UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIF 318 – Quantum Mechanics
[Mekanik Kuantum]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **THIRTEEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in English or in Bahasa Malaysia.

Read the instructions carefully before answering.

In the event of any discrepancies in the exam questions, the English version shall be used.

*Sila pastikan kertas peperiksaan ini mengandungi **TIGA BELAS** muka surat yang bercetak sebelum anda menjawab sebarang soalan.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca setiap arahan dengan teliti sebelum menjawab.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Answer **ALL** questions.
*Jawab **SEMUA** soalan.*

1. (a) Planck successfully described the frequency distribution of a black body radiation by deriving the equation
Planck berjaya memerihalkan taburan frekuensi suatu jasad hitam dengan menerbitkan persamaan

$$\rho(v)dv = \frac{8\pi h}{c^3}\frac{v^3}{\left(e^{hv/kT}-1\right)}dv$$

where ρ is the energy density of the black body radiation. What were the differences between his postulates and those used by Rayleigh-Jeans and other classical physicists?
di sini ρ ialah ketumpatan tenaga sinaran jasad hitam. Apakah perbezaan antara postulat-postulat beliau dengan postulat yang digunakan oleh Rayleigh-Jeans dan ahli fizik klasik lain?

(6 marks/*markah*)

(b) Describe two experiments proving the existence of matter waves.
Perihalkan dua ujikaji yang membuktikan kewujudan gelombang jirim.

(8 marks/*markah*)

(c) In quantum mechanics, the motion of a particle in space is represented by a wave function. Give reasons for doing this.
Dalam mekanik kuantum, gerakan suatu zarah dalam ruang diwakili oleh suatu fungsi gelombang. Berikan sebab mengapa ini dilakukan.

(6 marks/*markah*)

2. (a) A complex number is given as $z = Ae^{icx}$ where A and c are constants. Calculate

Suatu nombor kompleks diberikan sebagai $z = Ae^{icx}$ *di sini A dan c adalah pemalar-pemalar. Hitung*

(i) $z*$

(ii) $z + z*$

(iii) $zz*$

(iv) z^2

(v) $|z|^2$

(10 marks/*markah*)

(b) Prove that two commutative operators have the same set of eigenfunctions.

Buktikan bahawa dua operator yang berkomut mempunyai set fungsi eigen yang sama.

(4 marks/*markah*)

(c) Evaluate the following commutators:

Nilaikan komutator-komutator berikut:

(i) $\left[\hat{H}_x, \hat{P}_x\right]$

(ii) $\left[\hat{P}_x^2, \hat{X}^2\right]$

(6 marks/*markah*)

3. (a) Time-independent Schrodinger equation is given by

Persamaan Schrodinger tak bersandar masa diberikan oleh

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}\left[V(x) - E\right]\psi(x)$$

Explain what will happen if $E < V_{\min}$, where $V_{\min}$ is the minimum of the potential $V(x)$.

Jelaskan apa yang akan berlaku jika $E < V_{\min}$, di sini $V_{\min}$ *adalah minimum keupayaan* $V(x)$.

(6 marks/*markah*)

(b) The solution of a time-independent Schrodinger equation is given by

Penyelesaian suatu persamaan Schrodinger tak bersandar masa diberikan oleh

$$\psi(x) = Ae^{-i(a/2)x^2}$$

for $-\infty < x < +\infty$, where a and A are constants. Given $a = \dfrac{k}{\hbar\omega}$.

bagi $-\infty < x < +\infty$, *di sini a dan A adalah pemalar-pemalar. Diberikan* $a = \dfrac{k}{\hbar\omega}$.

(i) Determine A.

Tentukan A.

(ii) Calculate the expectation value of position $\langle x \rangle$.

Hitung nilai jangkaan kedudukan $\langle x \rangle$.

(iii) Calculate the expectation value of momentum $\langle p \rangle$.

Hitung nilai jangkaan momentum $\langle p \rangle$.

(14 marks/*markah*)

4. (a) Discuss the ‘tunneling effect’ or ‘barrier penetration’. Give examples.

Bincangkan ‘kesan penerowongan’ atau ‘penerobosan sawar’. Berikan contoh-contoh.

(6 marks/*markah*)

(b) Figure 1 shows an entity with energy E moving in the positive x direction towards a step potential V_0. Given that $E < V_0$. Determine

Rajah 1 menunjukkan suatu entiti dengan tenaga E bergerak dalam arah x positif menuju suatu keupayaan bertangga V_0. *Diberikan* $E < V_0$. *Tentukan*

(i) the reflection coefficient R

pekali pantulan R

(ii) the transmission coefficient T.

pekali transmissi T.

