Final Examination
2017/2018 Academic Session

May/June 2018

JIM213 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **TEN** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEPULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

1. (a). Find a solution to the given second-order initial value problem

$$y'' - 7y' + 12y = 0 \quad , y(0) = 1 , \quad y'(0) = 12.$$

The general solution for this differential equation is governed by $y = c_1 e^{4x} + c_2 e^{3x}$.

(30 marks)

(b). (i). Find the general solution of following separable equation

$$y' = e^{5x-2y}.$$

(ii). Hence, find the particular solution of 1. (b). (i). if $y(0) = 0$.

(20 marks)

(c). Show that the integrating factor of

$$\left(x^2 + 1\right) y' + 2xy = x^2 + 1 - y$$

is $\mu = \alpha e^{\tan^{-1}\sqrt{\alpha - 1}}$ where $\alpha = x^2 + 1$.

(20 marks)

(d). Determine whether the following equation

$$\left(e^x \cos y + 2\cos x\right) dy + \left(e^x \sin y - 2y \sin x\right) dx = 0$$

is exact or not. If it is exact, find the solution.

(30 marks)

2. (a). Solve the following linear differential equation

$$\left(t \ln t\right) \frac{dw}{dt} + w = te^{3t}.$$

(20 marks)

(b). Solve the Bernoulli equation $\frac{dy}{dx} + y = y^3$ by using an appropriate substitution.

(30 marks)

(c). (i). Twenty minutes after being served a cup of coffee, it is still too hot to drink at $160^0 F$. Two minutes later, the temperature has dropped to $158^0 F$. Find the temperature of coffee when it is served if the room temperature is $69^0 F$. [Hint: Use Newton's Law of Cooling, $\frac{dT}{dt} = k\left(T - T_m\right)$].

(ii). A 250-volt electromotive force is applied to an RC series circuit where the resistance is 1100 ohms and the capacitance is 6×10^{-6} farad. Find the charge $q(t)$ on capacitor if the initial charge, $q(0)=0$. Find the current $I(t)$.

[Hint: Use Kirchhoff's Second Law, $RI + \frac{1}{C}q = E\left(t\right)$].

(50 marks)

3. (a). (i). Solve the following homogeneous second order of linear differential equation

$$y'' - 5y' + 6y = 0 .$$

(ii). Hence, solve the following non-homogeneous second order of linear differential equation

$$y'' - 5y' + 6y = x + 4 + 2e^{3x}$$

by using Superposition Approach.

(50 marks)

(b). Solve the Euler differential equation

$$x^2 \frac{d^2 y}{dx^2} - 4x\frac{dy}{dx} + 6y = g(x) .$$

if

(i). $g(x) = 0$

(ii). $g(x) = x^6$

Hence, solve the initial value problem of $x^2 \frac{d^2 y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^6$ if the initial conditions are $y(1) = \frac{13}{12}$ and $y'(1) = \frac{13}{2}$.

(50 marks)

4. (a). (i). Use Table 1 to find the Laplace transform of $f(t)$

$$f(t) = \left(\frac{e^{-t}}{2} - e^{t} \right)^2$$

(ii) . Evaluate the inverse Laplace transform of $F(s)$ where

$$F(s) = \frac{3}{s^2} - \frac{\sqrt{2}}{s+3} + \frac{s}{4\left(s^2 - 25\right)} .$$

(60 marks)

(b). Show that the solution of

$$y'' + 4y' + 4y = te^{-2t} \quad , \; y(0) = 1 \, , \; y'(0) = 1$$

is

$$Y(s) = \frac{1}{(s+2)^4} + \frac{s+5}{(s+2)^2} .$$

(40 marks)

5. Given a system of homogenous linear differential equations

$$\frac{dx}{dt} - 3x - 2y - 4z = 0$$
$$\frac{dy}{dt} - 2x - 2z = 0$$
$$\frac{dz}{dt} - 4x - 2y - 3z = 0$$

(a). Write the system of equations in the form

$$\frac{dX}{dt} = AX,$$

where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and identify the matrix A.

(15 marks)

(b). Show that

$$V_1 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

is an eigenvector of the matrix A found in (a). State the corresponding eigenvalue.

(35 marks)

(c). Find two other eigenvectors and state the corresponding eigenvalues.

(30 marks)

(d). Hence write down the general solution of the system as given in (a).

(20 marks)

1. (a). Cari penyelesaian untuk masalah nilai awal peringkat kedua

$$y'' - 7y' + 12y = 0 \quad , y(0) = 1 , \quad y'(0) = 12.$$

Penyelesaian umum untuk persamaan pembezaan ini adalah $y = c_1 e^{4x} + c_2 e^{3x}$.

