UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 417 – Partial Differential Equations
[Persamaan Pembezaan Separa]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **SIX** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Solve the given boundary value problem using Laplace transforms:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=0, \qquad u(0,t)=0 \quad t>0, \quad \text{and} \quad u(x,0)=0 \quad x>0.$$

(50 marks)

(b) Show that

$$u(x,t)=e^{-t}[\sin(x-t)+H(t-x)\sin(t-x)]$$

is the solution for the following boundary value problem:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}+u=0, \qquad u(0,t)=0 \quad t>0, \quad \text{and} \quad u(x,0)=\sin(x) \quad x>0.$$

(50 marks)

2. Given a function

$$f(x)=\begin{cases} 0, & -\pi \le x \le 0, \\ \pi - x, & 0 < x \le \pi, \end{cases}$$

$$f(x+2\pi)=f(x).$$

(a) Sketch the graph of function $f(x)$.

(30 marks)

(b) Determine the Fourier series expansion of $f(x)$.

(70 marks)

3. Given a partial differential equation

$$V_{xx}=V_{yy}+5V_{y}.$$

(a) State the order and type of the equation above.

(20 marks)

(b) Show that

$$V(x,y)=e^{-6x}(Ae^{4y}+Be^{-9y})$$

is a general solution of the equation above.

(40 marks)

(c) Determine the values of *A* and *B* from (b) if the solution satisfies the initial conditions:

$$V(x,0) = e^{-6x},$$
$$V_y(0,0) = 0.$$

(40 marks)

4. By using the method of separation of variables, solve the following boundary value problem:

$$\frac{\partial u}{\partial t} - 2kt\frac{\partial^2 u}{\partial x^2} = 0 \ , \ 0 < x < \pi, \ \ t > 0$$

$$u(x,0) = 2\sin 2x - 5\sin 3x \ , \ 0 \le x \le \pi \ ,$$

$$u(0,t) = u(\pi,t) = 0 \ , \ t \ge 0 \ .$$

(100 marks)

5. Reduce the equation to canonical form and then solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} + 1 = 0,$$

in $0 \le x \le 1, \quad y > 0, \quad$ with $\ u = \dfrac{\partial u}{\partial y} = x \ $ on $\ y = 0$.

(100 marks)

1. (a) Selesaikan masalah nilai sempadan yang diberi menggunakan jelmaan Laplace:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=0, \qquad u(0,t)=0 \quad t>0, \quad \text{dan} \quad u(x,0)=0 \quad x>0.$$

(50 markah)

(b) Tunjukkan bahawa

$$u(x,t)=e^{-t}[\sin(x-t)+H(t-x)\sin(t-x)]$$

adalah penyelesaian kepada masalah nilai sempadan berikut:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}+u=0, \qquad u(0,t)=0 \quad t>0, \quad \text{dan} \quad u(x,0)=\sin(x) \quad x>0.$$

(50 markah)

2. Diberi fungsi

$$f(x)=\begin{cases} 0, & -\pi \le x \le 0, \\ \pi - x, & 0 < x \le \pi, \end{cases}$$

$f(x+2\pi)= f(x)$.

(a) Lakarkan graf fungsi $f(x)$.

(30 markah)

(b) Tentukan kembangan siri Fourier bagi $f(x)$.

(70 markah)

3. Diberi persamaan pembezaan separa

$$V_{xx} = V_{yy} + 5V_y .$$

(a) Nyatakan peringkat dan jenis persamaan di atas.

(20 markah)

(b) Tunjukkan

$$V(x, y) = e^{-6x}(Ae^{4y} + Be^{-9y})$$

ialah penyelesaian am bagi persamaan di atas.

(40 markah)

(c) Tentukan nilai-nilai A dan B daripada (b) jika penyelesaiannya menepati nilai awal:

$$V(x,0) = e^{-6x},$$
$$V_y(0,0) = 0.$$

(40 markah)

4. Dengan menggunakan kaedah pemisahan pembolehubah, selesaikan masalah nilai sempadan berikut :

$$\frac{\partial u}{\partial t} - 2kt\frac{\partial^2 u}{\partial x^2} = 0 \ , \ 0 < x < \pi, \ \ t > 0$$

$$u(x,0) = 2\sin 2x - 5\sin 3x \ , \ 0 \le x \le \pi \ ,$$

$$u(0,t) = u(\pi,t) = 0 \ , \ t \ge 0 \ .$$

(100 markah)

5. Turunkan persamaan berikut ke dalam bentuk kanonikal dan selesaikan

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} + 1 = 0,$$

dalam $0 \le x \le 1, \quad y > 0,$ dengan $u = \frac{\partial u}{\partial y} = x$ pada $y = 0$.

(100 markah)

Table 1/Jadual 1

Elementary Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	t^n, $n =$ positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$
4.	t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin\ at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos\ at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh\ at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh\ at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$

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