UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIM 105 – Basic Mathematics
[Matematik Asas]

Duration: 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.

1. (a) Given that $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

Find A^{-1}.

(30 marks)

(b) Solve the following system of linear equations using Cramer's rule.

$$x + 2y = 1$$
$$y + z = 2$$
$$y + 2z = 3$$

(30 marks)

(c) Solve the following matrix equation using Gauss-Jordan elimination method.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 6 \end{bmatrix}$$

(40 marks)

2. (a) Find the values of *k* for the singular matrix $\begin{bmatrix} 3 & 2 & 2k \\ k & 0 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

(40 marks)

(b) Evaluate

(i) $\int 2\, e^{2x+1}\, dx$

(ii) $\int \left(\cos x + \frac{1}{x} - 4x^3 \right) dx$

(iii) $\int \frac{x^3}{x^4 + 2}\, dx$

(iv) $\int x^3 \ln x\, dx$

(60 marks)

3. (a) Compute

(i) $\int_0^1 3\left(3x-1\right)^4\,dx$

(ii) $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$

(iii) $\int_0^1 x\left(x^2+1\right)^3\,dx$

(iv) $\int_0^{\frac{\pi}{4}} \cos 2x \sin^3 2x\,dx$

(60 marks)

(b) Find the area bounded by the curve $y=x^3$ and the line $y=4x$ from $x=0$ to $x=2$.

(40 marks)

4. (a) Given that $f\left(x\right)=\begin{cases} x^2-1, & x<0 \\ 2x+1, & x\geq 0 \end{cases}$

(i) Evaluate $\lim_{x\to 0} f\left(x\right)$.

(ii) Determine if $f\left(x\right)$ is continuous at $x=0$.

(40 marks)

(b) Find $\frac{dy}{dx}$ if

(i) $y=e^x\left(2x-5\right)^7$

(ii) $y=\frac{\sin 3x}{2x^2}$

(iii) $yx^4-3x^2-y^3=0$

(40 marks)

(c) Use the definition of derivative to compute $f'\left(x\right)$ where $f\left(x\right)=3x^2+4$.

(20 marks)

5. (a) Find the equation of the tangent line of the curve $y = 2x^2 + 1$ at the point $x = 1$.

(30 marks)

(b) Given the function $f(x) = x^3 - 3x^2$.

(i) Find all critical points.

(ii) Determine all the local maximum and minimum.

(iii) Determine the intervals where the graph of $f(x)$ is concave up and concave down.

(iv) Find the inflection point.

(v) Sketch the graph of $f(x)$ showing the features above.

(70 marks)

1. (a) Diberi $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

Dapatkan A^{-1}.

(30 markah)

(b) Selesaikan sistem persamaan linear berikut dengan menggunakan Petua Cramer.

$$\begin{aligned} x + 2y &= 1 \\ y + z &= 2 \\ y + 2z &= 3 \end{aligned}$$

(30 markah)

(c) Selesaikan persamaan matriks berikut dengan menggunakan kaedah penghapusan Gauss-Jordan.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 6 \end{bmatrix}$$

(40 markah)

2. (a) Dapatkan nilai-nilai *k* bagi matriks singular $\begin{bmatrix} 3 & 2 & 2k \\ k & 0 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

(40 markah)

(b) Nilaikan

(i) $\int 2\, e^{2x+1}\, dx$

(ii) $\int \left(\cos x + \frac{1}{x} - 4x^3 \right) dx$

(iii) $\int \frac{x^3}{x^4 + 2}\, dx$

(iv) $\int x^3 \ln x\, dx$

(60 markah)

3. (a) Hitungkan

(i) $\int_0^1 3\left(3x-1\right)^4\,dx$

(ii) $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$

(iii) $\int_0^1 x\left(x^2+1\right)^3\,dx$

(iv) $\int_0^{\frac{\pi}{4}} \cos 2x \sin^3 2x\,dx$

(60 markah)

(b) Cari luas rantau yang dibatasi oleh lengkung $y=x^3$ dan garis $y=4x$ dari $x=0$ ke $x=2$.

(40 markah)

4. (a) Diberi $f\left(x\right)=\begin{cases} x^2-1, & x<0 \\ 2x+1, & x\geq 0 \end{cases}$

(i) Nilaikan $\lim\limits_{x\to 0} f\left(x\right)$.

(ii) Tentukan sama ada $f\left(x\right)$ selanjar pada $x=0$.

(40 markah)

(b) Cari $\frac{dy}{dx}$ jika

(i) $y=e^x\left(2x-5\right)^7$

(ii) $y=\frac{\sin 3x}{2x^2}$

(iii) $yx^4-3x^2-y^3=0$

(40 markah)

(c) Gunakan takrif terbitan untuk mendapatkan $f'\left(x\right)$ di mana $f\left(x\right)=3x^2+4$.

(20 markah)

5. (a) Dapatkan persamaan garis tangen bagi lengkung $y = 2x^2 + 1$ pada titik $x = 1$.

(30 markah)

(b) Diberi fungsi $f(x) = x^3 - 3x^2$.

(i) Cari semua titik genting.

(ii) Tentukan semua maksimum setempat dan minimum setempat.

(iii) Tentukan selang di mana graf $f(x)$ adalah cekung ke atas dan cekung ke bawah.

(iv) Cari titik lengkok balas.

(v) Lakarkan graf $f(x)$ dengan menunjukkan ciri-ciri di atas.

(70 markah)

- oooOooo -