UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIM 215 – Introduction to Numerical Analysis
[Pengantar Analisis Berangka]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **ELEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggerís.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. (a) State one advantage and one disadvantage of the following methods:

   (i) bisection method.
   (ii) fixed-point iteration.
   (iii) Newton-Raphson method.

(15 marks)

(b) Use Newton-Raphson method to estimate the root of $x^2 - 5\cos x = 0$, employing an initial guess of $x_0 = 1.0$ accurate to 0.01%.

(20 marks)

(c) Given that $f(1.4) = 0.65949$, $f(1.6) = 1.20321$, $f(1.8) = 1.90443$.

   (i) Construct a Lagrange interpolating polynomial of degree two.
   (ii) Use the polynomial in (i) to approximate $f(1.5)$.
   (iii) The above data are generated using the function $f(x) = x^2 \ln x$, compute the absolute error for the result obtained in (ii). Also, find an error bound for the approximation in (ii).

(65 marks)

2. (a) Consider the following table of data:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.1987	0.2955	0.3894	0.4794	0.5646	0.6442	0.7174

   (i) Find $f'(0.3)$ and $f''(0.3)$ using five-point midpoint formula with step size $h = 0.1$.
   (ii) Find $\int_0^{0.6} f(x)\, dx$ using composite trapezoidal rule with step size $h = 0.2$.
   (iii) Repeat question (ii) by using step size $h = 0.1$.
   (iv) Use the results in (ii) and (iii) to find an improved approximation using Richardson extrapolation.

(40 marks)

(b) Given the following table of data:

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$	1.3416	1.8974	2.3238	2.6833	α	3.2863	3.5496

Find the value of α if the composite Simpson's rule with $n=6$ gives $\int_{0.2}^{1.4} f(x)\,dx = 3.1338$.

(15 marks)

(c) Use Romberg integration to approximate the definite integral

$$\int_0^1 e^{x^2}\,dx .$$

Continue the calculation until two successive diagonal entries differ by less than 5×10^{-2}.

(45 marks)

3. (a) A basic RL series electrical circuit contains a power source with voltage of V volts, a resistor with resistance of R ohms and an inductor with inductance of L henrys. Given that the current I amperes that flows through the circuit satisfies the differential equation

$$L\frac{dI}{dt} + RI = V .$$

Let $L=5$ henrys, $R=10$ ohms, $V=20$ volts and I at $t=0$ second is 0 ampere. By taking step size $h=0.2$, find the current I at time $t=0.4$ seconds using

(i) modified Euler method.

(ii) 4th order Runge-Kutta method.

(70 marks)

(b) The solutions of an initial value problem

$$y' = t^2 y - 2y$$

from $t = 0$ to $t = 1.5$ are given in the table below:

t	0	0.5	1.0	1.5
y	1	0.3835	0.1889	0.1534

Approximate $y(2.0)$ by using the Adams fourth-order predictor-corrector method.

(30 marks)

4. (a) Give two reason why pivoting strategy is used when solving the linear system by Gaussian elimination method.

(10 marks)

(b) Solve the system of equations

$$\begin{aligned} 2x - 3y + 4z &= -4 \\ x + 5z &= 4 \\ 2y - 3z &= 1 \end{aligned}$$

using LU decomposition technique.

(35 marks)

(c) The linear system $AX = B$ is given by

$$\begin{bmatrix} 3 & -1 & -1 \\ 6 & 8 & 1 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -1.1 \\ 0.1 \end{bmatrix}$$

(i) Show that the matrix A is strictly diagonally dominant.

(ii) Solve the above linear system using Gauss-Seidel iterative method, starting with $\mathbf{x}^{(0)} = (0,0,0)^t$ and iterating until $\varepsilon = \left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty = 0.005$.

(55 marks)

5. (a) Given a matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$

(i) Use the Geršgorin Circle Theorem to determine the bound for the eigenvalues, and the spectral radius of the matrix A.

