Final Examination
2017/2018 Academic Session

May/June 2018

JIF318 – Quantum Mechanics
[Mekanik Kuantum]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **ELEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in English or in Bahasa Malaysia.

Read the instructions carefully before answering.

In the event of any discrepancies in the exam questions, the English version shall be used.

Sila pastikan kertas peperiksaan ini mengandungi ***SEBELAS*** *muka surat yang bercetak sebelum anda menjawab sebarang soalan.*

Jawab ***SEMUA*** *soalan. Anda dibenarkan menjawab soalan* ***sama ada*** *dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca setiap arahan dengan teliti sebelum menjawab.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Answer **ALL** questions.
*Jawab **SEMUA** soalan.*

1. (a). Explain the black-body radiation phenomenon. Describe why classical physics was unable to explain the results of the black-body radiation and how quantum concepts were able to do so.
Jelaskan fenomena sinaran jasad hitam. Perihalkan mengapa fizik klasik tidak dapat menjelaskan keputusan-keputusan sinaran jasad hitam dan bagaimana konsep kuantum dapat menjelaskannya.

(11 marks/*markah*)

(b). Discuss **TWO (2)** other experiments that show the failure of classical physics and require quantum physics to explain the results.
*Bincangkan **DUA (2)** ujikaji lain yang menunjukkan kegagalan fizik klasik dan memerlukan fizik kuantum untuk menjelaskan keputusan-keputusannya.*

(9 marks/*markah*)

2. (a). Show that the eigenvalues for a Hermitian operator must be real numbers.
Tunjukkan bahawa nilai-nilai eigen bagi suatu operator Hermitian mestilah nombor-nombor nyata.

(8 marks/*markah*)

(b). Are the operator for position, $\hat{X}$, and the operator for momentum along the *x*-axis, $\hat{P}_x$, commutative? Show.
Adakah operator bagi kedudukan, $\hat{X}$, dan operator bagi momentum di sepanjang paksi x, $\hat{P}_x$, berkomut? Tunjukkan.

(7 marks/*markah*)

(c). If $\psi = e^{2x}$ is the eigenfunction for the operator $\frac{d^2}{dx^2}$, determine the corresponding eigenvalue.

Jika $\psi = e^{2x}$ *adalah fungsi eigen bagi operator* $\frac{d^2}{dx^2}$ *, tentukan nilai eigen yang berkaitan.*

(5 marks/*markah*)

3. (a). Given a general one-dimensional wave function

Diberikan suatu fungsi gelombang umum satu dimensi adalah

$$\Psi(\vec{x},t) = Be^{i(\vec{k}.\vec{x}-\omega t)}$$

Show that the operators for energy and momentum are

Tunjukkan bahawa operator-operator bagi tenaga dan momentum adalah

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$
$$\hat{P} = -i\hbar\frac{\partial}{\partial x}$$

(10 marks/*markah*)

(b). The Schrodinger equation is

Persamaan Schrodinger ialah

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

where the symbols and terms used have the usual meaning. Solve the Schrodinger equation for the wave function $\Psi\ (x,\ t)$ of a particle inside a one-dimensional square-well potential of width *a* where the potential V is

dengan simbol-simbol dan sebutan-sebutan yang digunakan mempunyai maksud-maksud yang lazim. Selesaikan persamaan Schrodinger ini bagi fungsi gelombang $\Psi\ (x,\ t)$ suatu zarah di dalam suatu keupayaan telaga persegi satu dimensi lebar a dan keupayaan V adalah

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le a \\ \infty & \text{for } x < 0 \text{ or } x > a \end{cases}$$

(10 marks/*markah*)

4. (a). What is a linear harmonic oscillator?

Apakah pengayun harmonik linear?

(2 marks/*markah*)

(b). What is the main difference between the classical and quantum understanding of the harmonic oscillator regarding the lowest energy level?

Apakah perbezaan utama antara fahaman klasik dan kuantum tentang paras tenaga terendah pengayun harmonik?

(2 marks/*markah*)

(c). (i). Based on the Uncertainty Principle, show that the ground state of the linear harmonic oscillator is non-zero.

Berdasarkan Prinsip Ketidakpastian, tunjukkan bahawa keadaan asas pengayun osillator linear adalah bukan sifar.

(8 marks/*markah*)

(ii). The first excited state of the linear harmonic oscillator is of the form

Keadaan teruja pertama bagi pengayun osilator linear adalah dalam bentuk

$$\varphi_{(x)} = cxe^{-\alpha x^2}$$

Determine the value of α and the corresponding energy for this state.

Tentukan nilai α dan tenaga bagi keadaan tersebut.

(8 marks/*markah*)

5. (a). What is quantum tunneling?

Apakah penerowongan kuantum?

