UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 316 – Introduction To Operations Research
[Pengantar Penyelidikan Operasi]

Duration : 3 hour
[Masa: 3 jam]

Please ensure that this examination paper contains **TEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEPULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Give a brief definition each of the following terms:

(i) unbinding constraint,
(ii) objective function,
(iii) unit worth of a resource,
(iv) optimal solution,
(v) artificial variable.

(25 marks)

(b) A company publishes 72 titles of books for children, teenagers and adults annually. The company's policy is set for at least 4 titles of books for adults published. The number of titles of books for children must be at least 3 times the number of titles of books for teenagers and the number of titles of books for teenagers must be at least 2 times the number of titles of books for adults. On average, the annual profit was RM8000.00 for a title of book for children, RM7000.00 for a title of book for teenagers and RM1000.00 for a title of book for adults.

(i) Determine the decision variables.
(ii) Formulate a Linear Programming model for this problem.

(30 marks)

(c) Use the graphical method to solve the following Linear Programming Model. Then find the optimal solutions:

Minimise

$$Z = 5x_1 + 3x_2$$

subject to

$$x_1 - x_2 \leq 10$$
$$5x_1 + 3x_2 \geq 210$$
$$x_1 \geq 15$$
$$x_2 \leq 45$$

and

$$x_1 \geq 0, \quad x_2 \geq 0$$

(45 marks)

2. (a) Consider the following Linear Programming Model:

Maximise

$$Z = 8x_1 + 5x_2 \qquad (\text{Profit})$$

subject to

$$2x_1 + 3x_2 \le a \qquad (\text{Resource 1})$$
$$11x_1 + 6x_2 \le b \qquad (\text{Resource 2})$$
$$x_1 + 5x_2 \le c \qquad (\text{Resource 3})$$

and

$$x_1 \ge 0, \quad x_2 \ge 0$$

For certain values of *a*, *b* and *c* (where *a*, *b* and *c* are constants), the optimal tableau is given below.

Basic Variables	Coefficient of:					Right Side
	x_1	x_2	x_3	x_4	x_5	
Z	*e*	0	1/3	2/3	*f*	*d*
x_2	0	*g*	11/21	– 2/21	0	3
x_1	1	*h*	– 2/7	1/7	0	6
x_5	0	*i*	– 7/3	1/3	*j*	4

Find the values of

(i) *a, b, c, d, e, f, g, h, i* and *j*.

(ii) shadow prices for all resources and describe their significance.

(40 marks)

(b) Given below is the initial tableau of phase 1 using the Two Phase Method of a minimisation Linear Programming problem where $Z = 2x_1 + 3x_2 + x_3$.

Phase 1

Basic Variables	Coefficient of:						Right Side
	x_1	x_2	x_3	x_4	x_5	$\overline{x}_6$	
Z	3	2	0	0	– 1	0	6
x_3	1	4	2	– 1	0	0	8
$\overline{x}_6$	3	2	0	0	– 1	1	6

(i) Determine the standard form of phase 1 of the Linear Programming model.

(ii) Give the original Linear Programming model.

(iii) Continue with the next iteration until the optimal solution is obtained, then state the optimal solution.

(60 marks)

3. The project of constructing a small bridge in a town consists of 10 major activities. Information pertaining to the project is given below:

		Duration (Weeks)		
Activity	Immediate Predecessor(s)	Optimistic (*o*)	Most Likely (*m*)	Pessismistic (*p*)
A	-	2	5	8
B	A	4	7	10
C	A	4	9	14
D	B	6	10	20
E	C	1	3	5
F	C	3	6	9
G	D, F	4	5	12
H	E	6	8	10

(a) Construct the project network.

(20 marks)

(b) Find the mean and variance of each activity.

(10 marks)

(c) Find the critical path.

(30 marks)

(d) What is the expected minimum time to complete the project?

(10 marks)

(e) A delay in which activity/activities should be avoided in order to prevent a delay in project completion?

