Final Examination
2017/2018 Academic Session

May/June 2018

JIM310 – Introductory Numerical Methods
[Pengantar Kaedah Berangka]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **TWENTY** printed pages before you begin the examination.

Answer **FOUR (4)** questions only. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

Sila pastikan bahawa kertas peperiksaan ini mengandungi ***DUA PULUH*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

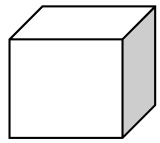
Jawab ***EMPAT (4)*** *soalan sahaja. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggerís.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. (a).



Given a cube with base area, $A = 32.49 \pm 1.14$ cm^2 and the height $h = 5.7 \pm 0.1$ cm.

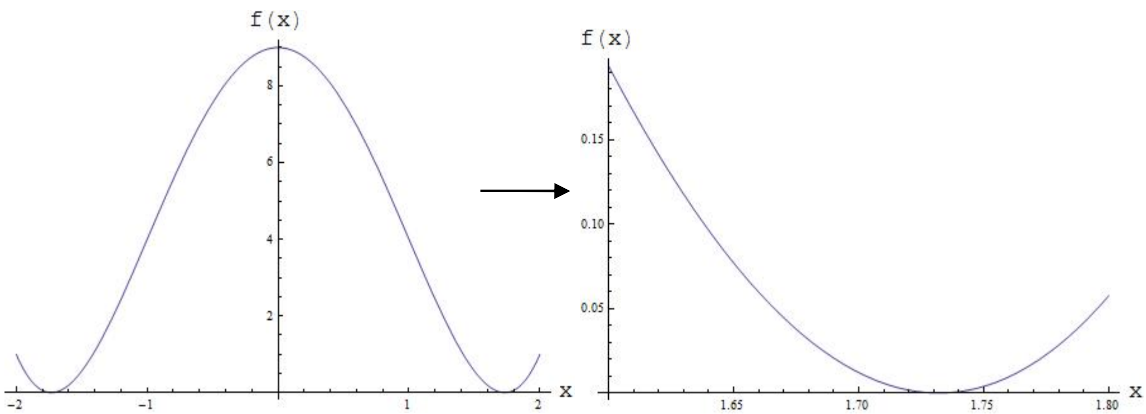
(i). Find the volume of a cube without error limit.

(ii). Find the volume of a cube with error limit.

(iii). Verify your answer in (a). (ii). using other alternative formula.

(30 marks)

(b).



The diagrams show the graph of function $f(x) = x^4 - 6x^2 + 9$

(i). Use Incremental Search method to find a solution of $x^4 - 6x^2 + 9 = 0$ in the interval $(1.6, 1.8)$. Divide the given interval to the four segments and do three iterations.

(ii). Use Secant method with three iterations to find a solution of $x^4 - 6x^2 + 9 = 0$ in the interval $(1.6, 1.8)$. Assuming that $x_0 = 1.6$ and $x_1 = 1.8$.

(iii). Given that the exact solutions are $x = \pm 1.7320508075688772$. Find the absolute error for both numerical methods obtained in (b). (i). and (b).(ii). between exact and approximate solutions.

(50 marks)

(c). Given that $f(x) := x^4 - 6x^2 + 9 = 0$ can be form as several fixed point function. Show that

(i). $x_{i+1} = \pm\sqrt[4]{6x_i^{\,2} - 9}$

(ii). $x_{i+1} = \pm\sqrt{\dfrac{x_i^{\,4} + 9}{6}}$

(iii). $x_{i+1} = \pm\sqrt{\dfrac{9}{6 - x_i^{\,2}}}$

in the form of $x_{i+1} = g(x_i)$ where $i = 0, 1, 2, ..., n.$

(20 marks)

2. (a). Given that the general solution of Lagrange Interpolating Polynomial is

$$P_n(x) = f(x_0)L_{n,0}(x) + f(x_1)L_{n,1}(x) + \quad ... \quad + f(x_n)L_{n,1}(x)$$

where

$$L_{n,i} = \frac{(x - x_0)(x - x_1)...(x - x_{i-1})(x - x_{i+1})...(x - x_n)}{(x_i - x_0)(x_i - x_1)...(x_i - x_{i-1})(x_i - x_{i+1})...(x_i - x_n)}$$

$$= \prod_{\substack{k=0 \\ k \neq i}}^{n} \frac{(x - x_k)}{(x_i - x_k)}$$

or can be form as $P_n(x) = \sum_{i=0}^{n} f(x_i)L_{n,i}(x)$.