(ii) Assume the initial eigenvector is $\mathbf{x}^{(0)} = (0.5, 0.5, 0.5)^t$. Use the Power method to approximate the most dominant eigenvalue and its corresponding eigenvector of the matrix A. Iterate until a tolerance of 10^{-1} is achieved.

(60 marks)

(b) Use Newton's method with initial values $\mathbf{x}^{(0)} = (1.2, 1.2)^t$ to find $\mathbf{x}^{(1)}$ for the following nonlinear system:

$$x_1^2 - x_1 + x_2 - 0.75 = 0$$
$$x_1^2 - 5x_1x_2 - x_2 = 0$$

(40 marks)

1. (a) Nyatakan satu kelebihan dan satu kelemahan bagi kaedah-kaedah berikut:

(i) kaedah pembahagian dua sama.

(ii) lelaran titik tetap.

(iii) kaedah Newton-Raphson.

(15 markah)

(b) Gunakan kaedah Newton-Raphson untuk menganggar punca bagi $x^2 - 5\cos x = 0$ dengan nilai awal $x_0 = 1.0$ dan tepat kepada 0.01%.

(20 markah)

(c) Diberi bahawa $f(1.4) = 0.65949$, $f(1.6) = 1.20321$, $f(1.8) = 1.90443$.

(i) Bina suatu polinomial interpolasi Lagrange berdarjah dua.

(ii) Guna polinomial dalam (i) untuk menganggarkan $f(1.5)$.

(iii) Data di atas dijana daripada fungsi $f(x) = x^2 \ln x$, kira ralat mutlak bagi jawapan yang diperolehi dalam (ii). Cari juga batas ralat bagi penghampiran dalam (ii).

(65 markah)

2. (a) Pertimbangkan data dalam jadual berikut:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.1987	0.2955	0.3894	0.4794	0.5646	0.6442	0.7174

(i) Cari $f'(0.3)$ dan $f''(0.3)$ dengan menggunakan rumus beza tengah lima titik dengan saiz langkah $h = 0.1$.

(ii) Cari $\int_0^{0.6} f(x)\, dx$ dengan menggunakan petua trapezium gubahan dengan saiz langkah $h = 0.2$.

(iii) Ulangi soalan (ii) dengan menggunakan saiz langkah $h = 0.1$.

(iv) Guna jawapan dari (ii) dan (iii) untuk mendapatkan penghampiran yang lebih baik dengan menggunakan kaedah ekstrapolasi Richardson.

(40 markah)

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(b) Diberi jadual data berikut:

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$	1.3416	1.8974	2.3238	2.6833	α	3.2863	3.5496

Cari nilai α jika petua Simpson gubahan dengan $n=6$ memberikan $\int_{0.2}^{1.4} f\left(x\right)\, dx = 3.1338$.

(15 markah)

(c) Gunakan pengamiran Romberg untuk menganggarkan kamiran tentu

$$\int_0^1 e^{x^2}\, dx .$$

Teruskan pengiraan sehingga perbezaan antara dua unsur pepenjuru berturut-turut adalah kurang daripada 5×10^{-2}.

(45 markah)

3. (a) Suatu siri asas RL litar elektrik mempunyai punca kuasa dengan voltan V volt, perintang dengan rintangan R ohm dan aruhan dengan keraruhan L henry. Diberi bahawa arus I ampere yang mengalir dalam litar tersebut diwakilkan oleh persamaa pembezaan

$$L\frac{dI}{dt} + RI = V .$$

Biar $L=5$ henry, $R=10$ ohm, $V=20$ volt dan I pada $t=0$ saat ialah 0 ampere. Pertimbangkan saiz langkah $h=0.2$, cari arus I pada masa $t=0.4$ saat dengan menggunakan

(i) kaedah Euler diubahsuai.

(ii) keadah Runge-Kutta berperingkat keempat.

(70 markah)

(b) Penyelesaian bagi masalah nilai awal

$$y' = t^2 y - 2y$$

dari t = 0 ke t = 1.5 diberikan dalam jadual di bawah:

t	0	0.5	1.0	1.5
y	1	0.3835	0.1889	0.1534

Anggarkan $y(2.0)$ dengan menggunakan kaedah Adams peramal-pembetul berperingkat keempat.

