Final Examination
2017/2018 Academic Session

May/June 2018

JIB323 – Biostatistics
[Biostatistik]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **FIFTEEN** printed pages before you begin the examination.

Ensure Appendix (formula and distribution tables) are enclosed with the question paper.

Answer **FIVE (5)** questions. You may answer **either** in Bahasa Malaysia or English.

All answers must be written in the answer booklet provided.

Each question is worth 20 marks and the mark for each sub question is given at the end of that sub question.

In the event of any discrepancies in the exam questions, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA BELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

Sila pastikan Appendiks (formula dan jadual taburan) disertakan bersama kertas soalan.

*Jawab **LIMA (5)** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris.*

Setiap jawapan mesti dijawab di dalam buku jawapan yang disediakan.

Setiap soalan bernilai 20 markah dan markah subsoalan diperlihatkan di penghujung subsoalan itu.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

…2/-

Answer FIVE (5) questions.
Jawab LIMA (5) soalan.

1. A researcher conducted a study on the effect of radiation on amphibian embryos by observing the time it took for a sample of eight different species of frogs' eggs to hatch. Table 1 shows the time (days) for the eggs to hatch.

Table 1: Time (days) for the eggs to hatch							
6	7	11	6	5	5	11	9

Find,

(a). mean

(5 marks)

(b). median

(2 marks)

(c). mode

(3 marks)

(d). range

(3 marks)

(e). standard deviation

(7 marks)

Seorang penyelidik menjalankan kajian tentang kesan radiasi terhadap embrio amfibia dengan memerhatikan perbezaan masa yang diperlukan untuk lapan sampel spesies telur katak yang berlainan menetas. Jadual 1 menunjukkan masa (hari) untuk telur menetas.

Jadual 1: Masa (hari) untuk telur menetas							
6	*7*	*11*	*6*	*5*	*5*	*11*	*9*

Cari,

(a). min

(5 markah)

(b). median

(2 markah)

(c). mod

(3 markah)

(d). julat

(3 markah)

(e). sisihan piawai

(7 markah)

2. Table 2 shows the number of males and females with their own eye colour.

Table 2: Number of males and females with different eye color					
	Black	Brown	Blue	Green	Grey
Male	25	15	12	20	10
Female	20	30	10	15	10

(a). What is the probability that a randomly selected person will have brown eyes?

(4 marks)

(b). What is the probability that a randomly selected person will have green eyes or will be female?

(4 marks)

(c). What is the probability that a randomly selected person will have grey eyes or will be male?

(4 marks)

(d). What is the probability of a person having blue eyes, given that they are female?

(4 marks)

(e). What is the probability of a person having black eyes, given that they are male?

(4 marks)

Jadual 2 menunjukkan bilangan lelaki dan perempuan yang mempunyai warna mata tersendiri.

Jadual 2: Bilangan lelaki dan perempuan yang mempunyai warna mata yang berbeza.					
	Hitam	*Coklat*	*Biru*	*Hijau*	*Kelabu*
Lelaki	*25*	*15*	*12*	*20*	*10*
Perempuan	*20*	*30*	*10*	*15*	*10*

(a). *Apakah kebarangkalian seseorang dipilih secara rawak akan mempunyai mata coklat?*

(4 markah)

(b). *Apakah kebarangkalian seseorang dipilih secara rawak akan mempunyai mata hijau atau perempuan?*

(4 markah)

(c). *Apakah kebarangkalian seseorang dipilih secara rawak akan mempunyai mata kelabu atau lelaki?*

(4 markah)

(d). *Apakah kebarangkalian seseorang mempunyai mata biru, sekiranya jantinanya perempuan?*

(4 markah)

(e). *Apakah kebarangkalian seseorang mempunyai mata hitam, sekiranya jantinanya lelaki?*

(4 markah)

3. In a study of adult green sea urchin, *Strongylocentrotus droebachiensis,* a researcher found the weights of the species are normally distributed with mean 52.0 g and standard deviation 17.2 g.

(a). Find the percentage of adult green sea urchin weighing between 45.0 and 55.0 g. Sketch a graph in this case.

(8 marks)

(b). Find the percentage of adult green sea urchin having weight exceeding 65.0 g. Sketch a graph in this case.

(8 marks)

(c). Calculate and interpret the 95 percentile for weight of adult green sea urchins.