(i). Write the formula of $P_2(x)$.

(ii). Determine the second Lagrange interpolating polynomial $P_2(x)$ that passes through the points $\left(2, \frac{1}{2}\right), \left(\frac{11}{4}, \frac{4}{11}\right)$ and $\left(4, \frac{1}{4}\right)$.

(iii). Approximate the solution of $f(3)$ using the second order Lagrange interpolating polynomial $P_2(x)$ in (a). (ii).

(iv). The data in part (a). (ii). are generated using the function $f(x) = \dfrac{1}{x}$, compute the absolute error and the percentage of relative error for the result obtained in (a). (iii).

(50 marks)

(b). Given that the general solution of Newton's Divided Difference is

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots$$
$$+ f[x_0, x_1, \dots, x_n](x - x_0)(x - x_1)\dots(x - x_{n-1})$$

where

$$f[x_0, x_1, \dots, x_n] = \frac{f[x_1, x_2, \dots, x_n] - f[x_0, x_1, \dots, x_{n-1}]}{x_n - x_0}$$

(i). Write down the formula of second Divided Difference $P_2(x)$.

(ii). Determine the second order Divided Difference $P_2(x)$ that passes through the point $\left(2, \frac{1}{2}\right), \left(\frac{11}{4}, \frac{4}{11}\right)$ and $\left(4, \frac{1}{4}\right)$.

(iii). Approximate the solution of $f(3)$ using the second order Divided Difference $P_2(x)$ in (b). (ii).

(iv). The above data are generated using the function $f(x) = \frac{1}{x}$, compute the absolute error and the percentage of relative error for the result obtained in (b). (iii).

(45 marks)

(c). Based on your result in (a). and (b). to approximate $f(3)$, which method is better. State your reason.

(5 marks)

3. (a). State three inadequate criteria to get "Best Fitting Line" for regression, regarding to the following equation

(i). $\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (y_i - \tilde{y}_i)$,

(ii). $\sum_{i=1}^{n} |e_i| = \sum_{i=1}^{n} |y_i - \tilde{y}_i|$,

(iii). $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2$,

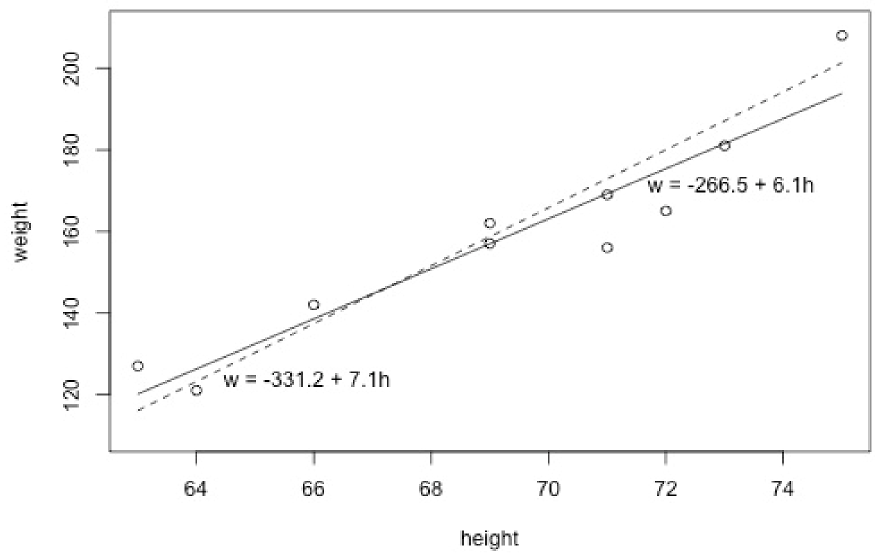
where $e = y_i - \tilde{y}_i$ is error or residual between the true value y_i and the approximate value $\tilde{y}_i$.

