UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIM 211 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **SIX** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggerís.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. (a) Determine the monotonicity and boundedness of the sequence

$$a_n = \ln\left(\frac{n+1}{n}\right).$$

(15 marks)

(b) Given that $y = (1+x)^{1/x}$.

(i) Use L'Hôpital's rule to show that $\lim\limits_{x\to 0} y = e$.

(ii) Consider $k = \frac{1}{x}$ in (i), show that $\lim\limits_{k\to\infty}\left(\frac{k}{k+1}\right)^k = \frac{1}{e}$.

(iii) Hence, show that $\sum\limits_{k=1}^{\infty}\frac{k!}{k^k}$ is convergent.

(40 marks)

(c) Test the alternating series for absolute convergence and conditional convergence.

$$\sum_{k=1}^{\infty}\frac{(-1)^k k}{k^2+3k}.$$

(45 marks)

2. (a) Find the interval of convergence of the power series $\sum\limits_{k=1}^{\infty}\frac{(2)^{k+1}x^k}{\sqrt{k}}$.

(55 marks)

(b) Show that the function $f(x,y) = \frac{8x^2-4y^2}{x^2+y^2}$ does not have a limit as $(x,y)\to(0,0)$.

(20 marks)

(c) Given that $f(x,y) = e^{-2x}\ln y$. Find

(i) $f_x(0,e)$ and $f_y(0,e)$.

(ii) the second-order partial derivatives of $f(x,y)$.

(25 marks)

3. (a) Let $z=\ln(x^2+y^2)$, where $x=3s^2-t^2$ and $y=-2st$. Find $\dfrac{\partial z}{\partial t}$.

(15 marks)

(b) Find the absolute extreme values of $f(x,y)=2x^2+y^2-4x-2y+2$ on the triangular region $D=\{(x,y):0\le x\le 2,\ 0\le y\le 2x\}$.

(50 marks)

(c) Using the Lagrange multiplier, find the maximum of $f(x,y)=x+y$ on $2x^2+2y^2=1$.

(35 marks)

4. (a) Evaluate $\iint_R e^{(x+y)}\,dx\,dy$ where $R=\{(x,y):0\le x\le 4,\ 0\le y\le 8\}$.

(20 marks)

(b) Evaluate the integral $\iint_R x^4+y^2\,dy\,dx$ where R is the region bounded by $y=x^2$ and $y=x^3$.

(45 marks)

(c) Change the Cartesian integral $\int_{-4}^{4}\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} e^{-(x^2+y^2)}\,dx\,dy$ into an equivalent polar integral. Hence, evaluate the integral.

(35 marks)

5. (a) Evaluate the integral $\int_1^5\int_y^{y^2}\int_0^{\ln x} ye^z\,dz\,dx\,dy$.

(20 marks)

(b) Use triple integral in cylindrical coordinates to find the volume of a solid that is bounded above by the paraboloid $z=1-(x^2+y^2)$ and bounded below by the xy-plane.

(30 marks)

(c) Use spherical coordinates to evaluate $\iiint_T dx\,dy\,dz$,

$$T=\left\{(x,y,z):0\le x\le 4,\ 0\le y\le\sqrt{16-x^2},\ 0\le z\le\sqrt{16-(x^2+y^2)}\right\}.$$

(50 marks)

1. (a) Tentukan keekanadaan dan keterbatasan bagi jujukan

$$a_n = \ln\left(\frac{n+1}{n}\right).$$

(15 markah)

(b) Diberi bahawa $y = (1+x)^{1/x}$.

(i) Guna petua L'Hôpital untuk tunjukkan bahawa $\underset{x\to 0}{\text{had}}\ y = e$.

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(ii) Pertimbangkan $k = \frac{1}{x}$ dalam (i), tunjukkan bahawa

$$\underset{k\to\infty}{\text{had}} \left(\frac{k}{k+1}\right)^k = \frac{1}{e}.$$

(iii) Seterusnya, tunjukkan bahawa $\sum_{k=1}^{\infty} \frac{k!}{k^k}$ adalah menumpu.

