



UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIM 201 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan.

1. (a) Consider the following matrices :

$$M = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix} \text{ and } (NM)^T = \begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}.$$

Find the matrix N .

(40 marks)

(b) If $\begin{vmatrix} -3g+2i & 2h & -i \\ -3d+2f & 2e & -f \\ -3a+2c & 2b & -c \end{vmatrix} = 12$, find $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$.

(40 marks)

- (c) Determine the type of solution for the homogenous system of linear equations below:

$$\begin{aligned} 2x + 2z &= 0 \\ x + y - 3z &= 0 \\ 3x - y + 4z &= 0 \end{aligned}$$

(20 marks)

2. Consider the following system of linear equations:

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 3 \\ 3x + 3y + kz &= k^2 - 6 \end{aligned}$$

- (a) Determine all values of k so that the system above will
- (i) have a unique solution;
 - (ii) have no solution;
 - (iii) have infinitely many solutions.

(60 marks)

- (b) If $k = 0$, solve for x using Cramer's rule.

(40 marks)

3. (a) Solve the system of linear equations by using Gauss-Jordan elimination method.

$$\begin{aligned}x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\5x_3 + 10x_4 + 15x_6 &= 0 \\2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 6\end{aligned}$$

(40 marks)

- (b) Solve the following systems of linear equations using the inverse matrix.

(i)
$$\begin{aligned}x_1 + x_2 + x_3 &= 0 \\2x_1 + 4x_2 &= 0 \\6x_2 - 5x_3 &= 0\end{aligned}$$

(ii)
$$\begin{aligned}x_1 + x_2 + x_3 &= 1 \\2x_1 + 4x_2 &= -2 \\6x_2 - 5x_3 &= 4\end{aligned}$$

(60 marks)

4. (a) Let W be a subspace spanned by $u_1 = (1,2,3,0)$, $u_2 = (1,2,0,0)$ and $u_3 = (1,0,0,1)$. Find
- (i) an orthogonal basis of W .
- (ii) an orthonormal basis of W .

(50 marks)

(b) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$.

- (i) Find elementary matrices E_1, E_2, E_3 , such that $E_3E_2E_1A = U$ where U is an upper triangular matrix.
- (ii) Determine the inverses of E_1, E_2, E_3 , and set $L = E_1^{-1}E_2^{-1}E_3^{-1}$. What type of matrix is L ? Verify that $A = LU$.

(50 marks)

5. (a) Consider the matrix

$$A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

- (i) Find the eigenvalues and corresponding eigenvectors of A .
- (ii) Is A diagonalizable? Give a reason for your answer.

(50 marks)

(b) Let $B = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. Find

- (i) the nullspace of B ,
- (ii) a basis for row space of B ,
- (iii) a basis for column space of B ,
- (iv) nullity of B ,
- (v) rank of B .

(50 marks)

1. (a) Pertimbangkan matriks berikut :

$$M = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix} \text{ dan } (NM)^T = \begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}.$$

Cari matriks N .

(40 markah)

(b) Jika $\begin{vmatrix} -3g+2i & 2h & -i \\ -3d+2f & 2e & -f \\ -3a+2c & 2b & -c \end{vmatrix} = 12$, cari $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$.

(40 markah)

- (c) Tentukan jenis penyelesaian kepada sistem persamaan linear homogen berikut:

$$\begin{aligned} 2x + 2z &= 0 \\ x + y - 3z &= 0 \\ 3x - y + 4z &= 0 \end{aligned}$$

(20 markah)

2. Pertimbangkan sistem persamaan linear yang berikut:

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 3 \\ 3x + 3y + kz &= k^2 - 6 \end{aligned}$$

- (a) Nyatakan semua nilai untuk k supaya sistem di atas akan

- (i) mempunyai penyelesaian yang unik;
- (ii) tiada penyelesaian;
- (iii) mempunyai penyelesaian yang tidak terhingga.

(60 markah)

- (b) Jika $k = 0$, selesaikan x menggunakan petua Cramer.

(40 markah)

3. (a) Selesaikan persamaan linear berikut menggunakan kaedah penghapusan Gauss-Jordan.

$$\begin{aligned}x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\5x_3 + 10x_4 + 15x_6 &= 0 \\2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 6\end{aligned}$$

(40 markah)

- (b) Selesaikan persamaan linear berikut menggunakan kaedah matriks songsang.

(i) $x_1 + x_2 + x_3 = 0$
 $2x_1 + 4x_2 = 0$
 $6x_2 - 5x_3 = 0$

(ii) $x_1 + x_2 + x_3 = 1$
 $2x_1 + 4x_2 = -2$
 $6x_2 - 5x_3 = 4$

(60 markah)

4. (a) Katakan W subruang direntangi oleh $u_1 = (1,2,3,0)$, $u_2 = (1,2,0,0)$ dan $u_3 = (1,0,0,1)$. Cari

- (i) asas ortogon W .
(ii) asas orthonormal W .

(50 markah)

(b) Diberi $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$.

- (i) Cari operasi baris permulaan E_1, E_2, E_3 , seperti $E_3E_2E_1A = U$ di mana U adalah matriks segitiga atas.
(ii) Tentukan songsangan ke atas E_1, E_2, E_3 , dan set $L = E_1^{-1}E_2^{-1}E_3^{-1}$. Apakah jenis matriks L ? Tentukan bahawa $A = LU$.

(50 marks)

5. (a) Pertimbangkan matriks

$$A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

- (i) Cari nilai eigen dan vektor eigen yang sepadan bagi matriks A .
- (ii) Adakah A terpepenjurukan? Berikan sebab jawapan anda.

(50 markah)

(b) Andaikan $B = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. Cari

- (i) ruang nol B ,
- (ii) suatu asas bagi ruang baris B ,
- (iii) suatu asas bagi ruang lajur B ,
- (iv) nol B ,
- (v) pangkat B .

(50 markah)