(10 marks)

(b). Data shows a set of heights and weights of 10 students.

Weight, pound	63	64	66	69	69	71	71	72	73	75
Height, inch	127	121	142	157	162	156	169	165	181	208

According to the data, two possibilities of lines can be drawn as follows.



(i). Assuming $w = -331.2 + 7.1h$ where w = weight (in pound) and h = height (in inch). (Rewrite the table with answers in the answer script).

i	x_i	y_i	$\tilde{y}_i$	$(y_i - \tilde{y}_i)$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

…6/-

(ii). Assuming $w = -266.5 + 6.1h$ where w = weight (in pound) and h = height (in inch). (Rewrite the table with answers in the answer script).

i	x_i	y_i	$\tilde{y}_i$	$(y_i - \tilde{y}_i)$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

Based on your result, which line has better fit? State your reason.

(45 marks)

(c). To get an equation of linear regression, least square method is used.

(i).

i	x_i	y_i	x_i^2	$x_i y_i$
1	63	127		
2	64	121		
3	66	142		
4	69	157		
5	69	162		
6	71	156		
7	71	169		
8	72	165		
9	73	181		
10	75	208		
Σ				

where $a_0 = \bar{y} - a_1\bar{x}$ and $a_1 = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$

…7/-

By using linear regression analysis, (Rewrite the table with answers in the answer script),

(ii). Find mean of height $\bar{x}$ and mean of weight $\bar{y}$ of 10 students.

(iii). Find the equation of line $w = a_0 + a_1 h$ using least-square method.

(iv). Estimate the weight of a student if his/her height is 177.8 cm. (Hint : 1 inch = 2.54 cm).

(v). Find the Total of Sum Square Error, $\sum_{i=1}^{n} e_i^2$ where $e = y_i - \tilde{y}_i$ for this linear regression.

i	x_i	y_i	$\tilde{y}_i = a_0 + a_1 x_i$	$y_i - \tilde{y}_i$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

Compare the result with the answer in (b). and conclude your observation.

(45 marks)

4. (a). Approximate first derivative of $f(x)=\ln x$ at $x_0=1.8$ using $h=0.05$ and

(i). Forward Difference Formula.

(ii). Center Difference Formula.

(iii). Three-Point Endpoint Formula.

(20 marks)

(b).

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	0.2955	β	0.4794	0.5646	0.6442

(i). Given that $f'(0.3)=0.8774$ by using five-point midpoint formula, find the value of β.

(ii). Using second derivative midpoint formula and the value β obtained, find the step size h that should be used to fulfill $f''(0.3)=-0.48$.

(iii). Find the area $\int_{0.1}^{0.5} f(x)\,dx$ for the given data using Trapezoidal rule $(n=1)$ and Simpson's rule $(n=2)$. Use the appropriate value of step size h for each aforementioned rule.

(50 marks)

(c). Approximate $\int_0^1 x^2 \ln\left(x^2+1\right) dx$ using $h=0.25$. Use

(i). Composite Trapezoidal rule.

(ii). Composite Simpson's rule.

(30 marks)

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5. (a). Given that

$$\begin{aligned} x - y + 3z &= 2 \\ 3x - 3y + z &= -1 \\ x + y &= 3 \end{aligned}$$

(i). Solve the given system of equations using Gaussian elimination method.

(ii). Determine whether row interchanges are necessary or not?

(iii). Why pivoting strategy in Gaussian Elimination method is important for solving system of linear equations ?

(15 marks)

(b). The linear system $AX = B$ is given by

$$\begin{pmatrix} 3 & -1 & -1 \\ \lambda & 8 & 1 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.8 \\ -1.1 \\ 0.1 \end{pmatrix}$$

(i). Determine the maximum integer λ that should be used in matrix A to become strictly diagonally dominant $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|,\ i = 1, 2, \ldots, n$.

