



Final Examination
2017/2018 Academic Session

May/June 2018

JIM316 – Introduction To Operations Research
[Pengantar Penyelidikan Operasi]

Duration : 3 hour
[Masa: 3 jam]

Please ensure that this examination paper contains **FOURTEEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT BELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

- 2 -

1. (a). An advertising company wishes to plan an advertising campaign in three different media – television, radio and magazines. The purpose of the advertising program is to reach as many potential customers as possible. Results of a market study are given below:

	Television		Radio	Magazines
	Daytime	Prime Time		
Cost of an advertising unit (RM)	40,000	75,000	30,000	15,000
Number of potential customers reached per unit	400,000	900,000	500,000	200,000
Number of women customers reached per unit	300,000	400,000	200,000	100,000

The company does not want to spend more than RM800,000 on advertising. It further requires that,

- (i). at least 2 million exposures take place among women;
- (ii). advertising on television be limited to RM500,000;
- (iii). at least 3 advertising units be bought on daytime television, and two units during prime time; and
- (iv). the number of advertising units on radio and magazine should be between 5 and 10.

Formulate a linear programming model for this problem.

(35 marks)

...3/-

- 3 -

- (b). Consider the following Linear Programming Model:

Maximise

$$Z = 90x_1 + 100x_2 \quad (\text{profit})$$

subject to

$$8x_1 + 18x_2 \leq 360 \quad (\text{resource 1})$$

$$2x_1 + 2x_2 \leq 50 \quad (\text{resource 2})$$

$$2x_1 + x_2 \leq 40 \quad (\text{resource 3})$$

and

$$x_1 \geq 0, x_2 \geq 0$$

(x_i is the amount of product i)

- (i). Use the graphical method to solve this model.
- (ii). Find the optimal solution.
- (iii). Find the shadow prices for all resources and describe their significance.
- (iv). If the objective function is changed to $Z = 120x_1 + 80x_2$ while all the other information does not change, find the new optimal solution.

(65 marks)

2. (a). Consider the following Linear Programming Model:

Maximise

$$Z = -x_1 + x_2 \quad (\text{profit})$$

subject to

$$-3x_1 + 2x_2 \leq 8 \quad (\text{resource 1})$$

$$x_1 - 2x_2 \geq -12 \quad (\text{resource 2})$$

$$x_2 \leq 8 \quad (\text{resource 3})$$

and

$$x_1 \geq 0, x_2 \geq 0$$

Use the Simplex Method to solve this model. Find

- (i). the optimal solution.
- (ii). the unit worth for all resources and describe their significance.

(50 marks)

...4/-

- 4 -

- (b). The following Linear Programming problem was solved using the Two-Phase Method.

$$Z = 2x_1 + 3x_2 + x_3$$

subject to

$$ax_1 + bx_2 + cx_3 \geq d$$

$$ex_1 + fx_2 \geq g$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

For certain values of a, b, c, d, e, f and g (where a, b, c, d, e, f and g are constants) the tableau is given below.

Phase 1

Basic Variable	Coefficient of:						Right Side
	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	
Z	3	2	0	0	-1	0	6
x_3	1	4	2	-1	0	0	d
$\bar{x}_6$	3	2	0	0	-1	1	g
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
Z	0	0	0	0	0	-1	0
x_2	0	1	3/5	-3/5	1/10	-1/10	h
x_1	1	0	-2/5	1/5	-2/5	2/5	i

Phase 2

Basic Variable	Coefficient of:					Right Side
	x_1	x_2	x_3	x_4	x_5	
Z	0	0	0	-1/2	-1/2	j
x_2	0	1	3/5	-3/5	1/10	9/5
x_1	1	0	-2/5	1/5	-2/5	4/5

- (i). Identify whether this Linear Programming problem is a maximisation or minimisation problem. Give reason for your choice.

- (ii). Calculate the values of $a, b, c, d, e, f, g, h, i$ and j .

(50 marks)