(30 markah)

4. (a) Beri dua sebab mengapa strategi pangsian digunakan apabila menyelesaikan sistem linear dengan kaedah penghapusan Gaussian.

(10 markah)

(b) Selesaikan sistem persamaan

$$\begin{aligned} 2x - 3y + 4z &= -4 \\ x \quad + 5z &= 4 \\ 2y - 3z &= 1 \end{aligned}$$

dengan menggunakan teknik penghuraian LU.

(35 markah)

(c) Diberi suatu sistem linear $AX = B$ sebagai

$$\begin{bmatrix} 3 & -1 & -1 \\ 6 & 8 & 1 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -1.1 \\ 0.1 \end{bmatrix}$$

(i) Tunjukkan bahawa matriks A adalah dominan pepenjuru tegas.

(ii) Selesaikan sistem linear di atas dengan menggunakan kaedah lelaran Gauss-Seidel, bermula dengan $\mathbf{x}^{(0)} = (0,0,0)^t$ dan mengulangi sehingga $\varepsilon = \left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty = 0.005$.

(55 markah)

5. (a) Diberi suatu matriks

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$

(i) Gunakan Teorem Bulatan Geršgorin untuk menentukan batas nilai eigen dan jejari spektrum bagi matriks A.

(ii) Andaikan vektor eigen awal ialah $\mathbf{x}^{(0)} = (0.5, 0.5, 0.5)^t$. Guna kaedah Kuasa untuk menganggarkan nilai eigen dominan dan vektor eigen yang sepadan bagi matriks A. Lelar sehingga toleransi 10^{-1} tercapai.

(60 markah)

(b) Dengan nilai awal $\mathbf{x}^{(0)} = (1.2, 1.2)^t$, guna kaedah Newton untuk mencari $\mathbf{x}^{(1)}$ bagi sistem tak linear berikut:

$$x_1^2 - x_1 + x_2 - 0.75 = 0$$
$$x_1^2 - 5x_1x_2 - x_2 = 0$$

(40 markah)

List of formula:

1. $f(x) = P(x) + \dfrac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$

2. $N_j(h) = N_{j-1}\left(\dfrac{h}{2}\right) + \dfrac{N_{j-1}\left(\dfrac{h}{2}\right) - N_{j-1}(h)}{4^{j-1}-1}$ for $j = 2,3,\ldots$

3. $f'(x_0) \approx \dfrac{f(x_0+h)-f(x_0)}{h}$

$f'(x_0) \approx \dfrac{f(x_0)-f(x_0-h)}{h}$

$f'(x_0) \approx \dfrac{f(x_0+h)-f(x_0-h)}{2h}$

$f'(x_0) \approx \dfrac{1}{2h}\left[-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right]$

$f'(x_0) \approx \dfrac{1}{2h}\left[f(x_0-2h)-4f(x_0-h)+3f(x_0)\right]$

$f'(x_0) \approx \dfrac{1}{12h}\left[f(x_0-2h)-8f(x_0-h)+8f(x_0+h)-f(x_0+2h)\right]$

4. $f''(x_0) \approx \dfrac{1}{h^2}\left[f(x_0)-2f(x_0+h)+f(x_0+2h)\right]$

$f''(x_0) \approx \dfrac{1}{h^2}\left[f(x_0)-2f(x_0-h)+f(x_0-2h)\right]$

$f''(x_0) \approx \dfrac{1}{h^2}\left[f(x_0-h)-2f(x_0)+f(x_0+h)\right]$

$f''(x_0) \approx \dfrac{1}{h^2}\left[2f(x_0)-5f(x_0+h)+4f(x_0+2h)-f(x_0+3h)\right]$

$f''(x_0) \approx \dfrac{1}{h^2}\left[2f(x_0)-5f(x_0-h)+4f(x_0-2h)-f(x_0-3h)\right]$

$f''(x_0) \approx \dfrac{1}{12h^2}\left[-f(x_0-2h)+16f(x_0-h)-30f(x_0)+16f(x_0+h)-f(x_0+2h)\right]$