(ii). Solve the above linear system using Jacobi iterative method and Gauss-Seidel iterative method, starting with $x^{(0)} = (0,0,0)^T$ and do iteration until $\mathbf{x}_i^{(3)}$.

(65 marks)

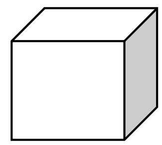
(c). Given $\mathbf{F}(\mathbf{x})$ is defined as

$$\begin{aligned} f_1(x,y) &= x^2 - 2x - y + 0.5 = 0, \\ f_2(x,y) &= x^2 + 4y^2 - 4 = 0, \end{aligned}$$

where $\mathbf{F}(\mathbf{x}) = (f_1, f_2)^T$ and $\mathbf{x} = (x, y)^T$. Find Jacobian matrix $J(\mathbf{x})$ and its inverse $J^{-1}(\mathbf{x})$ for any value of $\mathbf{x}$.

(20 marks)

1.



(a). Diberi luas sebuah kubus adalah $A = 32.49 \pm 1.14\,\text{cm}^2$ dan tinggi $h = 5.7 \pm 0.1$ cm.

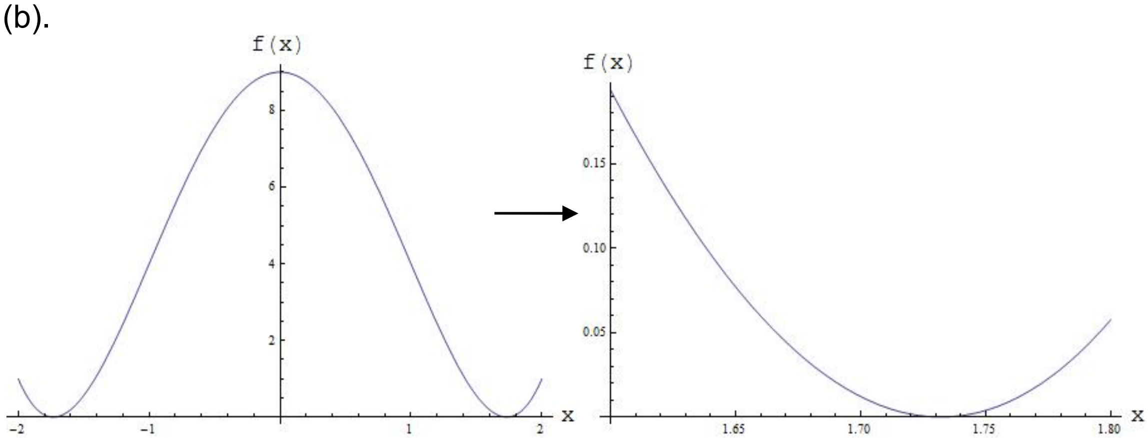
(i). Cari isipadu bagi kubus tanpa ralat.

(ii). Cari isipadu bagi kubus dengan kiraan ralat.

(iii). Sahkan jawapan anda di (a). (ii). menggunakan kaedah alternatif lain.

(30 markah)

(b).



Rajah menunjukkan graf fungsi bagi $f(x) = x^4 - 6x^2 + 9$.

(i). Guna kaedah Carian Tambahan untuk menyelesaikan $x^4 - 6x^2 + 9 = 0$ dalam selang $(1.6\,,1.8)$. Bahagikan selang yang diberi kepada 4 segmen dan lelarkan 3 kali.

(ii). Guna kaedah Sekan dengan tiga lelaran untuk mencari satu penyelesaian $x^4 - 6x^2 + 9 = 0$ dalam selang $(1.6\,,1.8)$. Andaikan $x_0 = 1.6$ dan $x_1 = 1.8$.

(iii). Diberi penyelesaian sebenar adalah $x = \pm 1.7320508075688772$. Cari ralat mutlak bagi kedua-dua kaedah berangka yang diperolehi dalam (b). (i). dan (b). (ii). antara penyelesaian sebenar dengan anggaran penyelesaian.

