UNIVERSITI SAINS MALAYSIA

Final Examination
2016/2017 Academic Session

May/June 2017

JIF 315 – Mathematical Methods
[Kaedah Matematik]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question carries 100 marks.

In the event of any discrepancies in the exam questions, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Table of Laplace Transform

[Jadual transformasi Laplace]

$f(t)$	$L\{f(t)\} = F(s)$
a	$\frac{a}{s}$
$t^n \,, \quad n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(at)$	$\frac{a}{s^2 + a^2}$
$\cos(at)$	$\frac{s}{s^2 + a^2}$
$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$e^{at} f(t)$	$F(s-a)$

Legendra Polynomial Function

[Jadual fungsi Legendra Polynomial]

$P_0(x) = 1,$ $P_1(x) = x,$

$P_2(x)=\frac{1}{2}(3x^2-1),$ $P_3(x)=\frac{1}{2}(5x^3-3x),$

$P_4(x)=\frac{1}{8}(35x^4-30x^2+3),$ $P_5(x)=\frac{1}{8}(63x^5-70x^3+15x)$

Answer ALL questions.

1. (a) Find the inverse Laplace

$$L^{-1}\left\{\frac{2}{(s+5)^4}\right\}$$

(20 marks)

(b) Find the Laplace transforms of the following function:

$$f(t)=\begin{cases} 0, & 0\le t<1 \\ t-1, & 1\le t<2 \\ 0, & t\ge 2 \end{cases}$$

(40 marks)

(c) Solve the following function using the Laplace transform method.

$$y''(t)+4y'(t)+4y(t)=t^2e^{-2t};\quad y(0)=0,\ y'(0)=0$$

(40 marks)

2. (a) The Legendre's Polynomial $P_n(x)$ is given by

$$(n+1)P_{n+1}(x)-(2n+1)xP_n(x)+P_{n-1}(x)=0$$

and

$$P_0(x)=1$$
$$P_1(x)=x$$

Show that

(i) $P_3(x)=\frac{1}{2}\left(5x^3-3x\right)$

(ii) $P_4(x)=\frac{1}{8}\left(35x^4-30x^2+3\right)$

(50 marks)

(b) Consider the following form of differential equation.

$$y''-\frac{2a-1}{x}y'+\left[b^2c^2x^{2c-2}+\frac{a^2-v^2c^2}{x^2}\right]y=0$$

has the solution

$$x^a J_v\left(bx^2\right)$$

Find the general solution of the following equation in terms of the Bessel function.

(i) $y''+\frac{1}{x}y'+\left(9x^4-\frac{9}{25x^2}\right)y=0$

(ii) $y''-\frac{3}{x}y'+\left(4x^2-\frac{60}{x^2}\right)y=0$

(50 marks)

3. Determine all the eigenvalues and the corresponding eigenfunction of the following Sturm-Liouville problem

$$-y''(x)=\lambda y(x), \qquad y(0)=0, \qquad y(1)+y'(1)=0$$

and find the associated eigenfunction expansion of function f(x). Consider all cases for λ.

(100 marks)

4. (a) Let $f(x)$ be a function of period 2π such that

$$f(x)=\begin{cases}\pi-x, 0<x<\pi\\ 0, \pi<x<2\pi\end{cases}$$

Find the Fourier series for $f(x)$.

(55 marks)

(b) Using the result in (a), show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} \ldots$$

(25 marks)

(c) Sketch the graph of the function $f(x)$ in the interval $-2\pi < x < 2\pi$

(20 marks)

5. (a) Consider the following form of differential equation.

$$f(x) = \begin{cases} -2 & -3 < x < 0 \\ 5e^x & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the Fourier transform of $f(x)$.

(50 marks)

(b) Consider a vibrating string of length L = 30 that satisfies the wave equation

$$4u_{xx} = u_{tt}, \quad 0 < x < 30, \quad t > 0$$

Assume that the ends of the string are fixed and that string is set in motion with no initial velocity from the initial position.

$$u(x,0) = f(x) = \begin{cases} \dfrac{x}{10}, & 0 \le x \le 10, \\ \dfrac{30 - x}{20}, & 10 < x \le 30 \end{cases}$$

Find the displacement *u(x ,t)* of the string and describe its motion through one period.