(10 marks/*markah*)

(b). A 30 eV electron is incident on a square barrier of height 40 eV. What is the probability that the electron will tunnel through the barrier if its width is 0.1 nm?

Suatu elektron 30 eV melanggar halangan empat segi dengan ketinggian 40 eV. Apakah kebarangkalian elektron tersebut dapat menerowong melalui halangan tersebut jika lebar 0.1 nm?

(10 marks/*markah*)

Constants:

Speed of light $c = 3.0 \times 10^{8}$ m s^{-1}

Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

Planck constant $h = 6.63 \times 10^{-34}$ J s

Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}

Permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}

Permeability of free space $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

Basic charge $e = 1.6 \times 10^{-19}$ C

Electron rest-mass $m_e = 9.1 \times 10^{-31}$ kg

Proton rest-mass $m_p = 1.6725 \times 10^{-27}$ kg ≡ 1.0072766 u

Neutron rest-mass $m_n = 1.6748 \times 10^{-27}$ kg ≡ 1.0086654 u

Bohr's radius $a = 5.3 \times 10^{-11}$ m

1 eV = 1.6×10^{-19} J

1 u ≡ 931 MeV c^{-2}

1 barn = 10^{-28} m^2

1 fm = 10^{-15} m

1 Ci = 3.7×10^{10} s^{-1}

USEFUL MATHEMATICS IN QUANTUM MECHANICS

Exponential series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

Trigonometric series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Differentiation and integration (Standard forms)

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$ $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+c$ $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}+c$
$\frac{d}{dx}\log x = \frac{1}{x}$ $\frac{d}{dx}\log(ax+b) = \frac{a}{ax+b}$	$\int \frac{dx}{x} = \log x + c$ $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)+c$
$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}e^{mx} = me^{mx}$	$\int e^x dx = e^x + c$ $\int e^{mx} dx = \frac{e^{mx}}{m}+c$
$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\sin mx = m\cos mx$	$\int \cos x\ dx = \sin x + c$ $\int \cos mx\ dx = \frac{\sin mx}{m}+c$
$\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\cos mx = -m\sin mx$	$\int \sin x\ dx = -\cos x + c$ $\int \sin mx\ dx = -\frac{\cos mx}{m}+c$
$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\tan mx = m\sec^2 mx$	$\int \sec^2 x\ dx = \tan x + c$ $\int \sec^2 mx\ dx = \frac{\tan mx}{m}+c$
$\frac{d}{dx}\cot x = -\mathrm{cosec}^2 x$ $\frac{d}{dx}\cot mx = -m\ \mathrm{cosec}^2 mx$	$\int \mathrm{cosec}^2 x\ dx = -\cot x + c$ $\int \mathrm{cosec}^2 mx\ dx = -\frac{\cot mx}{m}+c$
$\frac{d}{dx}\sinh x = \cosh x$	$\int \cosh x\ dx = \sinh x + c$
$\frac{d}{dx}\cosh x = \sinh x$	$\int \sinh x\ dx = \cosh x + c$

Integration by parts

$$\int u\upsilon\,dx = u\int \upsilon\,dx - \int\left\{\int \upsilon\,dx\right\}\frac{du}{dx}\,dx$$

Integration common in Quantum Mechanics

$$f(x) = \int_0^\infty x^n e^{-ax^2}\,dx$$

n	$f(n)$	n	$f(n)$
0	$\frac{1}{2}\sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4}\sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8}\sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16}\sqrt{\frac{\pi}{a^7}}$	7	$\frac{3}{a^4}$

If n is even, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\,dx = 2f(x)$

If n is odd, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\,dx = 0$

Other standard integrals

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$$\int_0^\infty \frac{x}{(e^x - 1)}\,dx = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^3}{(e^x - 1)}\,dx = \frac{\pi^4}{15}$$

Pythagorean identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

Sum & difference formulas

$$\sin(u \pm \upsilon) = \sin u \cos \upsilon \pm \cos u \sin \upsilon$$
$$\cos(u \pm \upsilon) = \cos u \cos \upsilon \mp \sin u \sin \upsilon$$
$$\tan(u \pm \upsilon) = \frac{\tan u \pm \tan \upsilon}{1 \mp \tan u \tan \upsilon}$$

Double angle formulas

$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

Power reducing/half angle formulas

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$
$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$
$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Sum-to-product formulas

$$\sin u + \sin \upsilon = 2\sin\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$

$$\sin u - \sin \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$

$$\cos u + \cos \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$

$$\cos u - \cos \upsilon = -2\sin\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$

Product-to-sum formulas

$$\sin u \sin \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) - \cos(u+\upsilon)\right]$$

$$\cos u \cos \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) + \cos(u+\upsilon)\right]$$

$$\sin u \cos \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) + \sin(u-\upsilon)\right]$$

$$\cos u \sin \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) - \sin(u-\upsilon)\right]$$

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