (ii). Are the looms equal in output? Use  $\alpha = 0.05$ .
- (iii). Estimate the variability between looms.
- (iv). Estimate the experimental error variance.

(60 marks)

- 7 -

1. (a). Seorang ahli kimia ingin menguji kesan empat bahan kimia terhadap kekuatan sejenis kain tertentu. Oleh kerana kemungkinan terdapat kebolehubahan dari satu pembekal kepada yang lain, ahli kimia tersebut menggunakan keempat-empat bahan kimia secara rawak kepada kain daripada setiap 5 pembekal. Hasil kekuatan tegangan kain diberikan seperti yang berikut.

Bahan Kimia	Pembekal				
	1	2	3	4	5
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

- (i). Rekabentuk ujikaji apakah yang digunakan oleh ahli kimia tersebut?  
(ii). Tuliskan model kesan untuk ujikaji ini dan andaianya.  
(iii). Analisa data ini dan dapatkan kesimpulan. Guna  $\alpha = 0.05$ .  
(55 markah)
- (b). Andai cerapan bagi bahan kimia jenis 2 dan pembekal 3 telah hilang bagi 1. (a). Analisiskan masalah ini dengan menganggarkan nilai hilang tersebut. Jalankan analisis yang sama seperti di (a). (iii). dan bandingkan dengan keputusan asal yang diperolehi.  
(45 markah)

- 8 -

2. Satu ujikaji faktoran dijalankan untuk mengkaji kesan empat faktor (A, B, C & D) ke atas kadar penapisan produk. Setiap faktor mempunyai dua aras (L dan H). Matriks rekabentuk dan data respon daripada satu replikasi ujikaji  $2^4$  ditunjukkan di bawah.

Run Number	Factor				Run Label	Filtration Rate (gal/h)
	A	B	C	D		
1	-	-	-	-	(1)	45
2	+	-	-	-	a	71
3	-	+	-	-	b	48
4	+	+	-	-	ab	65
5	-	-	+	-	c	68
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	-	abc	65
9	-	-	-	+	d	43
10	+	-	-	+	ad	100
11	-	+	-	+	bd	45
12	+	+	-	+	abd	104
13	-	-	+	+	cd	75
14	+	-	+	+	acd	86
15	-	+	+	+	bcd	70
16	+	+	+	+	abcd	96

- (a). Bolehkan anda jalankan ANOVA jika model penuh digunakan?  
Terangkan.

(10 markah)

- 9 -

- (b). Tentusah penggangar kesan dan hasil tambah kuasa dua bagi factor A, B dan C yang diberi di dalam jadual dibawah.

Model Term	Effect Estimate	Sum of Squares	Percent Contribution
A	21.625	1870.56	32.6397
B	3.125	39.0625	0.681608
C	9.875	390.062	6.80626
D	14.625	855.563	14.9288
AB	0.125	0.0625	0.00109057
AC	-18.125	1314.06	22.9293
AD	16.625	1105.56	19.2911
BC	2.375	22.5625	0.393696
BD	-0.375	0.5625	0.00981515
CD	-1.125	5.0625	0.0883363
ABC	1.875	14.0625	0.245379
ABD	4.125	68.0625	1.18763
ACD	-1.625	10.5625	0.184307
BCD	-2.625	27.5625	0.480942
ABCD	1.375	7.5625	0.131959

(30 markah)

- (c). Diberi jadual di (b)., gunakan analisis plot kebarangkalian normal untuk mencari kesan yang signifikan.

(30 markah)

- (d). Gunakan plot kesan utama dan plot interaksi untuk menentukan aras faktor yang patut dijalankan oleh pengeluaran untuk memaksimumkan kadar penapisan.

(30 markah)

- 10 -

3. (a). (i). Binakan satu rekabentuk  $2^3$  dengan  $ABC$  terbaur di dalam replikasi pertama dan kedua, dan  $BC$  terbaur di dalam replikasi ketiga.
- (ii). Tentukan setiap sumber variasi dan darjah kebebasannya untuk ANOVA di bahagian (a). (i).