5. $\int_{x_0}^{x_1} f(x)\,dx = \dfrac{h}{2}\left[f(x_0)+f(x_1)\right]$

$\int_{x_0}^{x_2} f(x)\,dx = \dfrac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]$

$\int_{x_0}^{x_3} f(x)\,dx = \dfrac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right]$

$\int_{x_0}^{x_n} f(x)\,dx = \dfrac{h}{2}\left[f(x_0)+2\sum_{j=1}^{n-1} f(x_j)+f(x_n)\right]$

$\int_{x_0}^{x_n} f(x)\,dx = \dfrac{h}{3}\left[f(x_0)+4\sum_{j=1}^{n/2} f(x_{2j-1})+2\sum_{j=1}^{(n/2)-1} f(x_{2j})+f(x_n)\right]$

6. $h_k = \frac{(b-a)}{2^{k-1}}$ for $k = 1,2,3,\dots$

$R_{1,1} = \frac{h_1}{2}\left[f(a) + f(b)\right]$

$R_{k,1} = \frac{1}{2}\left[R_{k-1,1} + h_{k-1}\sum_{i=1}^{2^{k-2}} f(a + (2i-1)h_k)\right]$ for $k = 2,3,\dots$

$R_{k,j} = R_{k,j-1} + \frac{1}{4^{j-1}-1}\left(R_{k,j-1} - R_{k-1,j-1}\right)$ for $k = j, j+1,\dots$

7. $x = \frac{1}{2}\left[(b-a)t + (a+b)\right]$

$\int_{-1}^{1} f(x)\ dx \approx f\left(0.5773503\right) + f\left(-0.5773503\right)$

$\int_{-1}^{1} f(x)\ dx \approx 0.5555556 f\left(0.7745967\right) + 0.8888889 f\left(0\right) + 0.5555556 f(-0.7745967)$

8. $y_{i+1} = y_i + h\,f(t_i, y_i)$

9. $y_{i+1}^0 = y_i + h\,f(t_i, y_i)$, $y_{i+1} = y_i + \frac{h}{2}\left[f(t_i, y_i) + f(t_{i+1}, y_{i+1}^0)\right]$

10. $y_{i+1} = y_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$k_1 = h\,f(t_i, y_i)$, $k_2 = h\,f\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_1\right)$, $k_3 = h\,f\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_2\right)$

$k_4 = h\,f\left(t_{i+1}, y_i + k_3\right)$

11. $y_{i+1} = y_i + \frac{h}{24}\left[55 f(t_i, y_i) - 59 f(t_{i-1}, y_{i-1}) + 37 f(t_{i-2}, y_{i-2}) - 9 f(t_{i-3}, y_{i-3})\right]$

$y_{i+1} = y_i + \frac{h}{24}\left[9 f(t_{i+1}, y_{i+1}) + 19 f(t_i, y_i) - 5 f(t_{i-1}, y_{i-1}) + f(t_{i-2}, y_{i-2})\right]$

12. $R_i = \left\{ z \in C \,\middle|\, \left|z - a_{ii}\right| \le \sum_{\substack{j-1 \\ j \ne i}}^{n} \left|a_{ij}\right| \right\}$

13. $\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)} + \mathbf{y}^{(k-1)}$ for $k \ge 1$, $\mathbf{y}^{(k-1)} = -J\left(\mathbf{x}^{(k-1)}\right)^{-1}\mathbf{F}\left(\mathbf{x}^{(k-1)}\right)$

$$J(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\mathbf{x}) & \frac{\partial f_1}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_1}{\partial x_n}(\mathbf{x}) \\ \frac{\partial f_2}{\partial x_1}(\mathbf{x}) & \frac{\partial f_2}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_2}{\partial x_n}(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1}(\mathbf{x}) & \frac{\partial f_n}{\partial x_2}(\mathbf{x}) & \cdots & \frac{\partial f_n}{\partial x_n}(\mathbf{x}) \end{bmatrix}$$

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