UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIM 311 – Vector Analysis
[Analisis Vektor]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **TEN** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEPULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. (a) Let $\underline{u} = -\underline{i} + 2\underline{j} + 3\underline{k}$ and $\underline{w} = 2\underline{i} - \underline{j} + \underline{k}$ where $\underline{i}, \underline{j}$ and $\underline{k}$ are mutually orthogonal unit vectors.

(i) Find $\underline{u} + 2\underline{w}$ and $\underline{u} - 2\underline{w}$.

(ii) Find $(2\underline{w} - \underline{u}) \cdot \underline{w}$ and $(\underline{u} + 2\underline{w}) \cdot \underline{u}$.

(iii) By explicitly calculating the vectors product, show that

$$(\underline{u} \times \underline{w}) \cdot (\underline{u} + 2\underline{w}) = 0.$$

(30 marks)

(b) Given $\underline{a}$, $\underline{b}$ and $\underline{c}$ are vectors in the Euclidean space with $\underline{a} \neq \underline{0}$ and $\underline{a} \times \underline{c} = \underline{b}$.

(i) Show that $\underline{a} \cdot \underline{b} = 0$.

(ii) If $\underline{c}$ can be written as $\underline{c} = \lambda\underline{a} + \alpha\underline{b} + \beta\underline{a} \times \underline{b}$, show that

$$\alpha = 0 \text{ and } \beta = -\frac{1}{|\underline{a}|^2}.$$

(30 marks)

(c) If $\|\underline{a}\| = \|\underline{b}\| = 3$ and the angle between $\underline{a}$ and $\underline{b}$ is 60^0, show that $\|\underline{a} - \underline{b}\| = 3$.

(20 marks)

(d) If $\underline{a} \times \underline{b} = \underline{a} - \underline{b}$, show that $\underline{a} = \underline{b}$.

(20 marks)

2. The lines L_1 and L_2 are given by the vector equations

$$\underline{r}_1 = 4\underline{i} - 4\underline{j} + 4\underline{k} + \lambda(3\underline{i} - \underline{j} + 2\underline{k})$$

and

$$\underline{r}_2 = 5\underline{i} - \underline{j} + 6\underline{k} + t(2\underline{i} + \underline{j} + 2\underline{k})$$

respectively.

(a) Rewrite the equations in parametric form and show that the point $(1,-3,2)$ lies on both L_1 and L_2.

(20 marks)

(b) Find the equation of the plane containing both L_1 and L_2, giving your answer in the scalar product form $\underline{r}\cdot\underline{n} = d$.

(35 marks)

(c) Find the perpendicular distance from the point (3,5,2) to this plane.

(45 marks)

3. (a) Given that

$$\phi(x,y,z) = 2x^2 + y^2 + z^2$$

(i) Find $\underline{\nabla}\phi$.

(ii) Deduce the magnitude and the direction of the greatest rate of change of $\phi(x,y,z)$ at the point $(1,1,0)$.

(iii) Further, calculate the outward pointing unit normal to the ellipsoid $2x^2 + y^2 + z^2 = 5$ at the point $(1,1,0)$. Use this to find the Cartesian equation of the tangent plane at this point.

(40 marks)

(b) Show that for any (smooth enough) scalar field ϕ

$$\underline{\nabla} \times \underline{\nabla} \phi = \underline{0}.$$

Deduce that the vector field

$$\underline{F} = (6x + 2y)\,\underline{i} + 2x\,\underline{j} + \underline{k}$$

can be expressed as the gradient of a scalar field ϕ.

(60 marks)

4. (a) A path C as shown in the Figure 1 below is composed of the straight line C_1 from $(4,2,0)$ to $(0,2,0)$, followed by the circular path C_2 in the *yz*-plane.

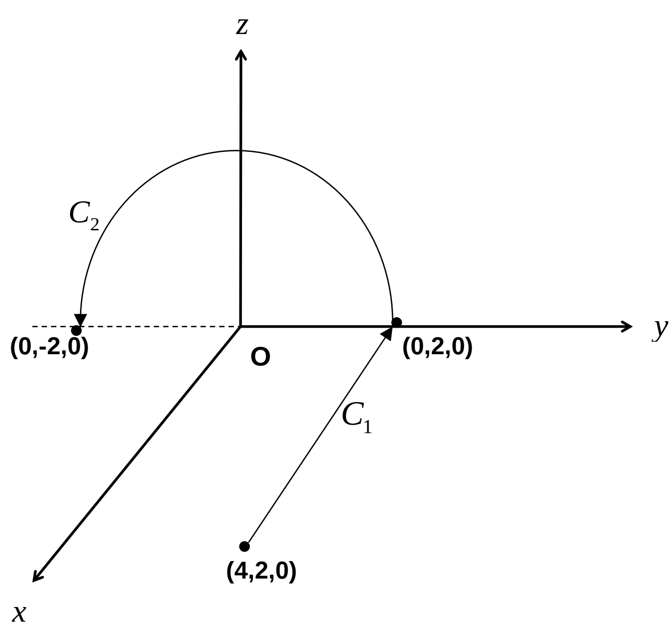


Figure 1

(i) Write the parametric equations for the two paths C_1 and C_2.