(50 markah)

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(c). Diberi $f(x) := x^4 - 6x^2 + 9 = 0$ boleh membentuk sebagai beberapa fungsi kaedah tetap. Tunjukkan bahawa

(i). $x_{i+1} = \pm\sqrt[4]{6x_i^{\,2} - 9}$

(ii). $x_{i+1} = \pm\sqrt{\dfrac{x_i^{\,4} + 9}{6}}$

(iii). $x_{i+1} = \pm\sqrt{\dfrac{9}{6 - x_i^{\,2}}}$

dalam bentuk $x_{i+1} = g(x_i)$ dengan $i = 0, 1, 2, ..., n.$

(20 markah)

2. (a). Diberi penyelesaian umum bagi Interpolasi Lagrange Polinomial adalah

$$P_n(x) = f(x_0)L_{n,0}(x) + f(x_1)L_{n,1}(x) + \quad ... \quad + f(x_n)L_{n,1}(x)$$

dengan

$$L_{n,i} = \frac{(x - x_0)(x - x_1)...(x - x_{i-1})(x - x_{i+1})...(x - x_n)}{(x_i - x_0)(x_i - x_1)...(x_i - x_{i-1})(x_i - x_{i+1})...(x_i - x_n)}$$
$$= \prod_{\substack{k=0 \\ k \neq i}}^{n} \frac{(x - x_k)}{(x_i - x_k)}$$

atau boleh diringkaskan sebagai $P_n(x) = \sum_{i=0}^{n} f(x_i)L_{n,i}(x)$.

(i). Tulis formula bagi $P_2(x)$.

(ii). Tentukan persamaan interpolasi Lagrange polinomial peringkat kedua $P_2(x)$ yang melalui koordinat berikut $\left(2, \frac{1}{2}\right), \left(\frac{11}{4}, \frac{4}{11}\right)$ dan $\left(4, \frac{1}{4}\right)$.

(iii). Anggarkan penyelesaian bagi $f(3)$ menggunakan interpolasi Lagrange polinomial peringkat kedua $P_2(x)$ di (a). (ii).

(iv). Data di bahagian (ii). dijana daripada fungsi $f(x)=\frac{1}{x}$, kira ralat mutlak dan peratus ralat relatif untuk hasil yang diperolehi di (a). (iii).

(50 markah)

(b). Diberi penyelesaian umum bagi Pembezaan Pembahagi Newton adalah

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \ldots$$
$$+ f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

dengan

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_1, x_2, \ldots, x_n] - f[x_0, x_1, \ldots, x_{n-1}]}{x_n - x_0}$$

(i). Tulis formula bagi Pembezaan Pembahagi Peringkat Kedua $P_2(x)$.

(ii). Tentukan persamaan Pembezaan Pembahagi Peringkat Kedua $P_2(x)$ yang melalui titik $\left(2, \frac{1}{2}\right), \left(\frac{11}{4}, \frac{4}{11}\right)$ dan $\left(4, \frac{1}{4}\right)$.

(iii). Anggarkan penyelesaian bagi $f(3)$ menggunakan Pembezaan Pembahagi Peringkat Kedua $P_2(x)$ di (b). (ii).

(iv). Data di atas dijana menggunakan fungsi $f(x)=\frac{1}{x}$, kira ralat mutlak dan peratus ralat relatif untuk hasil yang diperolehi di (b). (iii).

(45 markah)

(c). Berdasarkan jawapan anda di (a). dan (b). untuk menganggarkan $f(3)$, kaedah manakah yang lebih baik. Nyatakan alasan anda.

(5 markah)

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3. (a). Nyatakan tiga kriteria yang tidak mencukupi untuk mendapat “Garisan Terbaik” untuk regresi, berdasarkan persamaan berikut.