(10 marks)

(f) Compute the probability of completing the project in 36 weeks.

(20 marks)

4. (a) Stock of a product will be replenished immediately when ordered. Demand for the product is 50 units per day. Any orders placed will be charged the cost of setup of RM500 while a unit of product held in stock for a day will be charged at a cost of 5 cents. Shortages are allowed with a cost of RM1.50 for a unit short per day. Determine the

(i) maximum order quantity,
(ii) maximum shortage allowed.

(50 marks)

(b) (i) The demand for a product in a store is fixed and amounts to 50 units per month. Each unit costs RM6. Ordering cost is RM5. If the holding cost per year is 20% of the unit cost, when should the shop owner replenish his stock, and what is the optimal stock size?

(ii) The same unit item in (b) (i) will be given a price of RM5 if the shop owner orders more than 50 units while orders of 75 units or more will be given a price of RM4 per unit. Holding cost for the three classes of order quantity is 10 cents, 8 cents and 7 cents per month, respectively. Find the total inventory cost for all three offers.

(50 marks)

1. (a) Berikan penerangan ringkas bagi istilah-istilah yang berikut:

(i) kekangan tak terikat
(ii) fungsi objektif
(iii) nilai seunit sumber
(iv) penyelesaian optimum
(v) pembolehubah buatan

(25 markah)

(b) Sebuah syarikat menerbitkan 72 judul buku untuk kanak-kanak, remaja dan dewasa setahun. Polisi syarikat menetapkan sekurang-kurangnya 4 judul buku untuk orang dewasa diterbitkan. Bilangan judul buku untuk kanak-kanak mestilah sekurang-kurangnya 3 kali ganda bilangan judul buku untuk remaja dan bilangan judul buku untuk remaja mestilah sekurang-kurangnya 2 kali ganda bilangan judul buku untuk orang dewasa. Secara purata, keuntungan tahunan adalah sebanyak RM8000.00 bagi satu judul buku untuk kanak-kanak, RM7000.00 bagi satu judul buku untuk remaja dan RM1000.00 bagi satu judul buku untuk orang dewasa.

(i) Tentukan pembolehubah keputusan.
(ii) Rumuskan model Pengaturcaraan Linear bagi masalah ini.

(30 markah)

(c) Gunakan kaedah bergraf untuk menyelesaikan Model Pengaturcaraan Linear yang berikut kemudian dapatkan penyelesaian-penyelesaian optimum.

Minimumkan

$$Z = 5x_1 + 3x_2$$

terhadap

$$x_1 - x_2 \leq 10$$
$$5x_1 + 3x_2 \geq 210$$
$$x_1 \geq 15$$
$$x_2 \leq 45$$

dan

$$x_1 \geq 0, \quad x_2 \geq 0$$

(45 markah)

2. (a) Pertimbangkan model Pengaturcaraan Linear yang berikut:

Maksimumkan

$$Z = 8x_1 + 5x_2 \qquad (\text{Keuntungan})$$

terhadap

$$2x_1 + 3x_2 \le a \qquad (\text{Sumber 1})$$
$$11x_1 + 6x_2 \le b \qquad (\text{Sumber 2})$$
$$x_1 + 5x_2 \le c \qquad (\text{Sumber 3})$$

dan

$$x_1 \ge 0, \quad x_2 \ge 0$$

Untuk nilai-nilai *a*, *b* dan *c* yang tertentu (di mana *a*, *b* and *c* adalah pemalar), tablo optimum diberikan di bawah.

Pemboleh-ubah Asas	Pekali:					Sebelah Kanan
	x_1	x_2	x_3	x_4	x_5	
Z	e	0	1/3	2/3	f	d
x_2	0	g	11/21	– 2/21	0	3
x_1	1	h	– 2/7	1/7	0	6
x_5	0	i	– 7/3	1/3	j	4

Cari nilai-nilai

(i) *a, b, c, d, e, f, g, h, i* dan *j*.

(ii) harga bayangan untuk semua sumber dan terangkan kepentingannya.