...5/-

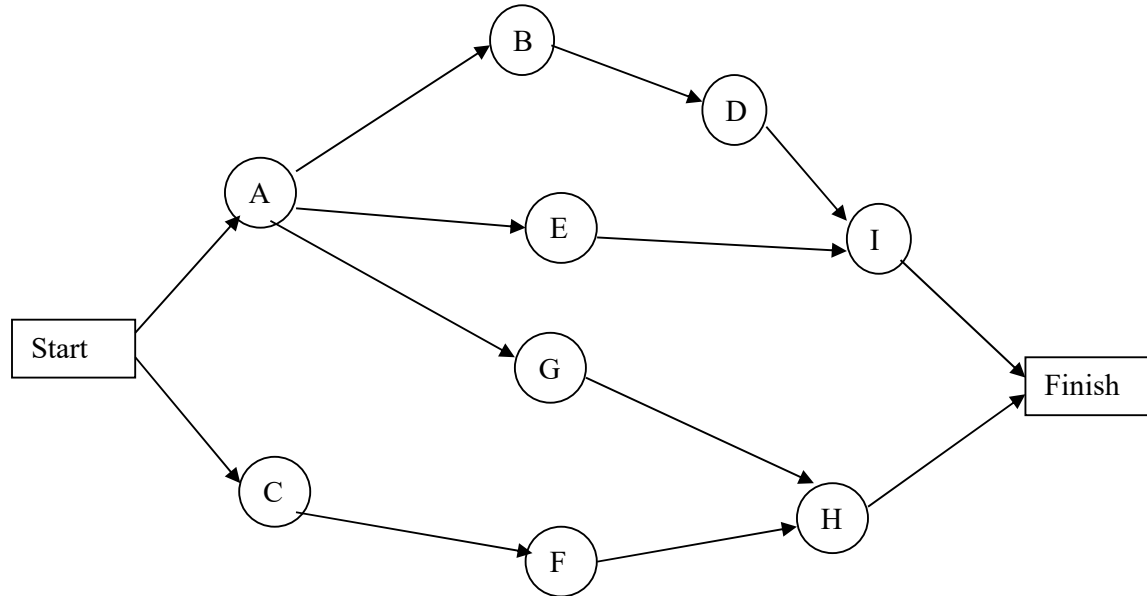
- 5 -

3. (a). Construct the project network for a project with the following activity list.

Activity	Immediate Predecessors
A	-
B	A
C	-
D	-
E	C
F	B, C, D
G	E
H	A
I	F, G, H
J	E

(25 marks)

- (b). Given below is a project network with the estimated duration of all activities in weeks using the PERT three-estimate approach.



...6/-

- 6 -

Activity	Duration (week)		
	Optimistic Estimate	Most Likely Estimate	Pessimistic Estimate
A	2	3	4
B	2	3	5
C	4	5	6
D	2	2	2
E	4	6	7
F	2	3	5
G	3	4	5
H	5	7	9
I	3	5	6

- (i). List the immediate predecessors of each activity.
- (ii). Using the duration of the most likely estimate,
 - [1]. find all the paths and path lengths through this project network.
 - [2]. what is the project earliest completion time?
 - [3]. find the earliest start time and earliest finish time for each activity.
 - [4]. find the latest start time and latest finish time for each activity.
 - [5]. find the slack for each activity. Which of the paths is a critical path?
 - [6]. if all other activities take the most likely estimated amount of time, what is the maximum duration of activity G without delaying the completion of the project?
- (iii). Find the estimate of the mean and variance of the duration of each activity.
- (iv). Find the probability that activity E will complete before the beginning of the 10th week.
- (v). Find the probability that the project will finish within 15 weeks.

(75 marks)

...7/-

4. (a). Give a brief description of the following terms.

- (i). Binding constraints.
- (ii). Redundant constraints.
- (iii). Feasible region.
- (iv). Corner point feasible solution.
- (v). Shadow prices.

(40 marks)

(b). The demand for a product is 600 units per week, and the items are withdrawn at a constant rate. The setup cost for placing an order to replenish inventory is RM25.00. The unit cost of each item is RM3.00, and the inventory holding cost is RM0.05 per item per week.

- (i). Assuming shortages are not allowed, determine how often to order and what size the order should be.
- (ii). If shortages are allowed but cost RM2.00 per item per week, determine how often to order and what size the order should be.

(60 marks)

1. (a). Sebuah syarikat pengiklanan ingin merancang kempen iklan dalam tiga media yang berbeza - televisyen, radio dan majalah. Tujuan program pengiklanan ialah untuk mencapai sebanyak mungkin pelanggan yang berpotensi. Hasil kajian pasaran diberikan di bawah:

	Televisyen		Radio	Majalah
	Siang hari	Waktu utama		
Kos unit pengiklanan (RM)	40,000	75,000	30,000	15,000
Bilangan pelanggan berpotensi dicapai per unit	400,000	900,000	500,000	200,000
Bilangan pelanggan wanita dicapai per unit	300,000	400,000	200,000	100,000

Syarikat itu tidak mahu membelanjakan lebih daripada RM800,000 untuk pengiklanan. Ia selanjutnya memerlukan,

- sekurang-kurangnya 2 juta pendedahan berlaku di kalangan wanita;
- pengiklanan di televisyen terhad kepada RM500,000;
- sekurang-kurangnya 3 unit pengiklanan dibeli di televisyen pada siang hari, dan dua unit semasa waktu utama; dan
- bilangan unit pengiklanan di radio dan majalah mestilah antara 5 dan 10.