(40 markah)

(c) Uji penumpuan multak dan penumpuan bersyarat bagi siri berselang-seli

$$\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2 + 3k}.$$

(45 markah)

2. (a) Cari selang penumpuan bagi siri kuasa $\sum_{k=1}^{\infty} \frac{(2)^{k+1} x^k}{\sqrt{k}}$.

(55 markah)

(b) Tunjukkan bahawa fungsi $f(x,y) = \frac{8x^2 - 4y^2}{x^2 + y^2}$ tidak mempunyai had apabila $(x,y) \to (0,0)$.

(20 markah)

(c) Diberi $f(x,y) = e^{-2x} \ln y$. Cari

(i) $f_x(0,e)$ dan $f_y(0,e)$.

(ii) terbitan separa peringkat kedua bagi $f(x,y)$.

(25 markah)

3. (a) Katakan $z = \ln(x^2 + y^2)$, di mana $x = 3s^2 - t^2$ dan $y = -2st$. Cari $\dfrac{\partial z}{\partial t}$.

(15 markah)

(b) Dapatkan nilai ekstremum mutlak bagi $f(x, y) = 2x^2 + y^2 - 4x - 2y + 2$ pada rantau segitiga $D = \{(x, y) : 0 \le x \le 2,\ 0 \le y \le 2x\}$.

(50 markah)

(c) Dengan menggunakan pendarab Lagrange, cari maksimum bagi $f(x, y) = x + y$ pada $2x^2 + 2y^2 = 1$.

(35 markah)

4. (a) Nilaikan $\iint_R e^{(x+y)}\, dx\, dy$ di mana $R = \{(x, y) : 0 \le x \le 4,\ 0 \le y \le 8\}$.

(20 markah)

(b) Nilaikan kamiran $\iint_R x^4 + y^2\, dy\, dx$ di mana R ialah rantau yang dibatasi oleh $y = x^2$ dan $y = x^3$.

(45 markah)

(c) Tukarkan kamiran Cartesan $\int_{-4}^{4} \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} e^{-\left(x^2+y^2\right)}\, dx\ dy$ ke dalam kamiran kutub yang sepadan. Seterusnya, nilaikan kamiran tersebut.

(35 markah)

5. (a) Nilaikan kamiran $\int_1^5 \int_y^{y^2} \int_0^{\operatorname{In} x} y e^z\ dz\ dx\ dy$.

(20 markah)

(b) Gunakan kamiran ganda tiga dalam koordinat silinder untuk mencari isipadu bongkah yang dibatasi dari atas oleh paraboloid $z = 1 - (x^2 + y^2)$ dan dibatasi dari bawah oleh satah - xy.

(30 markah)

(c) Gunakan koordinat sfera untuk menilai $\iiint_T dx\ dy\ dz$,

$$T = \left\{(x, y, z) : 0 \le x \le 4,\ \ 0 \le y \le \sqrt{16 - x^2},\ \ 0 \le z \le \sqrt{16 - (x^2 + y^2)}\right\} .$$

(50 markah)

List of formula:

1. $P_n(x) = g(a) + g'(a)(x-a) + \frac{g''(a)}{2!}(x-a)^2 + \cdots + \frac{g^{(n)}(a)}{n!}(x-a)^n$

2. $A = \frac{\partial^2 f}{\partial x^2}(x_0, y_0)$, $B = \frac{\partial^2 f}{\partial y \partial x}(x_0, y_0)$, $C = \frac{\partial^2 f}{\partial y^2}(x_0, y_0)$, $D = AC - B^2$

3. $\frac{\partial z}{\partial x} = -\frac{f_x}{f_z}$, $\frac{\partial z}{\partial y} = -\frac{f_y}{f_z}$

4. $df = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z$

5. $\nabla f(x, y, z) = \frac{\partial f}{\partial x}\underline{i} + \frac{\partial f}{\partial y}\underline{j} + \frac{\partial f}{\partial z}\underline{k}$

6. $\nabla f(x, y) = \lambda \nabla g(x, y)$

7. $x = r\cos\theta$, $y = r\sin\theta$, $dA = r\,dr\,d\theta$

8. $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

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