(i). $\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (y_i - \tilde{y}_i)$,

(ii). $\sum_{i=1}^{n} |e_i| = \sum_{i=1}^{n} |y_i - \tilde{y}_i|$,

(iii). $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2$,

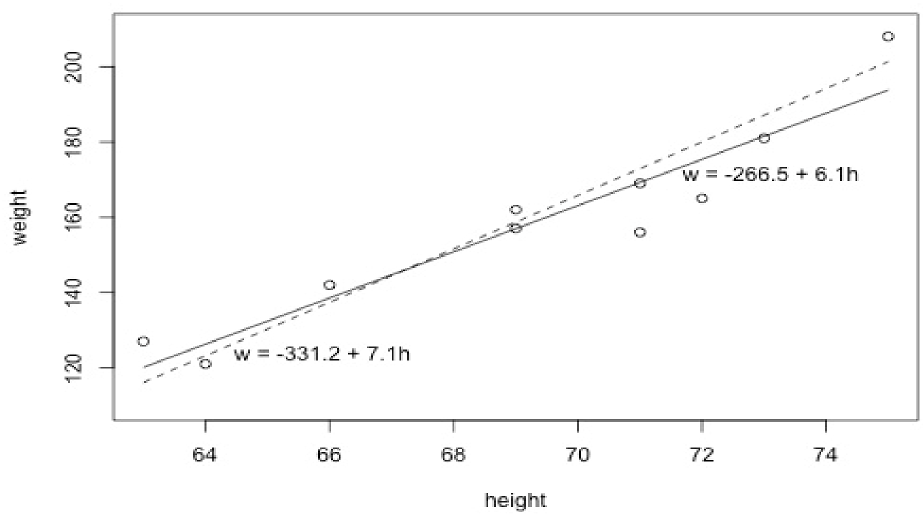
dengan $e = y_i - \tilde{y}_i$ adalah ralat antara nilai sebenar y_i dengan nilai anggaran $\tilde{y}_i$.

(10 markah)

(b). Data menunjukkan satu set tinggi dan berat bagi 10 pelajar .

Berat, paun	63	64	66	69	69	71	71	72	73	75
Tinggi, inci	127	121	142	157	162	156	169	165	181	208

Berdasarkan data, dua kemungkinan garis boleh dilukis seperti berikut.



(i). Andaikan $w = -331.2 + 7.1h$ dengan w = berat (dalam paun) dan h = tinggi (dalam inci). (Tulis semula jadual bersama jawapan di dalam buku jawapan).

i	x_i	y_i	$\tilde{y}_i$	$(y_i - \tilde{y}_i)$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

(ii). Andaikan $w = -266.5 + 6.1h$ dengan w = berat (dalam paun) dan h = tinggi (dalam inci). (Tulis semula jadual bersama jawapan di dalam buku jawapan).

i	x_i	y_i	$\tilde{y}_i$	$(y_i - \tilde{y}_i)$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

Berdasarkan keputusan anda, manakah garis yang mempunyai bentuk terbaik? Nyatakan alasan anda.

(45 markah)

(c). Untuk dapatkan persamaan linear regresi, kaedah kuasa dua terkecil digunakan.

(i).

i	x_i	y_i	x_i^2	$x_i y_i$
1	63	127		
2	64	121		
3	66	142		
4	69	157		
5	69	162		
6	71	156		
7	71	169		
8	72	165		
9	73	181		
10	75	208		
$\sum$				

dengan $a_0 = \bar{y} - a_1\bar{x}$ dan $a_1 = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$

Dengan menggunakan analisis linear regresi, (Tulis semula jadual bersama jawapan di dalam buku jawapan),

(ii). Cari purata ketinggian $\bar{x}$ and purata berat $\bar{y}$ bagi 10 pelajar.

(iii). Cari persamaan bagi garis $w = a_0 + a_1 h$ menggunakan kaedah kuasa dua terkecil.

(iv). Ramalkan berat bagi seorang pelajar yang mempunyai tinggi 177.8 cm. (Petunjuk : 1 inci = 2.54 cm).