(iii) Discuss the results obtained with those expected from classical physics.

Bincangkan keputusan yang diperolehi dengan keputusan yang dijangkakan dari fizik klasik.

(14 marks/*markah*)

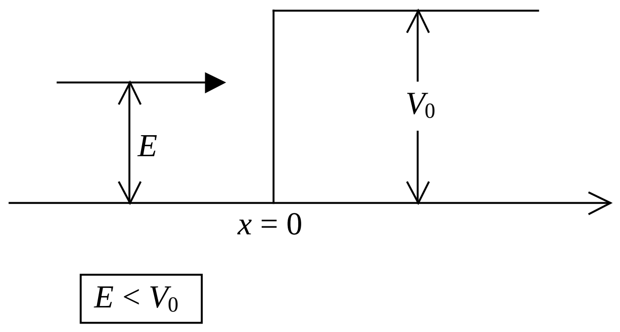

Figure 1

5. (a) Consider a 1-D infinite square-well potential of width *a* where
Pertimbangkan suatu keupayaan telaga persegi infinit berkelebaran a dengan

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \\ \infty & \text{for} \quad x < 0 \,\&\, x > a \end{cases}$$

For $V >> E$, show that the energy in the well is quantized and is given by
Untuk V >> E, tunjukkan bahawa tenaga dalam telaga adalah terkuanta dan diberikan oleh

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2, \qquad n = 1, 2, 3, ...$$

(12 marks/*markah*)

(b) The energy levels of a particle in a 3-D cubical box is given by
Paras-paras tenaga suatu zarah dalam suatu kotak kubus 3-D diberikan oleh

$$E = \left(\frac{\hbar^2 \pi^2}{2ma^2}\right)\left(n_x^2 + n_y^2 + n_z^2\right)$$

where *m* is the mass of the particle, *a* is the length of the side of the box, and *n* is the energy state of the particle. Describe how this equation indicates the existence of the energy degeneracy for a particle state in 3-D. Why is this property does not exist for a 1-D situation?
dengan m ialah jisim zarah, a ialah panjang sisi kotak dan n ialah keadaan tenaga zarah. Perihalkan bagaimana persamaan ini menunjukkan kewujudan tenaga terdegenerat bagi satu keadaan zarah dalam 3-D. Mengapa sifat ini tidak wujud bagi situasi 1-D?

(8 marks/*markah*)

Constants:

Speed of light $c = 3.0 \times 10^{8}$ m s^{-1}

Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

Planck constant $h = 6.63 \times 10^{-34}$ J s

Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}

Permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}

Permeability of free space $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

Basic charge $e = 1.6 \times 10^{-19}$ C

Electron rest-mass $m_e = 9.1 \times 10^{-31}$ kg

Proton rest-mass $m_p = 1.6725 \times 10^{-27}$ kg $\equiv$ 1.0072766 u

Neutron rest-mass $m_n = 1.6748 \times 10^{-27}$ kg $\equiv$ 1.0086654 u

Bohr's radius $a = 5.3 \times 10^{-11}$ m

1 eV = 1.6×10^{-19} J

1 u $\equiv$ 931 MeV c^{-2}

1 barn = 10^{-28} m^{2}

1 fm = 10^{-15} m

1 Ci = 3.7×10^{10} s^{-1}

USEFUL MATHEMATICS IN QUANTUM MECHANICS

Exponential series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

Trigonometric series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Differentiation and integration (Standard forms)