(40 markah)

(b) Diberikan di bawah tablo permulaan fasa 1 menggunakan Kaedah Dua Fasa bagi masalah peminimuman Pengaturcaraan Linear di mana $Z = 2x_1 + 3x_2 + x_3$.

Fasa 1

Pemboleh-ubah Asas	Pekali:						Sebelah Kanan
	x_1	x_2	x_3	x_4	x_5	$\overline{x}_6$	
Z	3	2	0	0	– 1	0	6
x_3	1	4	2	– 1	0	0	8
$\overline{x}_6$	3	2	0	0	– 1	1	6

(i) Tentukan bentuk piawai fasa 1 bagi Model Pengaturcaraan Linear.

(ii) Berikan model asal Pengaturcaraan Linear.

(iii) Teruskan dengan lelaran seterusnya sehingga penyelesaian optimum diperolehi, kemudian nyatakan penyelesaian optimum itu.

(60 markah)

3. Projek membina jambatan di sebuah bandar terdiri daripada 10 aktiviti utama. Maklumat berkaitan projek tersebut adalah seperti berikut:

		Tempoh (Minggu)		
Aktiviti	Kegiatan Pendahuluan (s)	Optimistik (*o*)	Paling boleh jadi (*m*)	Pesimistik (*p*)
A	-	2	5	8
B	A	4	7	10
C	A	4	9	14
D	B	6	10	20
E	C	1	3	5
F	C	3	6	9
G	D, F	4	5	12
H	E	6	8	10

(a) Bina rangkaian projek.

(20 markah)

(b) Cari min dan varians bagi setiap aktiviti.

(10 markah)

(c) Cari lintasan genting.

(30 markah)

(d) Apakah jangkaan masa minimum untuk menyiapkan projek?

(10 markah)

(e) Kelewatan di dalam aktiviti/aktiviti-aktiviti manakah perlu dielakkan untuk mengelakkan kelewatan dalam penyiapan projek?

(10 markah)

(f) Kira kebarangkalian projek disiapkan dalam masa 36 minggu.

(20 markah)

4. (a) Stok bagi sejenis barangan akan diisi semula secara serta merta apabila dipesan. Permintaan bagi barangan ini berlaku dengan kadar 50 unit sehari. Setiap pesanan yang dibuat akan dikenakan kos penyediaan sebanyak RM500 sementara seunit barangan yang ditangguh di dalam stok selama sehari akan dikenakan kos sebanyak 5 sen. Kekurangan dibenarkan berlaku dengan kos RM1.50 bagi setiap unit barangan yang berkurangan selama sehari. Tentukan

(i) kuantiti pesanan optimum,

(ii) saiz kekurangan maksimum yang dibenarkan.

(50 markah)

(b) (i) Permintaan suatu barangan bagi sebuah kedai adalah tetap dan berjumlah 50 unit sebulan. Seunit barangan berharga RM6. Kos pesanan ialah RM5. Jika kos simpanan setahun ialah 20% nilai seunit barangan, bilakah pekedai itu perlu menambah stoknya dan apakah saiz stok yang optimum?

(ii) Seunit barangan yang sama di (b)(i) akan diberi harga RM5 jika pekedai memesan lebih daripada 50 unit. Sementara pesanan berjumlah 75 unit atau lebih akan diberi harga RM4 seunit. Kos simpanan bagi ketiga-tiga kelas kuantiti pesanan masing-masing ialah 10 sen, 8 sen dan 7 sen sebulan. Dapatkan jumlah kos inventori bagi ketiga-tiga tawaran.

(50 markah)

Lampiran

Table E The Standard Normal Distribution

z	*.00*	*.01*	*.02*	*.03*	*.04*	*.05*	*.06*	*.07*	*.08*	*.09*
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: Use 0.4999 for z values above 3.09.

Source: Frederick Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, Table A–1 (Reading, Mass.: Addison-Wesley, 1973). Reprinted with permission of the copyright owners.