UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 311 – Vector Analysis
[Analisis Vektor]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Determine the values of α, β if

$$(\alpha \underline{i} + \beta \underline{j} + \underline{k}) \times (2\underline{i} + 2\underline{j} + 3\underline{k}) = \underline{i} - \underline{j}.$$

(30 marks)

(b) If $\underline{w} = a\underline{u} + b\underline{v} + c(\underline{u} \times \underline{v})$, and $|\underline{u}| = 2$, $|\underline{v}| = 3$ and the angle between $\underline{u}$ and $\underline{v}$ is $\frac{\pi}{3}$, show that

$$|\underline{w}|^2 = 4a^2 + 6ab + 9b^2 + 27c^2.$$

(30 marks)

(c) Show that if the two straight lines

$$\underline{r}_1 = \underline{a} + \lambda \underline{b}$$

and

$$\underline{r}_2 = \underline{c} + \mu \underline{d}$$

intercept, then

$$(\underline{a} - \underline{c}) \cdot \underline{b} \times \underline{d} = 0$$

but

$$\underline{b} \times \underline{d} \neq \underline{0}.$$

What can you conclude if $\underline{b} \times \underline{d} = \underline{0}$?

(40 marks)

2. Consider four points A, B, C and D with Cartesian coordinates $(1, 2, -4)$, $(0, 3, -2)$, $(5, 2, 0)$ and $(4, 5, 8)$ respectively.

(a) Write down the vectors $\underline{u} = \overrightarrow{AB}$ and $\underline{v} = \overrightarrow{AC}$.

(15 marks)

(b) Show that a unit vector normal to the plane containing the points *A*, *B* and *C* is given by $\underline{\hat{n}} = \dfrac{\underline{i} + 3\underline{j} - \underline{k}}{\sqrt{11}}$.

(15 marks)

(c) Verify by explicitly calculating the scalar products that $\underline{\hat{n}} \cdot \underline{u} = 0$ and $\underline{\hat{n}} \cdot \underline{v} = 0$.

(15 marks)

(d) What is the Cartesian equation of the plane through *A*, *B* and *C*?

(20 marks)

(e) Show that *D* lies in this plane.

(15 marks)

(f) Find the line of intersection of the plane through *A*, *B*, *C* and *D* with the plane $x + z = 0$.

(20 marks)

3. (a) Let $\phi = \phi(x, y, z)$ be the scalar field

$$\phi(x, y, z) = e^x + y^2 - z^3.$$

(i) Compute the gradient $\nabla\phi$ of the scalar field ϕ.

(ii) Calculate the directional derivative of ϕ in the direction of $7\underline{j} + 24\underline{k}$ at the point $(5, 50, 5)$.

(iii) Find an equation of the tangent plane to the surface $e^x + y^2 - z^3 = 0$ at the point $(0, 0, 1)$.

(60 marks)

(b) For the scalar field $\phi(x,y,z) = 2x^2y^2z^2$ and vector field $\underline{F} = 2z\underline{i} + x^2\underline{j} + x\underline{k}$, find

(i) divergence and curl of $\underline{F}$, and

(ii) $(\underline{F} \times \underline{\nabla})\phi$ at the point $(1,-1,1)$

(40 marks)

4. (a) A path C as shown in the diagram below is composed of two paths C_1 and C_2.

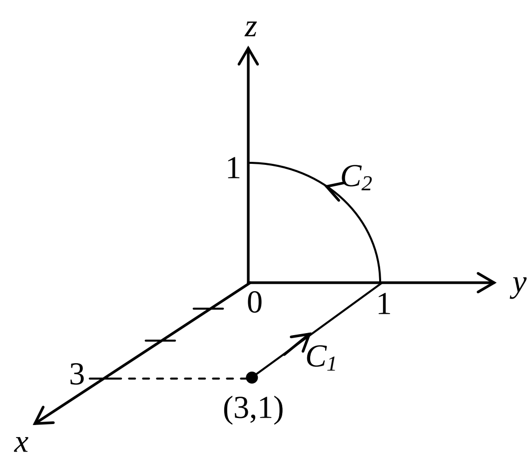


Fig 1.