(4 marks)

Dalam kajian landak laut hijau dewasa, Strongylocentrotus droebachiensis*, seorang penyelidik mendapati berat spesies tersebut bertaburan normal dengan min 52.0 g dan sisihan piawai 17.2 g.*

(a). Cari peratusan spesies tersebut yang mempunyai berat antara 45.0 g dan 55.0 g. Lakarkan satu graf dalam kes ini.

(8 markah)

(b). Cari peratusan spesies tersebut yang mempunyai berat melebihi 65.0 g. Lakarkan satu graf dalam kes ini.

(8 markah)

(c). Kira dan tafsir 95 persentil untuk berat spesies tersebut dewasa.

(4 markah)

4. A marine biologist obtained the weights of salmon caught in Juneau, Alaska. The last digits of those weights are shown in Table 3. Test the hypothesis that the digits do not occur with the same frequency. Based on the results, what can we conclude about the procedure used to obtain the weights?

Ahli biologi marin memperoleh berat salmon yang ditangkap di Juneau, Alaska. Angka terakhir dari berat tersebut ditunjukkan dalam Jadual 3. Uji hipotesis bahawa angka tidak berlaku dengan kekerapan yang sama. Berdasarkan keputusan, apakah yang boleh kita simpul mengenai prosedur yang digunakan untuk mendapatkan berat?

Table 3: Last digit and frequency of salmon weights.

Jadual 3:Angka terakhir dan frekuensi berat salmon.

Last digit *Angka terakhir*	Frequency *Frekuensi*
0	20
1	18
2	19
3	17
4	21
5	22
6	14
7	18
8	20
9	21

(20 marks/*markah*)

5. Three strains of cultured *Staphyloccoccus aureus,* were observed for 24 hours at 37°C. Table 4 shows bacteria counts in million. At 5% significance level, do the data provide sufficient evidence to conclude that a difference exists in mean bacteria counts among the three strains?

   Tiga strain kultur Staphyloccoccus aureus*, diperhatikan selama 24 jam pada 37°C. Jadual 4 menunjukkan kiraan bakteria dalam juta. Pada aras keertian 5%, adakah data memberikan bukti yang cukup untuk menyimpulkan perbezaan wujud dalam min kiraan bakteria untuk ketiga-tiga strain?*

Table 4:Bacteria counts in million for each strains *Jadual 4:Kiraan bakteria dalam juta untuk setiap strain*		
Strain A	Strain B	Strain C
9	3	10
27	32	47
22	37	50
30	45	52
16	12	26

(20 marks/*markah*)

6. Table 5 shows three different species of *Iris* sepal length. Use the Kruskal-Wallis test with a 0.05 significance level to test the hypothesis that the three different species come from populations having unequal median sepal lengths.

   Jadual 5 menunjukkan panjang sepal tiga spesies Iris *yang berlainan. Gunakan ujian Kruskal-Wallis dengan aras keertian 0.05 untuk menguji hipotesis bahawa tiga spesies yang berlainan datang dari populasi yang mempunyai panjang sepal median yang berbeza.*

Table 5: Three different species of *Iris* sepal length *Jadual 5: Panjang sepal tiga spesies* Iris *yang berlainan*		
Iris setosa	*Iris versicolor*	*Iris virginica*
5.1	7.4	6.3
4.9	6.4	5.8
4.7	6.9	7.1
4.6	5.5	6.3
5.0	6.5	6.5

(20 marks/*markah*)

APPENDIX

Descriptive statistics

$\bar{x} = \frac{\Sigma x}{n}$ Mean

$\bar{x} = \frac{\Sigma f \cdot x}{\Sigma f}$ Mean (frequency table)

$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$ Standard deviation

$s = \sqrt{\frac{n(\Sigma x^2) - (\Sigma x)^2}{n(n - 1)}}$ Standard deviation (shortcut)

$s = \sqrt{\frac{n[\Sigma(f \cdot x^2)] - [\Sigma(f \cdot x)]^2}{n(n - 1)}}$ Standard deviation (frequency table)

$\text{variance} = s^2$

$variance\ s^2 = \frac{n(\Sigma x^2) - (\Sigma x)^2}{n(n-1)}$

Coefficient of variation $CV = \frac{s}{\bar{x}} \times 100$

Probability

$P(A \text{ or } B) = P(A) + P(B)$ if A, B are mutually exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
if A, B are not mutually exclusive