(ii) Hence evaluate the path integral (line integral) of

$$\int_C (x - 4y + z)\,ds$$

where C is the piecewise smooth path as shown in the Figure 1.

(50 marks)

(b) Given the vector field

$$\underline{F} = (1+2xy)e^z \underline{i} + x^2 e^z \underline{j} + \left(x^2 y + x\right) e^z \underline{k}$$

and let L be the oriented line segment from $(1,0,0)$ to $(1,0,2\pi)$.

(i) Compute the path integral $\int_L \underline{F} \cdot d\underline{r}$.

(ii) Let C be the part of the helix $\underline{r}(t) = \cos t\, \underline{i} + \sin t\, \underline{j} + t\, \underline{k}$ which lies between the plane $z = 0$ and $z = 2\pi$. Explain why

$$\int_C \underline{F} \cdot d\underline{r} = \int_L \underline{F} \cdot d\underline{r}$$

(50 marks)

5. (a) Let $\underline{F} = 2xy\underline{i} + x^2 \underline{j}$. Show that the integral of $\underline{F}$ around the circumference of the square $[0,1]\times[0,1]$ in the xy-plane is zero by

(i) direct evaluation,

(ii) showing that $\underline{F}$ is a gradient of a function $\phi(x,y) = x^2 y$,

(iii) using Green's Theorem.

(40 marks)

(b) State Divergence Theorem.

Using the Divergence Theorem, evaluate

$$\iint_S \underline{F} \cdot d\underline{S}$$

where the vector field $\underline{F}$ is given by

$$\underline{F} = 2xy\underline{i} + e^z \underline{j} + x^3 \underline{k}$$

and S is the surface of the cube defined by

$$0 \le x \le 1,\ 0 \le y \le 1,\ \text{and } 0 \le z \le 1$$

with positive (outward) orientation.

(60 marks)

1. (a) Katakan $\underline{u} = -\underline{i} + 2\underline{j} + 3\underline{k}$ dan $\underline{w} = 2\underline{i} - \underline{j} + \underline{k}$ di mana $\underline{i}, \underline{j}$ dan $\underline{k}$ adalah vektor-vektor unit yang ortogonal antara satu sama lain.

(i) Cari $\underline{u} + 2\underline{w}$ dan $\underline{u} - 2\underline{w}$.

(ii) Cari $(2\underline{w} - \underline{u}) \cdot \underline{w}$ dan $(\underline{u} + 2\underline{w}) \cdot \underline{u}$

(iii) Dengan menghitung hasildarab vektor, tunjukkan bahawa $(\underline{u} \times \underline{w}) \cdot (\underline{u} + 2\underline{w}) = 0$.

(30 markah)

(b) Diberi $\underline{a}$, $\underline{b}$ dan $\underline{c}$ adalah vektor dalam ruang Euklidan dengan $\underline{a} \neq \underline{0}$ dan $\underline{a} \times \underline{c} = \underline{b}$.

(i) Tunjukkan bahawa $\underline{a} \cdot \underline{b} = 0$.

(ii) Jika $\underline{c}$ boleh ditulis sebagai $\underline{c} = \lambda\underline{a} + \alpha\underline{b} + \beta\underline{a} \times \underline{b}$, tunjukkan bahawa $\alpha = 0$ dan $\beta = -\dfrac{1}{|\underline{a}|^2}$.

(30 markah)

(c) Jika $\|\underline{a}\| = \|\underline{b}\| = 3$ dan sudut di antara $\underline{a}$ dan $\underline{b}$ adalah 60^0, tunjukkan bahawa $\|\underline{a} - \underline{b}\| = 3$.

(20 markah)

(d) Jika $\underline{a} \times \underline{b} = \underline{a} - \underline{b}$, tunjukkan bahawa $\underline{a} = \underline{b}$.

(20 markah)

2. Garis lurus L_1 dan L_2, masing-masing diberi oleh persamaan vektor

$$\underline{r}_1 = 4\underline{i} - 4\underline{j} + 4\underline{k} + \lambda(3\underline{i} - \underline{j} + 2\underline{k})$$

dan

$$\underline{r}_2 = 5\underline{i} - \underline{j} + 6\underline{k} + t(2\underline{i} + \underline{j} + 2\underline{k}) .$$

(a) Tulis semula persamaan tersebut dalam bentuk parametrik dan tunjukkan bahawa titik $(1,-3,2)$ terletak di atas kedua-dua L_1 dan L_2.

(20 markah)

(b) Cari persamaan satah yang mengandungi kedua-dua L_1 dan L_2, dengan memberi jawapan anda dalam bentuk hasil darab skalar $\underline{r} \cdot \underline{n} = d$.

(35 markah)

(c) Cari jarak serenjang dari titik $(3,5,2)$ ke satah tersebut.

(45 markah)

3. (a) Diberi

$$\phi(x, y, z) = 2x^2 + y^2 + z^2$$

(i) Cari $\underline{\nabla}\phi$.