(i) Write the parametric equations for the two curves C_1 and C_2.

(ii) Hence evaluate the path integral (line interal) of $\int_C (3x - 2y + z)ds$

where C is the piecewise smooth path as shown in the Fig 1.

(50 marks)

(b) Show that the path integral

$$\int_C \underline{F} . d\underline{r}$$

where $\underline{F} = y\underline{i} + (x+z)\underline{j} + y\underline{k}$ and C is a path given by

$$\underline{r} = \left(\frac{t^2+1}{t^2-1}\right)\underline{i} + \cos \pi t\, \underline{j} + 2t \sin \pi t \underline{k}, \quad 0 \le t \le \frac{1}{2},$$

is independent of the path.

Hence evaluate the path integral.

(50 marks)

5. (a) State the Stokes' theorem.

If $\underline{r}$ is the position vector, using the Stokes' theorem, show that

$$\oint_C \underline{r} \, . \, d\underline{r} = 0$$

where C is a smooth closed curve.

(50 marks)

(b) State the Gauss' theorem.

Using the Gauss theorem, evaluate

$$\iint_S \underline{F} . d\underline{S}$$

where the vector field $\underline{F}$ is given by

$$\underline{F} = y^2 z\underline{i} + y^3 \underline{j} + xz\underline{k}$$

and S is the boundary of the cube defined by

$$-1 \le x \le 1, \; -1 \le y \le 1, \text{ and } 0 \le z \le 2.$$

(50 marks)

1. (a) Tentukan nilai bagi α, β jika

$$\left(\alpha \underline{i} + \beta \underline{j} + \underline{k}\right) \times \left(2\underline{i} + 2\underline{j} + 3\underline{k}\right) = \underline{i} - \underline{j}\ .$$

(30 markah)

(b) Jika $\underline{w} = a\underline{u} + b\underline{v} + c\left(\underline{u} \times \underline{v}\right)$, and $\left|\underline{u}\right| = 2$, $\left|\underline{v}\right| = 3$ dan sudut di antara $\underline{u}$ dan $\underline{v}$ adalah $\dfrac{\pi}{3}$, tunjukkan bahawa

$$\left|\underline{w}\right|^2 = 4a^2 + 6ab + 9b^2 + 27c^2\ .$$

(30 markah)

(c) Tunjukkan bahawa jika dua garis lurus

$$\underline{r}_1 = \underline{a} + \lambda \underline{b}$$

dan

$$\underline{r}_2 = \underline{c} + \mu \underline{d}$$

bersilang, maka

$$\left(\underline{a} - \underline{c}\right) \cdot \underline{b} \times \underline{d} = 0$$

tetapi

$$\underline{b} \times \underline{d} \neq \underline{0}\ .$$

Apa yang anda dapat simpulkan jika $\underline{b} \times \underline{d} = \underline{0}$?

(40 markah)

2. Pertimbangkan empat titik *A*, *B*, *C* dan *D* dengan masing-masing koordinat Cartesian $\left(1,2,-4\right)$, $\left(0,3,-2\right)$, $\left(5,2,0\right)$ dan $\left(4,5,8\right)$.

(a) Tuliskan vektor $\underline{u} = \overrightarrow{AB}$ dan $\underline{v} = \overrightarrow{AC}$.

(15 markah)

(b) Tunjukkan bahawa vektor unit normal kepada satah yang mengandungi titik-titik *A*, *B* dan *C* diberi oleh $\underline{\hat{n}} = \dfrac{\underline{i} + 3\underline{j} - \underline{k}}{\sqrt{11}}$.

(15 markah)

(c) Tentusahkan dengan menghitung hasildarab skalar bahawa $\underline{\hat{n}} \cdot \underline{u} = 0$ dan $\underline{\hat{n}} \cdot \underline{v} = 0$.

(15 markah)

(d) Apakah persamaan Cartesian bagi satah yang melalui *A*, *B* dan *C*?

(20 markah)

(e) Tunjukkan bahawa *D* terletak dalam satah ini.