(40 markah)

- (b). Satu ujikaji dijalankan untuk menyiasat kesan faktor A, B, C dan D pada pembolehubah respon. Untuk menjimatkan masa, dia memutuskan untuk menyiasat faktor-faktor ini dengan menggunakan rekabentuk  $2^{5-2}_{II}$  dengan  $I = ABD$  dan  $J = BCE$ . Keputusan yang diperolehi ialah  $de = 95$ ,  $ae = 134$ ,  $b = 158$ ,  $abd = 190$ ,  $cd = 92$ ,  $ac = 187$ ,  $bce = 155$ , dan  $abcde = 185$ .
- (i). Tentusahkan bahawa gabungan rawatan yang diberikan adalah betul.
- (ii). Andai kesan interaksi tiga faktor atau lebih boleh diabaikan dan anggaran kesan, SS dan % sumbangan faktor adalah seperti di bawah. Kenal pasti faktor-faktor yang penting dengan menggunakan plot separuh normal.

Term	Effect	SumSqr	%Contribution
A	49	4802	43.9502
B	45	4050	37.0675
C	10.5	220.5	2.01812
D	-18	648	5.93081
E	-14.5	420.5	3.84862
AC	13.5	364.5	3.33608
AE	-14.5	420.5	3.84862

- (iii). Andaikan bahawa pecahan kedua ditambahkan kepada pecahan pertama, contohnya  $ade = 136$ ,  $e = 93$ ,  $ab = 187$ ,  $bd = 153$ ,  $acd = 139$ ,  $c = 99$ ,  $abce = 191$ , and  $bcde = 150$ . Bagaimanakah caranya pecahan kedua ini diperoleh?

(60 markah)

...11/-

- 11 -

4. (a). Pertimbangkan sebuah syarikat yang membeli bahan mentah secara berkelompok (B) daripada 3 pembekal (A) yang berasingan. Ketulenan bahan mentah ini agak berbeza, menyebabkan masalah dalam mengilangkan keluaran yang siap. Empat kelompok bahan mentah dipilih secara rawak daripada setiap pembekal dan 3 penentuan ketulenan dibuat pada setiap kelompok.
- (i). Rekabentuk apakah yang digunakan di sini?
- (ii). Diberi  $SS_T=148$ ,  $SS_A = 15.06$ ,  $SS_{B(A)}=69.92$ , adakah ubahan ketulenan timbul disebabkan perbezaan di antara atau dalam pembekal? Guna  $\alpha = 0.05$ .

(40 markah)

- (b). Sebuah kilang tekstil memiliki alat tenun dengan jumlah yang besar. Setiap alat tenun seharusnya memberikan keluaran kain yang sama pada setiap minit. Untuk menyiasat andaian ini, lima alat tenun dipilih secara rawak dan keluarannya direkodkan pada waktu yang berlainan. Data berikut diperolehi:

Alat Tenun	Keluaran (lb/min)					
	1	14.0	14.1	14.2	14.0	14.1
2	13.9	13.8	13.9	14.0	14.0	
3	14.1	14.2	14.1	14.0	13.9	
4	13.6	13.5	14.0	13.9	13.7	
5	13.8	13.6	13.9	13.8	14.0	

- (i). Terangkan mengapa ini adalah ujikaji kesan rawak.
- (ii). Adakah alat tenun sama dari segi keluaran? Guna  $\alpha = 0.05$ .
- (iii). Anggarkan variasi diantara alat tenun.
- (iv). Anggarkan ralat varians ujikaji.

(60 markah)

**- 12 -****APPENDIX****Randomized Complete Block Design:**

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_{Treatments} = \frac{1}{b} \sum_{i=1}^a y_{i..}^2 - \frac{y_{..}^2}{N}$$

$$SS_{Blocks} = \frac{1}{a} \sum_{j=1}^b y_{.j}^2 - \frac{y_{..}^2}{N}$$

$$SS_E = SS_T - SS_{Treatments} - SS_{Blocks}$$

$$x = \frac{ay'_{i..} + by'_{.j} - y'_{..}}{(a-1)(b-1)}$$

**Nested Design:**

$$SS_A = \frac{1}{bn} \sum_{i=1}^a y_{i..}^2 - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \frac{1}{n} \sum_{i=1}^a \sum_{j=1}^b y_{ij..}^2 - \frac{1}{bn} \sum_{i=1}^a y_{i..}^2$$

$$SS_E = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{1}{n} \sum_{i=1}^a \sum_{j=1}^b y_{ij..}^2$$

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

**Single Factor Design:**

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_{Treatments} = \frac{1}{n} \sum_{i=1}^a y_{i..}^2 - \frac{y_{..}^2}{N}$$

$$SS_E = SS_T - SS_{Treatments}$$

## **Contrast for $2^k$ Design :**

$$\text{Contrast}_{AB\ldots K} = (a \pm 1)(b \pm 1) \dots (k \pm 1)$$

with the sign in parentheses negative if the factor is included & positive if the factor is not included.

