Final Examination
2017/2018 Academic Session

May/June 2018

JIM211 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **EIGHT** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggerís.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

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1. (a). Given

$$2x + u^2 - v^2 = 0$$
$$y - uv = 0$$

Find $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$.

(35 marks)

(b). Find the local extreme values and saddle points of the function $f(x, y)$, if there is any, given that

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 - 6xy .$$

(35 marks)

(c). Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

(30 marks)

2. (a). Evaluate

(i). $\int_{-1}^{1}\int_{0}^{2}\left(1 - 6x^2 y\right) dxdy$,

(ii). $\iint_{R}\left(2x + 3y\right)\, dydx$,

over the region R enclosed by the curves $x = y^2$ and $y = x^2$.

(60 marks)

(b). Evaluate $\int_{0}^{1}\int_{x}^{1}\int_{0}^{y-x} dzdydx.$

(20 marks)

(c). Evaluate $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2 dzdydx$ using cylindrical coordinates.

(20 marks)

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3. (a). State the definition for each of the following:

(i). decreasing sequence,

(ii). convergence of a sequence,

(iii). Cauchy sequence.

(30 marks)

(b). Prove that the following sequence

(i). $a_n = \frac{2n+1}{3n-1}$ is decreasing,

(ii). $b_n = \frac{2n-1}{3n}$ is convergence,

(iii). $c_n = \frac{n}{n+1}$ is Cauchy.

(70 marks)

4. (a). State the Pinching Theorem for sequences. Hence, use the theorem to evaluate

$$\lim_{x\to\infty} \frac{\cos n\pi}{n}.$$

(30 marks)

(b). State L'Hôspital's Rule for the indeterminate form $\frac{0}{0}$ and $\infty \cdot 0$.

Calculate the following limits:

(i). $\lim_{x\to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2}$

(ii). $\lim_{x\to 0^+} \sqrt{x}\ \ln x$

(iii). $\lim_{x\to\infty} \left(x \sin \frac{1}{x} \right)$

(70 marks)

5. (a). Determine whether each of the following series converges or diverges:

(i). $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

(ii). $\sum_{n=1}^{\infty} \frac{1}{n+1}$

(iii). $\sum_{n=1}^{\infty} \frac{5}{5n-1}$

(iv). $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(60 marks)

(b). Find the Taylor's polynomial of degree 3 about $x = 0$ and the remainder for the function f given by

$$f(x) = x \sin x .$$

(40 marks)

1. (a). Diberi

$$2x + u^2 - v^2 = 0$$
$$y - uv = 0$$

Cari $\frac{\partial u}{\partial x}$ dan $\frac{\partial v}{\partial x}$.

(35 markah)

(b). Cari nilai ekstremum tempatan dan titik pelana bagi fungsi $f(x, y)$ jika ada, diberi bahawa

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 - 6xy .$$

(35 markah)

(c). Guna kaedah pendarab Lagrange untuk mencari nilai maksimum dan minimum bagi fungsi $f(x, y) = 3x + 4y$ pada bulatan $x^2 + y^2 = 1$.

(30 markah)

2. (a). Nilaikan

(i). $\int_{-1}^{1}\int_{0}^{2}\left(1 - 6x^2 y\right) dxdy$,

(ii). $\iint_{R}\left(2x + 3y\right) dydx$,

bagi rantau R yang dibatasi oleh lengkung $x = y^2$ dan $y = x^2$.

(60 markah)

(b). Nilaikan $\int_{0}^{1}\int_{x}^{1}\int_{0}^{y-x} dzdydx.$

(20 markah)

(c). Nilaikan $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2\, dzdydx$ menggunakan koordinat silinder.

(20 markah)

3. (a). Nyatakan definisi bagi setiap yang berikut:

(i). jujukan menyusut,

(ii). jujukan menumpu,

(iii). jujukan Cauchy.

(30 markah)

(b). Buktikan jujukan berikut:

(i). $a_n = \dfrac{2n+1}{3n-1}$ adalah menyusut,

(ii). $b_n = \dfrac{2n-1}{3n}$ adalah menumpu,

(iii). $c_n = \dfrac{n}{n+1}$ adalah Cauchy.

(70 markah)

4. (a). Nyatakan Teorem Menyepit bagi jujukan. Justeru itu, guna teorem untuk mencari nilai

$$\lim_{x\to\infty} \frac{\cos n\pi}{n}.$$

(30 markah)

(b). Nyatakan petua L'Hôspital bagi bentuk tak tentu $\dfrac{0}{0}$ dan $\infty \cdot 0$. Kira had yang berikut:

(i). $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2}$

(ii). $\displaystyle\lim_{x\to 0^+} \sqrt{x}\ln x$

(iii). $\displaystyle\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right)$

(70 markah)

5. (a). Tentukan sama ada setiap siri berikut menumpu atau mencapah:

(i). $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

(ii). $\sum_{n=1}^{\infty} \frac{1}{n+1}$

(iii). $\sum_{n=1}^{\infty} \frac{5}{5n-1}$

(iv). $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(60 markah)

(b). Cari polinomial Taylor darjah 3 sekitar $x = 0$ dan baki bagi fungsi f yang diberi oleh

$$f(x) = x \sin x .$$

(40 markah)

APPENDIX

List of formula:

1. $A=\frac{\partial^2 f}{\partial x^2}(x_0,y_0)$, $B=\frac{\partial^2 f}{\partial y\partial x}(x_0,y_0)$, $C=\frac{\partial^2 f}{\partial y^2}(x_0,y_0)$, $D=AC-B^2$

2. $\nabla f(x,y)=\lambda\nabla g(x,y)$

3. $x=r\cos\theta$, $y=r\sin\theta$, $z=z$

4. $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots+\frac{f^{(n)}}{n!}(a)(x-a)^n+R_n(x)$

   with $R_n(x)=\frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n\,dt$

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