



UNIVERSITI SAINS MALAYSIA

Final Examination  
2016/2017 Academic Session

May/June 2017

**JIM 101 – Calculus**  
*[Kalkulus]*

Duration: 3 hours  
*[Masa: 3 jam]*

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Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

*Baca arahan dengan teliti sebelum anda menjawab soalan.*

*Setiap soalan diperuntukkan 100 markah.*

*Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan.*

1. (a) If  $x$  is a real number, show that

$$\frac{2}{3} \leq \frac{x^2 + 1}{x^2 + x + 1} \leq 2.$$

(40 marks)

- (b) Given a complex number

$$z = \frac{2i - 2\sqrt{3}}{i}.$$

Find the modulus and argument.

(20 marks)

- (c) Convert each of the following points into the given coordinate system

(i) from polar coordinates  $\left(5, \frac{\pi}{2}\right)$  into cartesian coordinates,

(ii) from cartesian coordinates  $(4, -8)$  into polar coordinates.

(40 marks)

2. (a) Evaluate the following limits if exist:

(i)  $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}.$

(ii)  $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}.$

(40 marks)

- (b) Find the values of  $a$  and  $b$  that make  $f$  continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ ax^2 - bx + 3, & 2 \leq x < 3 \\ 2x - a + b, & x \geq 3 \end{cases}$$

(40 marks)

- (c) The function  $f'$  defined by the formula

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

is called the derivatives of  $f$  with respect to  $x$ , Use this definition to find  $f'$  for the function  $f(x) = x^2$ .

(20 marks)

3. (a) Find  $\frac{dy}{dx}$  for each of the following functions:

(i)  $y = (5x^3 - 4x + 9)^{17}$

(ii)  $y = \frac{\sin^2 3x}{1 - \sin 3x}$

(iii)  $y = \ln \sqrt{\frac{2x-3}{2x+3}}$

(iv)  $y = \tanh\left(\frac{3x}{2x-5}\right)$

(50 marks)

- (b) Consider the curve defined by  $x^3 + y^3 = \frac{9}{2}xy$ . Find the equation of the tangent line at the point (1,2).

(30 marks)

- (c) Find the values of  $p$ ,  $q$ ,  $r$ , and  $s$  so that the function

$$h(x) = px^3 + qx^2 + rx + s$$

has a local minimum at (0,0) and a local maximum at (1,1).

(20 marks)

4. (a) Evaluate the following integrals:

(i)  $\int (2x + 3)^{10} dx$

(ii)  $\int x e^{-x^2} dx$

(iii)  $\int_1^2 x^2 \ln(x) dx$

(iv)  $\int_4^5 \left( \frac{\sqrt{x^2 - 16}}{x^2} \right) dx$

(70 marks)

(b) Find the area enclosed by  $f(x) = x^2 - 4$ , and the  $x$ -axis from  $x = 0$  to  $x = 3$ .

(30 marks)

5. (a) Identify the curve  $4x^2 - 16x + y^2 + 2y - 8 = 0$ .

(40 marks)

(b) For each of the following hyperbolas, find the distance of the two foci, vertex, and asymptotes.

(i)  $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{25} = 1$ .

(ii)  $-3x^2 + y^2 - 12x = 0$ .

(60 marks)

1. (a) Sekiranya  $x$  adalah nombor nyata, tunjukkan

$$\frac{2}{3} \leq \frac{x^2 + 1}{x^2 + x + 1} \leq 2.$$

(40 markah)

- (b) Diberi nombor kompleks

$$z = \frac{2i - 2\sqrt{3}}{i}.$$

Tentukan modulus dan hujah.

(20 markah)

- (c) Tukarkan sistem koordinat berikut

(i) daripada koordinat polar  $\left(5, \frac{\pi}{2}\right)$  kepada koordinat cartesian,

(ii) daripada koordinat cartesian  $(4, -8)$  kepada koordinat polar.

(40 markah)

2. (a) Dapatkan nilai had berikut jika wujud:

(i)  $\text{had}_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}.$

(ii)  $\text{had}_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}.$

(40 markah)

- (b) Cari nilai untuk  $a$  dan  $b$  yang menjadikan  $f$  selanjur di mana-mana.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2 \\ ax^2 - bx + 3, & 2 \leq x < 3 \\ 2x - a + b, & x \geq 3 \end{cases}$$

(40 markah)

(c) Fungsi  $f'$  yang ditakrifkan oleh rumus

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

dipanggil terbitan fungsi  $f$  terhadap  $x$ . Gunakan takrifan ini untuk mencari  $f'$  bagi fungsi  $f(x) = x^2$ .

(20 markah)

3. (a) Cari  $\frac{dy}{dx}$  bagi setiap fungsi berikut:

(i)  $y = (5x^3 - 4x + 9)^{17}$

(ii)  $y = \frac{\sin^2 3x}{1 - \sin 3x}$

(iii)  $y = \ln \sqrt{\frac{2x-3}{2x+3}}$

(iv)  $y = \tanh\left(\frac{3x}{2x-5}\right)$

(50 markah)

(b) Pertimbangkan lengkung yang ditakrifkan oleh  $x^3 + y^3 = \frac{9}{2}xy$ . Cari persamaan garis tangen pada titik (1,2).

(30 markah)

(c) Cari nilai bagi  $p$ ,  $q$ ,  $r$ , dan  $s$  supaya fungsi

$$h(x) = px^3 + qx^2 + rx + s$$

mempunyai minimum setempat pada (0,0) dan maksimum setempat pada (1,1).

(20 markah)

4. (a) Cari nilai kamiran berikut:

(i)  $\int (2x + 3)^{10} dx$

(ii)  $\int xe^{-x^2} dx$

(iii)  $\int_1^2 x^2 \ln(x) dx$

(iv)  $\int_4^5 \left( \frac{\sqrt{x^2 - 16}}{x^2} \right) dx$

(70 markah)

(b) Cari luas yang dibatasi oleh lengkung  $f(x) = x^2 - 4$ , dan satah- $x$  dari  $x = 0$  sehingga  $x = 3$ .

(30 markah)

5. (a) Kenal pasti lengkung  $4x^2 - 16x + y^2 + 2y - 8 = 0$ .

(40 markah)

(b) Untuk setiap hiperbola berikut, cari jarak antara dua fokus, verteks dan asimptot.

(i)  $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{25} = 1$ .

(ii)  $-3x^2 + y^2 - 12x = 0$ .

(60 markah)