For a  $2^5$  design,      Contrast<sub>ABDE</sub> = (a-1)(b-1)(c+1)(d-1)(e-1)  
                              = ....(expand)

### **Effect Estimate for $2^k$ Design :**

$$AB\dots K = \frac{2}{n2^k} (\text{Contrast}_{AB\dots K})$$

## **Sum of Squares for $2^k$ Design :**

$$SS_{AB\dots K} = \frac{1}{n^2 k} (\text{Contrast}_{AB\dots K})^2$$

where n is number of replicates

IV Percentage Points of the  $F$  Distribution<sup>a</sup>

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49	9.58	9.63	9.67	9.71	9.76	9.80	9.85
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41	3.43	3.43	3.44	3.45	3.46	3.47	3.48
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.45	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.88	1.88	1.88	1.87	1.87	1.87
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.75	1.75	1.74	1.74	1.74
7	1.57	1.70	1.72	1.72	1.71	1.71	1.71	1.70	1.70	1.69	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.65	1.65
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.62	1.62	1.61	1.60	1.59	1.59	1.58	1.58	1.58
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.55	1.54	1.54	1.53	1.53	1.53
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52	1.52	1.51	1.51	1.50	1.49	1.48
11	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.54	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.47	1.46	1.45
12	1.46	1.56	1.56	1.55	1.54	1.53	1.53	1.52	1.51	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43
13	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42	1.42	1.42
14	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.38
15	1.43	1.52	1.52	1.51	1.49	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.43	1.43	1.42	1.41	1.40	1.39	1.38
16	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35
17	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
18	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
19	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.29
21	1.40	1.48	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28
22	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28
23	1.39	1.47	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.28	1.27
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.26
25	1.39	1.47	1.46	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.32	1.31	1.29	1.27	1.27	1.25
26	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.26	1.25
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.33	1.32	1.31	1.30	1.28	1.27	1.26	1.24
28	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.25	1.24
29	1.38	1.45	1.45	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.27	1.25	1.23	
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.24	
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.22	1.21	
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.27	1.25	1.24	1.22	1.21	1.19	1.17	
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.14	
$\infty$	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.22	1.19	1.18	1.16	1.14	1.12	1.08	

Degrees of Freedom for the Denominator ( $\nu_2$ ) $\nu =$  Degrees of freedom.<sup>a</sup>Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

IV Percentage Points of the  $F$  Distribution (Continued)

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.16	5.15	5.14	5.13	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.96	1.92	1.89	1.84	1.80	1.74	1.70	1.67	1.64	1.61	1.57	1.53
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
$\infty$	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

Degrees of Freedom for the Numerator ( $\nu_1$ )Degrees of Freedom for the Denominator ( $\nu_2$ )

IV Percentage Points of the  $F$  Distribution (Continued) $F_{0.05; \nu_1, \nu_2}$ 

$\nu_1$	Degrees of Freedom for the Numerator ( $\nu_1$ )																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
$\nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
161.4	199.5	215.7	224.6	230.2	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3		
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.47	19.48	19.49	19.50	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.30	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.07	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25	
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	

D.F. of Denominator ( $\nu_2$ )

1b. Invertible factor of the F Distribution (Casella et al.)

IV Percentage Points of the  $F$  Distribution (Continued)

$\nu_1$	$\nu_2$	Degrees of Freedom for the Numerator ( $\nu_1$ )																	
1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.47	39.48	39.49	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.23	6.18	6.12	6.07	6.02	
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14	
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.98	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

Degrees of Freedom for the Denominator ( $\nu_2$ )

IV Percentage Points of the  $F$  Distribution (Continued)

$\nu_1$	Degrees of Freedom for the Numerator ( $\nu_1$ )												$\nu_2$						
$\nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	4.052	4.9995	5.403	5.625	5.764	5.859	5.928	5.982	6.022	6.056	6.106	6.157	6.209	6.235	6.261	6.287	6.313	6.339	6.366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.48	99.49	99.50	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.00	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.35	2.26	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.34	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Degrees of Freedom for the Denominator ( $\nu_2$ )IV Percentage Points of the  $F$  Distribution (Continued)