Final Examination
2017/2018 Academic Session

May/June 2018

JIF315 – Mathematical Methods
[Kaedah Matematik]

Time: 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question carries 100 marks.

In the event of any discrepancies in the exam questions, the English version shall be used.

Sila pastikan bahawa kertas peperiksaan ini mengandungi ***SEMBILAN*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

Jawab ***SEMUA*** *soalan. Anda dibenarkan menjawab soalan* ***sama ada*** *dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Table of Laplace Transform
[Jadual transformasi Laplace]

$f(t)$	$L\{f(t)\}=F(s)$
a	$\frac{a}{s}$
t^n, $n=1,2,3,*$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$sin(at)$	$\frac{a}{s^2+a^2}$
$cos(at)$	$\frac{s}{s^2+a^2}$
$sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$e^{at}\ sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\ cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\left[F(s)\right]$
$e^{at} f(t)$	$F(s-a)$

Legendra Polynomial Function

[Jadual fungsi Legendra Polynomial]

$P_0(x) = 1,$ $P_1(x) = x,$

$P_2(x) = \frac{1}{2}(3x^2 - 1),$ $P_3(x) = \frac{1}{2}(5x^3 - 3x),$

$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3),$ $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

1. (a). Find the Laplace transform of the following function.

$$g(t) = 4\cos(4t) - 15\sin(4t) + 7\cos(10t)$$

(30 marks)

(b). Find the inverse Laplace transform for the following function.

$$F(S) = \frac{30}{s^7} + \frac{8}{s-4} + \frac{4}{s-3}$$

(30 marks)

(c). Solve the following function using the Laplace transform method.

$$y''(t) + 4y'(t) + 4y(t) = t^2 e^{-2t}; \quad y(0) = 0,\ y'(0) = 0$$

(40 marks)

2. (a). Consider the following form of differential equation

$$y'' + \frac{1 - \frac{1}{2}a}{x} y' + \left[b^2 c^2 x^{2c-1} + \frac{a^2 - n^2 c^2}{x^2} \right] y = 0$$

which has a solution

$$y = x^a z_n \left(bx^c \right)$$

where z stands for any linear combination of J_n and Y_n; a, b, c and n are constants. Find the general solutions of the following equations in terms of Bessel functions.

(i). $y'' + \frac{1}{9} xy = 0$

(ii). $y'' - \frac{3}{x} y' + 25xy = 0$

(50 marks)

(b). The Legendre's equation is given as follows

$$\left(1-x^2\right)y''-2xy'+n\left(n+1\right)y=0$$

where n is a constant. Express the following equations in terms of the Legendre's Polynomial:

(i). $2x^3-5x^2+6x+1$

(ii). $-x^3-4x^2-5x+5$

(50 marks)

3. Find all the eigenvalues and eigenfunctions of the following Sturm-Liouville problem. Consider all cases of λ.

$$y''+\lambda y=0,$$
$$y\left(0\right)-y\left(\pi\right)=0,$$
$$y'\left(0\right)-y'\left(\pi\right)=0$$

(100 marks)

4. Consider the following function.

$$f(x)=|x| \text{ on } -\pi<x<\pi$$

(a). Find the Fourier series for $f\left(x\right)$.

(55 marks)

(b). Using the result in (a)., show that

$$\frac{\pi}{2}-\frac{4}{\pi}\left(\cos x+\frac{1}{9}\cos 3x+\frac{1}{25}\cos 5x+...\right)$$

(25 marks)

(c). Sketch the graph of the function $f\left(x\right)$.

(20 marks)

5. (a). Consider the following form of differential equation.

$$f(x) = \begin{cases} -2, & -3 < x < 0 \\ 5e^{x}, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine the Fourier transform of $f(x)$.

(50 marks)

(b). An insulated wire has a length of 1 *m*. Both ends of the wire are embedded in ice (temperature is 0 °C). Let k = 0.003 and the initial heat distribution is

$$u(x,0) = 50x(1-x)$$

Find the temperature for the function $u\ (x,\ t)$.

(50 marks)

1. (a). *Cari tranformasi Laplace bagi fungsi berikut.*

$$g(t) = 4\cos(4t) - 15\sin(4t) + 7\cos(10t)$$

(30 markah)

(b). *Cari tranformasi Laplace songsang bagi fungsi berikut.*

$$F(S) = \frac{30}{s^7} + \frac{8}{s-4} + \frac{4}{s-3}$$

(30 markah)

(c). *Selesaikan fungsi berikut dengan menggunakan kaedah transformasi Laplace.*

$$y''(t) + 4y'(t) + 4y(t) = t^2 e^{-2t}; \quad y(0) = 0,\ y'(0) = 0$$

(40 markah)

2. (a). *Pertimbangkan persamaan pembezaan berikut.*

$$y'' + \frac{1 - \frac{1}{2}a}{x} y' + \left[b^2 c^2 x^{2c-1} + \frac{a^2 - n^2 c^2}{x^2} \right] y = 0$$

dengan penyelesaian

$$y = x^a z_n \left(b x^c \right)$$

dengan z *bermaksud mana-mana kombinasi linear* J_n *dan* Y_n; a, b, c *dan* n *adalah pemalar. Cari penyelesaian am bagi persamaan berikut dalam sebutan fungsi Bessel.*

(i). $$y'' + \frac{1}{9} xy = 0$$

(ii). $$y'' - \frac{3}{x} y' + 25xy = 0$$

(50 markah)

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(b). *Satu persamaan Legendre adalah seperti berikut:*

$$\left(1-x^2\right)y''-2xy'+n\left(n+1\right)y=0$$

dengan n *adalah pemalar. Ungkapkan persamaan-persamaan berikut dalam sebutan polinomial Legendre:*

(i). $2x^3-5x^2+6x+1$

(ii). $-x^3-4x^2-5x+5$

(50 markah)

3. *Cari semua nilai eigen dan fungsi eigen bagi masalah Sturm-Liouville tersebut. Pertimbangkan semua kes untuk* λ.

$$y''+\lambda y=0,$$
$$y\left(0\right)-y\left(\pi\right)=0,$$
$$y'\left(0\right)-y'\left(\pi\right)=0$$

(100 markah)

4. *Pertimbangkan fungsi berikut:*

$$f(x)=|x| \quad \text{on } -\pi<x<\pi$$

(a). *Cari siri Fourier bagi* $f\left(x\right)$.

(55 markah)

(b). *Dengan menggunakan keputusan di (a), tunjukkan*

$$\frac{\pi}{2}-\frac{4}{\pi}\left(\cos x+\frac{1}{9}\cos 3x+\frac{1}{25}\cos 5x+...\right)$$

(25 markah)

(c). *Lakarkan graf bagi fungsi* $f\left(x\right)$.

(20 markah)

5. (a). *Pertimbangkan persamaan yang berikut:*

$$f(x)=\begin{cases}-2, & -3<x<0\\ 5e^{x}, & 0<x<3\\ 0, & \text{lain-lain}\end{cases}$$

Tentukan transformasi Fourier bagi $f(x)$.

(50 markah)

(b). *Satu dawai bertebat mempunyai panjang 1 m. Kedua-dua hujung dawai tertanam di dalam ais (suhu 0 °C). Bagi* k *= 0.003, dan pengedaran haba awal adalah*

$$u(x,0)=50x(1-x)$$

Cari suhu untuk fungsi $u(x, t)$.

(50 markah)

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