Final Examination
2017/2018 Academic Session

May/June 2018

JIM201 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

(a). Calculate

(i). $|A|$

(ii). $\operatorname{adj}(A)$

(iii). A^{-1}

(iv). $|\operatorname{adj}(A)|$

(v). reduced row-echelon form of A

(vi). rank of A

(vii). $|A^T|$

(viii). $|A^{-1}|$

(ix). Solution of X if $AX = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(70 marks)

(b). Given that matrix $B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ such that $B^2 + pB + qI = 0$ where I is an identity matrix 2×2 and 0 is a zero matrix 2×2. Determine the values of p and q.

(30 marks)

...3/-

2. (a). Given the system of linear equations

$$x + y + z = 0$$
$$2x + y + 4z = 1$$
$$5x + 7y - 8z = 6$$

Solve the system using the method of

(i). Cramer's rule.

(ii). Gauss-Jordan elimination.

(60 marks)

(b). Determine the values of constant a if the system of linear equations

$$x + y - z = 1$$
$$2x + 3y + az = 3$$
$$x + ay + 3z = 2$$

has

(i). a unique solution,

(ii). infinitely many solutions,

(iii). no solution.

(40 marks)

3. (a). State the conditions that a subset W of a vector space V is a subspace of V.

(20 marks)

(b). Show that each of the following vector is a subspace

(i). $S = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} \middle| y \in R \right\} \in R^2$.

(ii). $S = \left\{ \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \middle| x \in R \right\} \in M_{2\times 2}$.

(iii). $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| x + y = z, \quad x, y, z \in R \right\} \in R^3$.

(40 marks)

(c). Give a counter example for each of the following vector which is not a subspace.

(i). $T = \left\{ \begin{pmatrix} 1 \\ y \end{pmatrix} \middle| y \in R \right\}$ of R^2.

(ii). $T = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| y \in R, y > 0 \right\}$ of R^2.

(40 marks)

4. (a). Let A be diagonalizable. Then there exist two matrices P and D such that

$$P^{-1}AP = D.$$

Show that

$$A^3 = PD^3P^{-1}.$$

(30 marks)

(b). Given the matrix

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

Find

(i). the characteristic polynomial,

(ii). eigenvalues,

(iii). eigenvectors,

(iv). the matrix P that diagonalize A.

(70 marks)

5. (a). State the definition for each of the following:

   (i). orthogonal basis.

   (ii). orthonormal basis.

(20 marks)

(b). Prove that set $S = \{V_1, V_2, V_3\}$ is an orthogonal basis for R^3, where

$$V_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \qquad V_2 = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}, \qquad V_3 = \begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$$

(40 marks)

(c). Use Gram-Schmidt process to find an orthonormal basis from the set

$$S = \{(1,1,1), (0,1,1), (0,0,1)\} \subseteq R^3.$$

(40 marks)

1. Pertimbangkan matrik

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

(a). Hitung

(i). $|A|$

(ii). $\text{adj}(A)$

(iii). A^{-1}

(iv). $|\text{adj}(A)|$

(v). bentuk echelon baris terturun A

(vi). pangkat A

(vii). $|A^T|$

(viii). $|A^{-1}|$

(ix). penyelesaian bagi X jika $AX = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(70 markah)

(b). Diberi matrik $B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ bahawa $B^2 + pB + qI = 0$ dengan I adalah matrik identiti 2×2 dan 0 adalah matrik sifar 2×2. Tentukan nilai p dan q.

(30 markah)

2. (a). Diberi sistem persamaan linear

$$x + y + z = 0$$
$$2x + y + 4z = 1$$
$$5x + 7y - 8z = 6$$

Selesaikan sistem menggunakan kaedah

(i). petua Cramer.

(ii). penghapusan Gauss-Jordan.

(60 markah)

(b). Tentukan nilai pemalar a jika sistem persamaan linear

$$x + y - z = 1$$
$$2x + 3y + az = 3$$
$$x + ay + 3z = 2$$

mempunyai

(i). penyelesaian unik,

(ii). penyelesaian tidak terhingga banyaknya,

(iii). tiada penyelesaian.

(40 markah)

3. (a). Nyatakan keadaan bahawa subset W bagi suatu ruang vektor V adalah suatu subruang bagi V.

(20 markah)

(b). Tunjukkan bahawa setiap vektor berikut adalah suatu subruang

(i). $S = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} \middle| y \in R \right\} \in R^2.$

(ii). $S = \left\{ \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \middle| x \in R \right\} \in M_{2\times 2}.$

(iii). $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| x + y = z, \quad x, y, z \in R \right\} \in R^3.$

(40 markah)

(c). Beri satu contoh lawan bagi setiap vektor berikut yang bukan suatu subruang.

(i). $T = \left\{ \begin{pmatrix} 1 \\ y \end{pmatrix} \middle| y \in R \right\} \in R^2$.

(ii). $T = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| y \in R \, , y > 0 \right\} \in R^2$.

(40 markah)

4. (a). Katakan A adalah terpepenjuru. Maka wujud dua matrik P dan D supaya

$$P^{-1}AP = D .$$

Tunjukkan bahawa

$$A^3 = PD^3P^{-1} .$$

(30 markah)

(b). Diberi matrik

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

Cari

(i). polinomial cirian,

(ii). nilai eigen,

(iii). vektor eigen,

(iv). matrik P yang pepenjurukan A.

(70 markah)

5. (a). Nyatakan definisi bagi setiap yang berikut:

   (i). asas orthogon.

   (ii). asas orthonormal.

(20 markah)

(b). Buktikan bahawa $S = \{V_1, V_2, V_3\}$ adalah asas orthogon bagi R^3, dengan

$$V_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \qquad V_2 = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}, \qquad V_3 = \begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$$

(40 markah)

(c). Guna proses Gram-Schmidt untuk mencari satu asas orthonormal daripada set

$$S = \{(1,1,1), (0,1,1), (0,0,1)\} \subseteq R^3.$$

(40 markah)

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