(30 markah)

(b). (i). Cari penyelesaian umum bagi persamaan yang boleh dipisahkan

$$y' = e^{5x-2y}.$$

(ii). Kemudian, cari penyelesaian khusus bagi 1. (b). (i). jika $y(0) = 0.$

(20 markah)

(c). Sahkan bahawa faktor pengamiran bagi

$$\left(x^2 + 1\right) y' + 2xy = x^2 + 1 - y$$

adalah $\mu = \alpha e^{\tan^{-1}\sqrt{\alpha - 1}}$ dengan $\alpha = x^2 + 1$.

(20 markah)

(d). Tentukan bahawa persamaan berikut adalah

$$\left(e^x \cos y + 2\cos x\right) dy + \left(e^x \sin y - 2y \sin x\right) dx = 0$$

adalah tepat atau tidak. Jika ia adalah persamaan tepat, cari penyelesaiannya.

(30 markah)

2. (a). Selesaikan persamaan pembezaan linear berikut

$$\left(t \ln t\right) \frac{dw}{dt} + w = te^{3t}.$$

(20 markah)

(b). Selesaikan persamaan Bernoulli $\frac{dy}{dx} + y = y^3$ dengan menggunakan penggantian yang sesuai.

(30 markah)

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(c). (i). Dua puluh minit selepas satu cawan kopi disediakan, ianya masih lagi panas pada $160^0 F$. Dua minit kemudian suhu menurun kepada $158^0 F$. Cari suhu kopi pada waktu ianya mula disediakan jika suhu persekitaran adalah $69^0 F$. [Petunjuk: Guna Hukum Newton terhadap penyejukan, $\frac{dT}{dt} = k\left(T - T_m\right)$].

(ii). Daya elektrik 250-volt diaplikasikan dalam satu litar bersiri RC dengan rintangan adalah 1100 ohms dan kapasitans adalah 6×10^{-6} farad. Cari caj $q(t)$ atas kapasitor jika nilai mula caj $q(0) = 0$. Cari arus $I(t)$. [Petunjuk: Guna Hukum Kedua Kirchhoff, $RI + \frac{1}{C} q = E\left(t\right)$].

(50 markah)

3. (a). (i). Selesaikan persamaan pembezaan linear peringkat kedua homogen

$$y'' - 5y' + 6y = 0 .$$

(ii). Kemudian, selesaikan persamaan pembezaan linear peringkat kedua tidak homogen

$$y'' - 5y' + 6y = x + 4 + 2e^{3x}$$

menggunakan Pendekatan Super Posisi.

(50 markah)

(b). Selesaikan persamaan pembezaan Euler

$$x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = g(x).$$

jika

(i). $g(x) = 0$

(ii). $g(x) = x^6$

Kemudian, selesaikan masalah nilai awal, $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^6$

jika syarat awal adalah $y(1) = \frac{13}{12}$ dan $y'(1) = \frac{13}{2}$.

(50 markah)

4. (a). (i). Guna Jadual 1 untuk mencari Jelmaan Laplace $f(t)$

$$f(t) = \left(\frac{e^{-t}}{2} - e^t\right)^2$$

(ii). Nilaikan Jelmaan Laplace songsang $F(s)$ dengan

$$F(s) = \frac{3}{s^2} - \frac{\sqrt{2}}{s+3} + \frac{s}{4\left(s^2 - 25\right)}.$$

(60 markah)

(b). Tunjukkan penyelesaian bagi

$$y'' + 4y' + 4y = te^{-2t} \quad , \; y(0) = 1, \; y'(0) = 1$$

adalah

$$Y(s) = \frac{1}{(s+2)^4} + \frac{s+5}{(s+2)^2}.$$

(40 markah)

5. Diberi satu sistem persamaan pembezaan linear homogen

$$\frac{dx}{dt} - 3x - 2y - 4z = 0$$
$$\frac{dy}{dt} - 2x - 2z = 0$$
$$\frac{dz}{dt} - 4x - 2y - 3z = 0$$

(a). Tulis sistem persamaan dalam bentuk

$$\frac{dX}{dt} = AX,$$

dengan $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ dan kenalpasti matrik A.

(15 markah)

(b). Tunjukkan bahawa

$$V_1 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

adalah vektor eigen bagi matrik A di (a). Nyatakan nilai eigen yang berkaitan.

(35 markah)

(c). Cari dua lagi vektor eigen dan nyatakan nilai eigen yang sepadan.

(30 markah)

(d). Kemudian tuliskan penyelesaian am bagi sistem yang diberi dalam (a).

(20 markah)

Table 1/Jadual 1
Elementary Laplace Transforms

$f(t) = L^{-1}\{F(s)\}$	$F(s) = L\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
14. $e^{ct} f(t)$	$f(s-c)$
15. $f'(t)$	$sF(s) - f(0)$
16. $f''(t)$	$s^2 F(s) - sf(0) - f'(0)$

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