$P(A \text{ and } B) = P(A) \cdot P(B)$ if A, B are independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if A, B are dependent

$P(\bar{A}) = 1 - P(A)$ Rule of complements

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ Conditional probability

Normal Distribution

$z = \frac{x - \bar{x}}{s}$ or $\frac{x - \mu}{\sigma}$ Standard score

$\mu_{\bar{x}} = \mu$ Central limit theorem

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ Central limit theorem (Standard error)

Test statistics (one population)

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$ Proportion—one population

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Mean—one population (σ known)

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$ Mean—one population (σ unknown)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Standard deviation or variance—one population

Multinomial and Contingency Tables

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Multinomial (df = $k - 1$)

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Contingency table [df = $(r - 1)(c - 1)$]

$$\text{where } E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

One-Way Analysis of Variance

$$df_{between} = k - 1$$
$$df_{within} = N - k$$
$$df_{total} = N - 1$$

$$\bar{\bar{x}} = \frac{\Sigma x}{N}$$ mean of all values combined

$$F = \frac{\text{variance between samples}}{\text{variance within samples}} \quad \text{or} \quad F = \frac{MS_{between}}{MS_{within}}$$

$$MS_{between} = \frac{SS_{between}}{k - 1} \qquad MS_{within} = \frac{SS_{within}}{N - k} \qquad MS_{total} = \frac{SS_{total}}{N - 1}$$

$$SS_{between} = n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + \ldots + n_k(\bar{x}_k - \bar{\bar{x}})^2$$

$$SS_{within} = \sum_1 (x_{i1} - \bar{x}_1)^2 + \sum_2 (x_{i2} - \bar{x}_2)^2 + \ldots + \sum_k (x_{ik} - \bar{x}_k)^2$$

$$SS_{total} = \sum_1 (x_{i1} - \bar{\bar{x}}_1)^2 + \sum_2 (x_{i2} - \bar{\bar{x}}_2)^2 + \ldots + \sum_k (x_{ik} - \bar{\bar{x}}_k)^2$$
$$= \Sigma(x - \bar{\bar{x}})^2$$

$$SS_{total} = SS_{between} + SS_{within}$$

Nonparametric tests

$$z = \frac{(x + 0.5) - (n/2)}{\sqrt{n}/2}$$ Sign test for $n > 25$

$$z = \frac{T - n(n+1)/4}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}}$$ Wilcoxon signed ranks (matched pairs and $n > 30$)

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{R - \dfrac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\dfrac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$ Wilcoxon rank-sum (two independent samples)

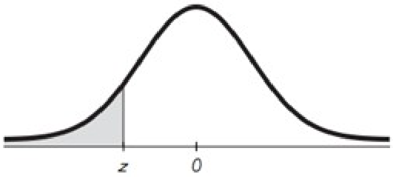
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \ldots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

Kruskal-Wallis (chi-square df $= k - 1$)

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$ Rank correlation

$\left(\text{critical value for } n > 30: \dfrac{\pm z}{\sqrt{n-1}}\right)$

$$z = \frac{G - \mu_G}{\sigma_G} = \frac{G - \left(\dfrac{2n_1 n_2}{n_1 + n_2} + 1\right)}{\sqrt{\dfrac{(2n_1 n_2)(2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}}$$ Runs test for $n > 20$



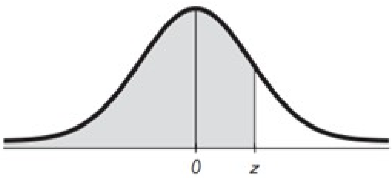
NEGATIVE *z* Scores

TABLE A-2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of *z* below −3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z score	Area
−1.645	0.0500
−2.575	0.0050



POSITIVE *z* Scores

TABLE A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949 *	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

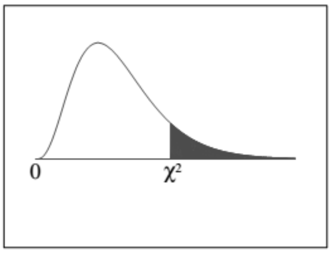
NOTE: For values of *z* above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

F Values for $\alpha = 0.05$

d_2 \ d_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

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TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
Large	2.576	2.326	1.960	1.645	1.282

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