(50 marks)

Jawab semua soalan.

1. (a) *Carikan transformasi Laplace songsang bgi fungsi yang berikut.*

$$L^{-1}\left\{\frac{2}{(s+5)^4}\right\}$$

(20 markah)

(b) *Carikan tranformasi Laplace bagi fungsi yang berikut.*

$$f(t)=\begin{cases}0, & 0\le t<1\\ t-1, & 1\le t<2\\ 0, & t\ge 2\end{cases}$$

(40 markah)

(c) *Selesaikan fungsi berikut menggunakan kaedah transformasi Laplace.*

$$y''(t)+4y'(t)+4y(t)=t^2e^{-2t};\quad y(0)=0,\ y'(0)=0$$

(40 markah)

2. (a) *Polinomial Legendre* $P_n(x)$ *adalah seperti berikut*

$$(n+1)P_{n+1}(x)-(2n+1)xP_n(x)+P_{n-1}(x)=0$$

dan

$$P_0(x)=1$$
$$P_1(x)=x$$

Tunjukkan

(i) $P_3(x)=\frac{1}{2}\left(5x^3-3x\right)$

(ii) $P_4(x)=\frac{1}{8}\left(35x^4-30x^2+3\right)$

(50 markah)

(b) *Pertimbangkan persamaan pembezaan berikut.*

$$y''-\frac{2a-1}{x}y'+\left[b^2c^2x^{2c-2}+\frac{a^2-v^2c^2}{x^2}\right]y=0$$

$$x^a J_v\left(bx^2\right)$$

Cari penyelesaian am bagi persamaan berikut dalam sebutan fungsi Bessel.

(i) $$y''+\frac{1}{x}y'+\left(9x^4-\frac{9}{25x^2}\right)y=0$$

(ii) $$y''-\frac{3}{x}y'+\left(4x^2-\frac{60}{x^2}\right)y=0$$

(50 markah)

3. *Cari semua nilai-eigen dan fungsi-eigen yang sepadan bagi masalah Sturm-Liouville berikut*

$$-y''(x)=\lambda y(x), \qquad y(0)=0, \qquad y(1)+y'(1)=0$$

dan cari pengembangan fungsi eigen yang berkaitan fungsi f (x). Pertimbangkan semua kes bagi λ .

(100 markah)

4. *Andaikan f(x) adalah fungsi tempoh 2π seperti*

$$f(x)=\begin{cases}\pi-x, 0<x<\pi \\ 0, \pi<x<2\pi\end{cases}$$

(a) *Cari siri Fourier bagi* $f(x)$.

(55 markah)

(b) *Dengan menggunakan keputusan di (a), tunjukkan bahawa*

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}\cdots$$

(25 markah)

(c) *Lakarkan graf bagi fungsi* $f(x)$ *dalam lingkungan* $-2\pi < x < 2\pi$

(20 markah)

5. (a) *Andaikan persamaan perbezaan seperti berikut*

$$f(x) = \begin{cases} -2, & -3 < x < 0 \\ 5e^{x}, & 0 < x < 3 \\ 0, & lain\text{-}lain \end{cases}$$

Tentukan transformasi Fourier bagi $f(x)$.

(50 markah)

(b) *Pertimbangkan tali bergetar dengan panjang, L = 30 yang mematuhi persamaan gelombang*

$$4u_{xx} = u_{tt}, \quad 0 < x < 30, \quad t > 0$$

Andaikan hujung tali adalah tetap dan tali ditetapkan dalam gerakan tanpa halaju awal dari kedudukan awal.

$$u(x,0) = f(x) = \begin{cases} \dfrac{x}{10}, & 0 \le x \le 10, \\ \dfrac{30-x}{20}, & 10 < x \le 30 \end{cases}$$

Cari sesaran u (x, t) tali dan merikan gerakan tersebut melalui satu tempoh.

(50 markah)

-oooOooo -