(v). Cari jumlah Ralat Kuasa Dua, $\sum_{i=1}^{n} e_i^2$ dengan $e = y_i - \tilde{y}_i$ untuk regresi linear ini.

i	x_i	y_i	$\tilde{y}_i = a_0 + a_1 x_i$	$y_i - \tilde{y}_i$	e_i^2
1	63	127			
2	64	121			
3	66	142			
4	69	157			
5	69	162			
6	71	156			
7	71	169			
8	72	165			
9	73	181			
10	75	208			
				$\sum e_i^2$	

Bandingkan keputusan anda dengan jawapan di (b). dan buat kesimpulan anda berdasarkan pemerhatian ini.

(45 markah)

4. (a). Anggarkan nilai terbitan $f(x) = \ln x$ pada $x_0 = 1.8$ menggunakan $h = 0.05$ dan

   (i). Formula Pembezaan Ke Hadapan.

   (ii). Formula Pembezaan Pusat.

   (iii). Formula Tiga Titik Hujung.

(20 markah)

(b).

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	0.2955	β	0.4794	0.5646	0.6442

(i). Diberi $f'(0.3) = 0.8774$ dengan menggunakan formula lima-titik tengah, cari nilai β.

(ii). Dengan menggunakan terbitan kedua formula titik tengah dan nilai β yang diperolehi, cari selang saiz h yang harus digunakan untuk memenuhi $f''(0.3) = -0.48$.

(iii). Cari luas $\int_{0.1}^{0.5} f(x)\, dx$ kepada data yang diberi menggunakan peraturan Trapezoid $(n=1)$ dan peraturan Simpson $(n=2)$. Guna nilai selang saiz h yang bersesuaian bagi setiap peraturan tersebut.

(50 markah)

(c). Anggarkan $\int_0^1 x^2 \ln\left(x^2+1\right) dx$ menggunakan $h = 0.25$. Guna

(i). peraturan gubahan Trapezoid.

(ii). peraturan gubahan Simpson.

(30 markah)

5. (a). Diberi

$$x - y + 3z = 2$$
$$3x - 3y + z = -1$$
$$x + y = 3$$

(i). Selesaikan sistem persamaan yang diberi menggunakan kaedah penghapusan Gaussian.

(ii). Tentukan sama ada pertukaran baris perlu atau tidak?

(iii). Kenapa strategi gerakan dalam kaedah Penghapusan Gaussian penting dalam menyelesaikan sistem persamaan linear?

(15 markah)

(b). Sistem linear $AX = B$ diwakili sebagai

$$\begin{pmatrix} 3 & -1 & -1 \\ \lambda & 8 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.8 \\ -1.1 \\ 0.1 \end{pmatrix}$$

(i). Tentukan integer maksimum λ yang perlu digunakan dalam matrik A untuk menjadi dominan pepenjuru ketat $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|,\ i = 1,2,...,n$.

(ii). Selesaikan sistem linear di atas menggunakan kaedah lelaran Jacobi dan kaedah lelaran Gauss-Seidel, bermula dengan $x^{(0)} = (0,0,0)^T$ dan lakukan lelaran sehingga $\mathbf{x}_i^{(3)}$.

(65 markah)

(c). Diberi $\mathbf{F}(\mathbf{x})$ ditakrifkan sebagai

$$f_1(x,y) = x^2 - 2x - y + 0.5 = 0,$$
$$f_2(x,y) = x^2 + 4y^2 - 4 = 0,$$

dengan $\mathbf{F}(\mathbf{x}) = (f_1, f_2)^T$ dan $\mathbf{x} = (x,y)^T$. Cari matrik Jacobi $J(\mathbf{x})$ dan matrix songsangannya $J^{-1}(\mathbf{x})$ untuk setiap nilai $\mathbf{x}$.