(ii) Deduksikan magnitud dan arah bagi kadar perubahan terbesar bagi $\phi(x, y, z)$ di titik $(1,1,0)$.

(iii) Seterusnya, hitung normal unit yang mengarah keluar dari elipsoid $2x^2 + y^2 + z^2 = 5$ di titik $(1,1,0)$. Guna maklumat ini untuk mencari persamaan Cartesan bagi satah tangen di titik berkenaan.

(40 markah)

(b) Tunjukkan bahawa untuk sebarang medan skalar ϕ (yang licin) ,

$$\underline{\nabla} \times \underline{\nabla}\phi = \underline{0}.$$

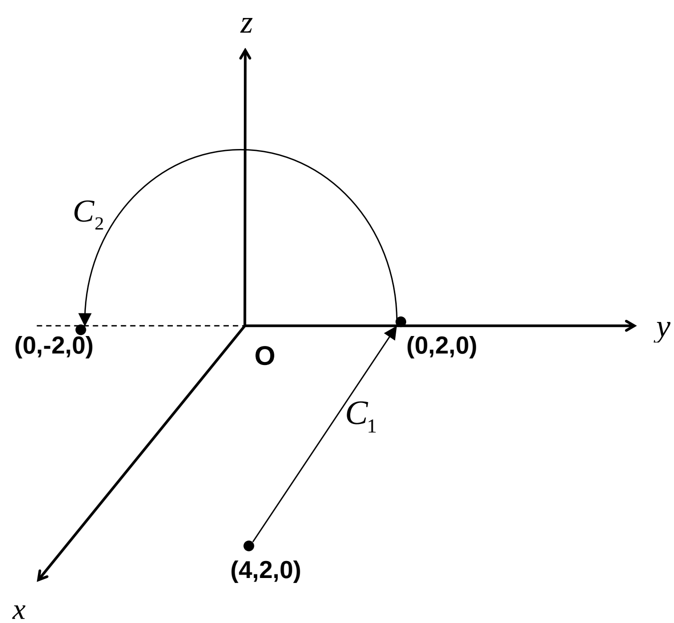
Deduksikan bahawa medan vektor

$$\underline{F} = (6x + 2y)\,\underline{i} + 2x\,\underline{j} + \underline{k}$$

dapat diungkapkan sebagai kecerunan suatu medan skalar ϕ.

(60 markah)

4. (a) Suatu lintasan C seperti yang ditunjukkan dalam Rajah 1 berikut terdiri daripada garis lurus C_1 dari $(4,2,0)$ ke $(0,2,0)$, diikuti dengan lintasan membulat C_2 dalam satah-*yz*.



Rajah 1

(i) Tuliskankan persamaan parametrik bagi kedua-dua lintasan C_1 dan C_2.

(ii) Dengan itu, nilaikan kamiran lintasan (kamiran garis)

$$\int_C (x - 4y + z)\,ds$$

di mana C adalah lintasan licin cebis demi cebis seperti yang ditunjukkan dalam Rajah 1.

(50 markah)

(b) Diberi medan vektor

$$\underline{F} = (1 + 2xy)\, e^z\, \underline{i} + x^2 e^z\, \underline{j} + (x^2 y + x)\, e^z\, \underline{k}$$

dan katakan L adalah bahagian garis lurus yang diorientasikan dari $(1,0,0)$ ke $(1,0,2\pi)$.

(i) Hitungkan kamiran lintasan $\int_L \underline{F} \cdot d\underline{r}$.

(ii) Katakan C adalah sebahagian dari heliks $\underline{r}(t) = \cos t\, \underline{i} + \sin t\, \underline{j} + t\, \underline{k}$ yang terletak di antara satah $z = 0$ dan $z = 2\pi$. Terangkan kenapa

$$\int_C \underline{F} \cdot d\underline{r} = \int_L \underline{F} \cdot d\underline{r} .$$

(50 markah)

5. (a) Katakan $\underline{F} = 2xy\,\underline{i} + x^2\,\underline{j}$. Tunjukkan bahawa kamiran bagi $\underline{F}$ di sekeliling lilitan segiempat sama $[0,1]\times[0,1]$ dalam satah-*xy* adalah sifar dengan

(i) melakukan pengiraan secara terus,

(ii) menunjukkan bahawa $\underline{F}$ adalah kecerunan bagi fungsi $\phi(x, y) = x^2 y$,

(iii) menggunakan Teorem Green.

(40 markah)

(b) Nyatakan Teorem Kecapahan.

Dengan menggunakan Teorem Kecapahan, nilaikan

$$\iint_S \underline{F} \cdot d\underline{S}$$

di mana medan vektor $\underline{F}$ diberi oleh

$$\underline{F} = 2xy\,\underline{i} + e^z\,\underline{j} + x^3\,\underline{k}$$

dan S adalah permukaan kubus yang ditakrifkan oleh

$$0 \le x \le 1, \quad 0 \le y \le 1, \quad \text{dan} \quad 0 \le z \le 1.$$

dengan orientasi positif (mengarah keluar)

(60 markah)

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