(15 markah)

(f) Cari persamaan garis bagi persilangan satah yang melalui *A*, *B*, *C* dan *D* dengan satah $x + z = 0$.

(20 markah)

3. (a) Biar $\phi = \phi(x, y, z)$ adalah medan skalar

$$\phi(x, y, z) = e^x + y^2 - z^3.$$

(i) Hitung kecerunan $\nabla\phi$ bagi medan skalar ϕ.

(ii) Hitung terbitan berarah bagi ϕ dalam arah $7\underline{j} + 24\underline{k}$ di titik $(5, 50, 5)$.

(iii) Cari persamaan satah tangen kepada permukaan $e^x + y^2 - z^3 = 0$ di titik $(0, 0, 1)$.

(60 markah)

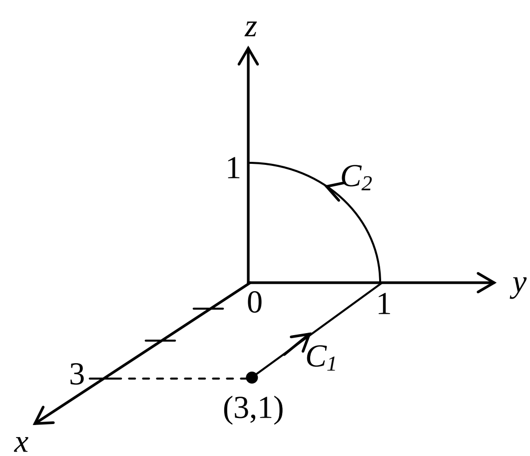
(b) Untuk medan skalar $\phi(x, y, z) = 2x^2y^2z^2$ dan medan vektor $\underline{F} = 2z\underline{i} + x^2\underline{j} + x\underline{k}$, cari

(i) kecapahan dan keikalan bagi $\underline{F}$, dan

(ii) $(\underline{F} \times \underline{\nabla})\phi$ di titik $(1,-1,1)$

(40 markah)

4. (a) Lintasan C seperti yang diberikan dalam rajah di bawah terdiri daripada dua lintasan C_1 dan C_2



Rajah 1.

(i) Tulis persamaan parametrik untuk dua lengkungan C_1 dan C_2.

(ii) Dengan itu, nilaikan kamiran lintasan (kamiran garis) bagi

$$\int_C (3x - 2y + z)\,ds$$

di mana C adalah lintasan yang licin cebis demi cebis seperti yang ditunjukkan dalam Rajah 1.

(50 markah)

(b) Tunjukkan kamiran lintasan

$$\int_C \underline{F} . d\underline{r}$$

di mana $\underline{F} = y\underline{i} + (x+z)\underline{j} + y\underline{k}$ dan C adalah lintasan yang diberikan oleh

$$\underline{r} = \left(\frac{t^2+1}{t^2-1}\right)\underline{i} + \cos\pi t\,\underline{j} + 2t\sin\pi t\,\underline{k}, \quad 0 \le t \le \frac{1}{2},$$

adalah tak bersandar kepada lintasan.

Dengan itu, nilaikan kamiran lintasan tersebut.

(50 markah)

5. (a) Nyatakan teorem Stokes.

Jika $\underline{r}$ adalah vektor kedudukan, dengan menggunakan teorem Stokes, tunjukkan bahawa

$$\oint_C \underline{r} \, . \, d\underline{r} = 0$$

di mana C adalah lengkung licin tertutup.

(50 markah)

(b) Nyatakan teorem Gauss.

Dengan menggunakan teorem Gauss, nilaikan

$$\iint_S \underline{F} . d\underline{S}$$

di mana medan vektor $\underline{F}$ adalah diberi oleh

$$\underline{F} = y^2 z\underline{i} + y^3\underline{j} + xz\underline{k}$$

dan S adalah sempadan bagi kubus yang ditakrifkan oleh

$$-1 \le x \le 1, \quad -1 \le y \le 1, \quad \text{dan} \quad 0 \le z \le 2.$$

(50 markah)

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