Rumuskan model Pengaturcaraan Linear bagi masalah ini.

(35 markah)

- 9 -

- (b). Pertimbangkan model Pengaturcaraan Linear yang berikut:

Maksimumkan

$$Z = 90x_1 + 100x_2 \quad (\text{keuntungan})$$

terhadap

$$8x_1 + 18x_2 \leq 360 \quad (\text{sumber 1})$$

$$2x_1 + 2x_2 \leq 50 \quad (\text{sumber 2})$$

$$2x_1 + x_2 \leq 40 \quad (\text{sumber 3})$$

dan

$$x_1 \geq 0, x_2 \geq 0$$

(x_i amaun produk i)

- (i). Gunakan kaedah bergraf untuk menyelesaikan model ini.
- (ii). Dapatkan penyelesaian optimum.
- (iii). Dapatkan harga bayangan bagi setiap sumber dan terangkan kepentingannya.
- (iv). Jika fungsi matlamat diubah kepada $Z = 120x_1 + 80x_2$ manakala semua maklumat lain tidak berubah, tentukan penyelesaian optimum yang baru.

(65 markah)

2. (a). Pertimbangkan model Pengaturcaraan Linear yang berikut:

Maksimumkan

$$Z = -x_1 + x_2 \quad (\text{keuntungan})$$

terhadap

$$-3x_1 + 2x_2 \leq 8 \quad (\text{sumber 1})$$

$$x_1 - 2x_2 \geq -12 \quad (\text{sumber 2})$$

$$x_2 \leq 8 \quad (\text{sumber 3})$$

dan

$$x_1 \geq 0, x_2 \geq 0$$

Gunakan kaedah Simpleks untuk menyelesaikan model ini. Cari

- (i). penyelesaian optimum.
- (ii). nilai seunit bagi setiap sumber dan terangkan kepentingannya.

(50 markah)

...10/-

- 10 -

- (b). Masalah Pengaturcaraan Linear yang berikut diselesaikan dengan menggunakan Teknik Dua-Fasa.

$$Z = 2x_1 + 3x_2 + x_3$$

terhadap

$$ax_1 + bx_2 + cx_3 \geq d$$

$$ex_1 + fx_2 \geq g$$

dan

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Untuk nilai-nilai tertentu bagi a, b, c, d, e, f dan g (di mana a, b, c, d, e, f dan g adalah pemalar), tablo penyelesaian diberikan di bawah.

Fasa 1

Pemboleh -ubah Asas	Pekali:						Sebelah kanan
	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	
Z	3	2	0	0	-1	0	6
x_3	1	4	2	-1	0	0	d
$\bar{x}_6$	3	2	0	0	-1	1	g
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
Z	0	0	0	0	0	-1	0
x_2	0	1	3/5	-3/5	1/10	-1/10	h
x_1	1	0	-2/5	1/5	-2/5	2/5	i

Fasa 2

Pemboleh -ubah Asas	Pekali:					Sebelah kanan
	x_1	x_2	x_3	x_4	x_5	
Z	0	0	0	-1/2	-1/2	j
x_2	0	1	3/5	-3/5	1/10	9/5
x_1	1	0	-2/5	1/5	-2/5	4/5

- (i). Tentukan sama ada masalah Pengaturcaraan Linear ini adalah masalah pemaksimuman atau peminimuman. Beri alasan atas pilihan anda.
- (ii). Hitung nilai-nilai bagi $a, b, c, d, e, f, g, h, i$ dan j .

(50 markah)