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$ $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+c$ $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}+c$
$\frac{d}{dx}\log x = \frac{1}{x}$ $\frac{d}{dx}\log(ax+b) = \frac{a}{ax+b}$	$\int \frac{dx}{x} = \log x + c$ $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)+c$
$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}e^{mx} = me^{mx}$	$\int e^x dx = e^x + c$ $\int e^{mx} dx = \frac{e^{mx}}{m}+c$
$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\sin mx = m\cos mx$	$\int \cos x\, dx = \sin x + c$ $\int \cos mx\, dx = \frac{\sin mx}{m}+c$
$\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\cos mx = -m\sin mx$	$\int \sin x\, dx = -\cos x + c$ $\int \sin mx\, dx = -\frac{\cos mx}{m}+c$
$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\tan mx = m\sec^2 mx$	$\int \sec^2 x\, dx = \tan x + c$ $\int \sec^2 mx\, dx = \frac{\tan mx}{m}+c$
$\frac{d}{dx}\cot x = -\mathrm{cosec}^2 x$ $\frac{d}{dx}\cot mx = -m\,\mathrm{cosec}^2 mx$	$\int \mathrm{cosec}^2 x\, dx = -\cot x + c$ $\int \mathrm{cosec}^2 mx\, dx = -\frac{\cot mx}{m}+c$
$\frac{d}{dx}\sinh x = \cosh x$	$\int \cosh x\, dx = \sinh x + c$
$\frac{d}{dx}\cosh x = \sinh x$	$\int \sinh x\, dx = \cosh x + c$

Integration by substitution

Some common types of substitutions:

(i) Integrals containing a term of the form $(ax+b)^n \rightarrow$ let $ax+b=z$.

(ii) Integrals containing a term in the form of $\sqrt{a^2-x^2}$ or $\left(\sqrt{a^2-x^2}\right)^n \rightarrow$ let $x=a\sin\theta$.

(iii) As in (ii) but with $\sqrt{a^2-x^2}$ replaced by $\sqrt{a^2+x^2} \rightarrow$ let $x=a\sinh\theta$ or $x=a\tan\theta$.

(iv) As in (ii) but with $\sqrt{a^2-x^2}$ replaced by $\sqrt{x^2-a^2} \rightarrow$ let $x=a\cosh\theta$.

(v) Integrals of *odd powers of sine or cosine* $\rightarrow$ let $\cos x=c$ or $\sin x=s$ respectively.

(vi) Integrals of the form $\int \frac{dx}{a+b\cos x}$ or $\int \frac{dx}{a+b\sin x} \rightarrow$ let $\tan\frac{x}{2}=t$.

(vii) Integrals of the form $\int \frac{dx}{x\sqrt{ax^2+bx+c}} \rightarrow$ let $x=\frac{1}{z}$;

or $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}} \rightarrow$ let $px+q=\frac{1}{z}$.

Integration by parts

$$\int u\upsilon\, dx = u\int \upsilon\, dx - \int\left\{\int \upsilon\, dx\right\}\frac{du}{dx}\, dx$$

Integration common in Quantum Mechanics

$$f(x) = \int_0^\infty x^n e^{-ax^2}\,dx$$

n	$f(n)$	n	$f(n)$
0	$\frac{1}{2}\sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4}\sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8}\sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16}\sqrt{\frac{\pi}{a^7}}$	7	$\frac{3}{a^4}$

If n is even, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\,dx = 2f(x)$

If n is odd, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\,dx = 0$

Other standard integrals

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$$\int_0^\infty \frac{x}{(e^x - 1)}\,dx = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^3}{(e^x - 1)}\,dx = \frac{\pi^4}{15}$$

Reciprocal identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \qquad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u}$$

Pythagorean identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

Quotient identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Co-function identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

Parity identities (even & odd)

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \qquad \cot(-u) = -\cot u$$
$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u$$

Sum & difference formulas

$$\sin(u \pm \upsilon) = \sin u \cos \upsilon \pm \cos u \sin \upsilon$$
$$\cos(u \pm \upsilon) = \cos u \cos \upsilon \mp \sin u \sin \upsilon$$
$$\tan(u \pm \upsilon) = \frac{\tan u \pm \tan \upsilon}{1 \mp \tan u \tan \upsilon}$$

Double angle formulas

$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

Power reducing/half angle formulas

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$
$$\cos^2 u = \frac{1+\cos(2u)}{2}$$
$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

Sum-to-product formulas

$$\sin u + \sin \upsilon = 2\sin\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\sin u - \sin \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u + \cos \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u - \cos \upsilon = -2\sin\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$

Product-to-sum formulas

$$\sin u \sin \upsilon = \frac{1}{2}[\cos(u-\upsilon) - \cos(u+\upsilon)]$$
$$\cos u \cos \upsilon = \frac{1}{2}[\cos(u-\upsilon) + \cos(u+\upsilon)]$$
$$\sin u \cos \upsilon = \frac{1}{2}[\sin(u+\upsilon) + \sin(u-\upsilon)]$$
$$\cos u \sin \upsilon = \frac{1}{2}[\sin(u+\upsilon) - \sin(u-\upsilon)]$$

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