Final Examination
2017/2018 Academic Session

May/June 2018

JIM105 – Basic Mathematics
[Matematik Asas]

Duration: 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1.   (a).   Matrix A is given as $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -3 \\ 2 & 2 & -1 \end{bmatrix}$.

         Find

         (i).   the cofactor matrix of A.

         (ii).   the inverse matrix of A.

                                                                  [50 marks]

       (b).   Consider the system of linear equations

$$x_1 - 2x_2 + 3x_3 - 1 = 0$$
$$x_1 + mx_2 + 2x_3 = 2$$
$$-2x_1 + m^2 x_2 - 4x_3 + 4 = 3m$$

         where m is a constant.

         (i).   Write the above system of linear equations in augmented matrix, $\left[A \mid B \right]$.

         (ii).   By using elementary row operations, show that the above augmented matrix can be reduced to $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & m+2 & -1 & 1 \\ 0 & 0 & m & 2m \end{bmatrix}$.

             Hence, solve the above system of linear equations for $m = 1$.

                                                                 [50 marks]

2.   (a).   Find the equations of the tangent to the circle $y^2 + x^2 - 25 = 0$ at the point $(3, -4)$.

                                                               [20 marks]

       (b).   (i).   If $y = \sin(x^2 + 1)$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$.

         (ii).   Find the gradient of a curve $xe^{xy} = e^{2x} - e^{3y}$ at $(0, 0)$.

                                                                [40 marks]

       (c).   Let $x^2 y^2 + 2xy + 4y = 4$. Determine the value of $\dfrac{d^2 y}{dx^2}$ at the point $(2, 2)$.

                                                                [40 marks]

3. (a). A function f is defined by

$$f(x) = \begin{cases} 34, & x = -4 \\ 0, & x = 2 \\ 17, & x = 4 \\ \dfrac{x^4 - 3x^2 - 4}{x^2 + x - 6}, & x \neq -4, x \neq -3, x \neq -2, x \neq 4 \end{cases}$$

Evaluate $\lim\limits_{x \to 2} f(x)$.

[40 marks]

(b). Find the following limits :

(i). $\lim\limits_{x \to \infty} \dfrac{2x^2 + x - 4}{1 - x^2}$.

(ii). $\lim\limits_{x \to 2} \dfrac{3 - \sqrt{x + 7}}{x^2 - 4}$.

(iii). $\lim\limits_{x \to 0} \dfrac{e^{2x} - e^{-2x}}{e^x - e^{-x}}$.

(iv). $\lim\limits_{x \to 0} \dfrac{x^2 - 4}{|x - 2|}$.

[60 marks]

4.   (a).   Find the derivative for $f(x) = 3\sqrt{x}$ using the first principle.

[30 marks]

(b).   If $y = x^{2x}$, find $\dfrac{dy}{dx}$.

[30 marks]

(c).   Given a piecewise function f as

$$f(x) = \begin{cases} x^2 + 5, & x < 2 \\ \sqrt{x-2}, & x \geq 2 \end{cases}$$

Evaluate $\displaystyle\int_{-1}^{6} f(x)\,dx$.

[40 marks]

5.   (a).   Find the area bounded by the curve $y = \sin x$, $y = \cos x$, $x = \dfrac{\pi}{2}$ and the y-axis.

[50 marks]

(b).   Given the curve $y = x^2 - 6x + 8$.

(i).   Find the coordinates of the critical points.

(ii).   Determine the minimum and maximum points.

[50 marks]

1.　(a).　Matriks A diberi sebagai $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -3 \\ 2 & 2 & -1 \end{bmatrix}$.

　　　Cari

　　　(i).　kofaktor bagi matriks A.

　　　(ii).　songsangan bagi matriks A.

[50 markah]

　　(b).　Pertimbangkan sistem persamaan linear

$$x_1 - 2x_2 + 3x_3 - 1 = 0$$
$$x_1 + mx_2 + 2x_3 = 2$$
$$-2x_1 + m^2 x_2 - 4x_3 + 4 = 3m$$

　　　di mana m adalah pemalar.

　　　(i).　Tulis sistem persamaan linear di atas dalam matriks imbuhan $\begin{bmatrix} A|B \end{bmatrix}$.

　　　(ii).　Dengan menggunakan operasi baris permulaan, tunjukkan bahawa matriks imbuhan di atas boleh diturunkan kepada $\begin{bmatrix} 1 & -2 & 3 & | & 1 \\ 0 & m+2 & -1 & | & 1 \\ 0 & 0 & m & | & 2m \end{bmatrix}$.

　　　Seterusnya, selesaikan sistem persamaan linear di atas bagi $m = 1$.

[50 markah]

2.   (a).   Cari persamaan tangen kepada bulatan $y^2 + x^2 - 25 = 0$ pada titik $(3, -4)$.

[20 markah]

(b).   (i).   Jika $y = \sin(x^2 + 1)$, cari $\dfrac{dy}{dx}$ dan $\dfrac{d^2 y}{dx^2}$.

(ii).   Cari kecerunan bagi lengkung $xe^{xy} = e^{2x} - e^{3y}$ pada $(0,0)$.

[40 markah]

(c).   Biar $x^2 y^2 + 2xy + 4y = 4$. Tentukan nilai bagi $\dfrac{d^2 y}{dx^2}$ pada titik $(2,2)$.

[40 markah]

3.   (a).   Fungsi f ditakrifkan oleh

$$f(x) = \begin{cases} 34, & x = -4 \\ 0, & x = 2 \\ 17, & x = 4 \\ \dfrac{x^4 - 3x^2 - 4}{x^2 + x - 6}, & x \neq -4, x \neq -3, x \neq -2, x \neq 4 \end{cases}$$

Nilaikan $\displaystyle\text{had}_{x \to 2} f(x)$.

[40 markah]

(b).   Cari had berikut :

(i). $\displaystyle\text{had}_{x \to \infty} \dfrac{2x^2 + x - 4}{1 - x^2}$.

(ii). $\displaystyle\text{had}_{x \to 2} \dfrac{3 - \sqrt{x + 7}}{x^2 - 4}$.

(iii). $\displaystyle\text{had}_{x \to 0} \dfrac{e^{2x} - e^{-2x}}{e^x - e^{-x}}$.

.   (iv). $\displaystyle\text{had}_{x \to 0} \dfrac{x^2 - 4}{|x - 2|}$.

[60 markah]

4. (a). Cari pembezaan bagi $f(x) = 3\sqrt{x}$ menggunakan prinsip pertama.

[30 markah]

(b). Jika $y = x^{2x}$, cari $\dfrac{dy}{dx}$.

[30 markah]

(c). Diberi fungsi cebis demi cebis f sebagai

$$f(x) = \begin{cases} x^2 + 5, & x < 2 \\ \sqrt{x-2}, & x \geq 2 \end{cases}$$

Nilaikan $\displaystyle\int_{-1}^{6} f(x)\,dx$.

[40 markah]

5. (a). Cari luas yang dibatasi oleh lengkung $y = \sin x$, $y = \cos x$, $x = \dfrac{\pi}{2}$ dan paksi-y.

[50 markah]

(b). Diberi lengkung $y = x^2 - 6x + 8$.

(i).   Cari koordinat bagi titik kritikal.

(ii). Tentukan titik minimum dan maksimum.

[50 markah]

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