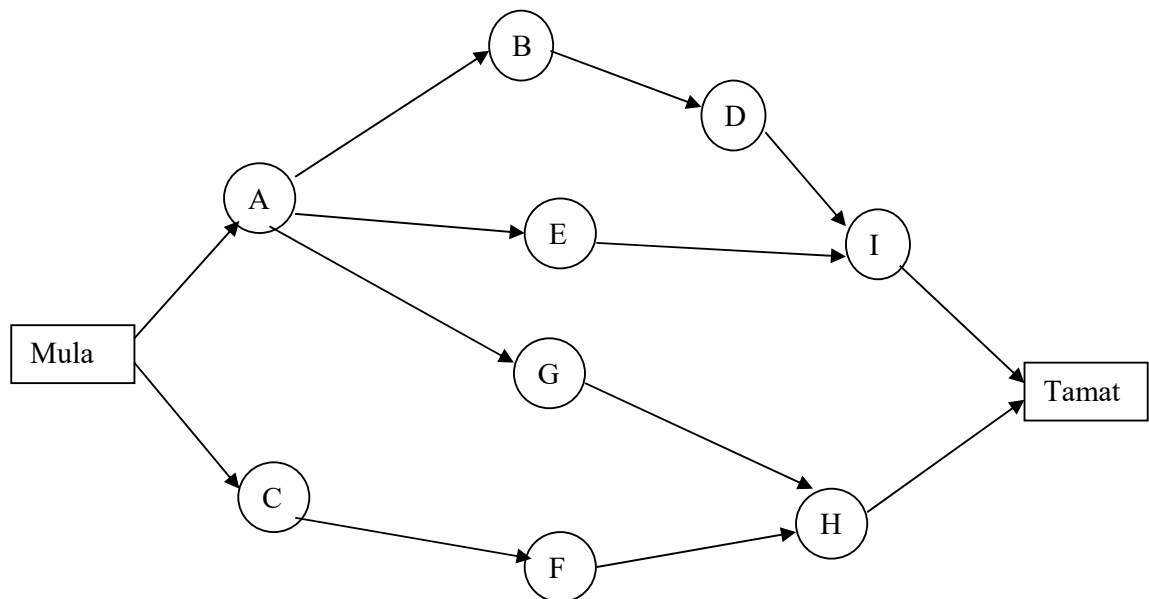
...11/-

3. (a). Bina rangkaian projek bagi suatu projek dengan senarai kegiatan berikut.

Kegiatan	Kegiatan Pendahulu
A	-
B	A
C	-
D	-
E	C
F	B, C, D
G	E
H	A
I	F, G, H
J	E

(25 markah)

- (b). Diberikan di bawah rangkaian projek dan anggaran jangka masa bagi semua kegiatan dalam minggu dengan menggunakan pendekatan tiga-anggaran PERT.



- 12 -

Kegiatan	Jangka Masa (minggu)		
	Masa Optimis	Masa Paling Boleh Jadi	Masa Pesimis
A	2	3	4
B	2	3	5
C	4	5	6
D	2	2	2
E	4	6	7
F	2	3	5
G	3	4	5
H	5	7	9
I	3	5	6

- (i). Senaraikan kegiatan pendahulu bagi setiap kegiatan.
- (ii). Dengan menggunakan masa paling boleh jadi sebagai jangka masa
 - [1]. dapatkan kesemua lintasan dan panjang lintasan dalam rangkaian projek ini.
 - [2]. berapa lamakah jangka masa siap terawal projek?
 - [3]. dapatkan masa permulaan terawal dan masa siap terawal bagi setiap kegiatan.
 - [4]. dapatkan masa permulaan terlewat dan masa siap terlewat bagi setiap kegiatan.
 - [5]. dapatkan apungan bagi setiap kegiatan. Lintasan yang manakah merupakan lintasan genting?
 - [6]. Jika semua kegiatan mengambil jangka masa paling boleh jadi seperti yang dianggarkan untuk siap, dapatkan jangka masa maksimum bagi kegiatan G tanpa menjejaskan masa siap projek.
- (iii). Dapatkan anggaran min dan varians bagi jangka masa setiap kegiatan.
- (iv). Dapatkan kebarangkalian bahawa kegiatan E boleh siap sebelum permulaan minggu ke-10.
- (v). Dapatkan kebarangkalian bahawa projek siap dalam tempoh 15 minggu.

(75 markah)

...13/-

4. (a). Berikan penerangan ringkas bagi istilah berikut:

- (i). Kekangan terikat.
- (ii). Kekangan membazir.
- (iii). Ruang tersaur.
- (iv). Titik sudut penyelesaian tersaur.
- (v). Harga bayangan.

(40 markah)

(b). Permintaan suatu barangan adalah 600 unit seminggu dan item dikeluarkan pada kadar malar. Kos penyediaan setiap kali pesanan dibuat untuk menambah inventori ialah RM25.00. Kos bagi seunit barangan ialah RM3.00, dan kos penangguhan inventori ialah RM0.05 setiap item seminggu.

- (i). Andaikan kekurangan tidak dibenarkan, tentukan kekerapan pesanan harus dibuat dan apakah saiz pengeluarannya.
- (ii). Jika kekurangan dibenarkan dengan kos RM2.00 setiap item seminggu, tentukan kekerapan pesanan harus dibuat dan saiz pengeluarannya.

(60 markah)

APPENDIX

Table E The Standard Normal Distribution										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: Use 0.4999 for z values above 3.09.

Source: Frederick Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, Table A-1 (Reading, Mass.: Addison-Wesley, 1973). Reprinted with permission of the copyright owners.