(20 markah)

List of formula:

Secant Method: $x_{i+1} = x_i - \dfrac{(x_i - x_{i-1})\, f(x_i)}{f(x_i) - f(x_{i-1})}$

Newton's method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

Fixed point method: $x_{i+1} = g(x_i)$

Lagrance Interpolating Polynomial: $P_n(x) = \sum_{i=0}^{n} f(x_i) L_{n,i}(x)$

where

$$L_{n,i} = \frac{(x-x_0)(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)(x_i-x_1)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)}$$
$$= \prod_{\substack{k=0 \\ k\neq i}}^{n} \frac{(x-x_k)}{(x_i-x_k)}$$

Newton's Divided Difference: $P_n(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + \ldots$
$+ f[x_0,x_1,\ldots,x_n](x-x_0)(x-x_1)\ldots(x-x_{n-1})$

where

$$f[x_0,x_1,\ldots,x_n] = \frac{f[x_1,x_2,\ldots,x_n] - f[x_0,x_1,\ldots,x_{n-1}]}{x_n - x_0}$$

Least Square Method (linear): $y = a_0 + a_1 x$

where

$$a_0 = y - a_1 x \quad \text{and} \quad a_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

Least Square Method (polynomial): $y = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$

Gauss-Seidel Method: $x_i^{(k)} = \dfrac{1}{a_{ii}}\left(-\sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k-1)} + b_i\right)$, for $i = 1, 2, \ldots, n$

Jacobi Iterative Method: $x_i^{(k)} = \sum_{\substack{j=1 \\ j\neq i}}^{n}\left(-\dfrac{a_{ij} x_j^{(k-1)}}{a_{ii}}\right) + \dfrac{b_i}{a_{ii}}$, for $i = 1, 2, \ldots, n$

Forward Difference : $f'(x_0) \approx \dfrac{f(x_0+h) - f(x_0)}{h}$

Backward Difference: $f'(x_0) \approx \dfrac{f(x_0) - f(x_0 - h)}{h}$

Center Difference: $f'(x_0) \approx \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$

Three-point Midpoint : $f'(x_0) \approx \dfrac{1}{2h}\left[f(x_0 + h) - f(x_0 - h) \right]$

Three-point Endpoint : $f'(x_0) \approx \dfrac{1}{2h}\left[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h) \right]$

Five-Point Endpoint: $f'(x_0) \approx \dfrac{1}{12h}\left[-25f(x_0) + 48f(x_0 + h) - 36f(x_0 + 2h) + 16f(x_0 + 3h) - 3f(x_0 + 4h) \right]$

Five-Point Midpoint: $f'(x_0) \approx \dfrac{1}{12h}\left[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h) \right]$

Second Derivative Midpoint: $f''(x_0) \approx \dfrac{1}{h^2}\left[f(x_0) - 2f(x_0 + h) + f(x_0 + 2h) \right]$

Trapezoidal rule: $\displaystyle\int_{x_0}^{x_1} f(x)\, dx = \frac{h}{2}\left[f(x_0) + f(x_1) \right]$

Simpson's rule: $\displaystyle\int_{x_0}^{x_2} f(x)\, dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2) \right]$

Simpson's $\dfrac{3}{8}$ rule : $\displaystyle\int_{x_0}^{x_3} f(x)\, dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right]$

Composite Trapezoidal Rule: $\displaystyle\int_{x_0}^{x_n} f(x)\, dx = \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n) \right]$

Composite Simpson's Rule: $\displaystyle\int_{x_0}^{x_n} f(x)\, dx = \frac{h}{3}\left[f(x_0) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + f(x_n) \right]$

Newton's Method: $\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} + \mathbf{y}^{(k-1)}$ for $k \geq 1$, $\mathbf{y}^{(k-1)} = -J\left(\mathbf{x}^{(k-1)}\right)^{-1} \mathbf{F}\left(\mathbf{x}^{(k-1)}\right)$

where $$J(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1}(\mathbf{x}) & \dfrac{\partial f_1}{\partial x_2}(\mathbf{x}) & \cdots & \dfrac{\partial f_1}{\partial x_n}(\mathbf{x}) \\ \dfrac{\partial f_2}{\partial x_1}(\mathbf{x}) & \dfrac{\partial f_2}{\partial x_2}(\mathbf{x}) & \cdots & \dfrac{\partial f_2}{\partial x_n}(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial f_n}{\partial x_1}(\mathbf{x}) & \dfrac{\partial f_n}{\partial x_2}(\mathbf{x}) & \cdots & \dfrac{\partial f_n}{\partial x_n}(\mathbf{x}) \end{bmatrix}$$

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