UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 215 – Introduction to Numerical Analysis
[Pengantar Analisis Berangka]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ***SEMBILAN*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

Jawab ***SEMUA*** *soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggerís.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

…2/-

1. (a) Given an equation

$$\sin x - x + 5 = 0 .$$

(i) Show that the above equation has at least one solution in the interval (4, 4.5).

(ii) Find a bound for the number of iterations that needed to achieve an approximate solution with accuracy 10^{-2}. Solve the above equation with this accuracy using the bisection method.

(iii) State one advantage and one disadvantage of the bisection method.

(40 marks)

(b) Construct a divided-difference table and hence approximate $f(0.5)$ if $f(0.2) = 0.7328$, $f(0.4) = 1.7902$, $f(0.6) = 3.2798$ and $f(0.8) = 5.3413$.

(15 marks)

(c) Consider the following table of data:

x	1.00	1.25	1.50	1.75	2.00
$f(x)$	1.20935	1.23549	1.22063	1.15776	1.04463

(i) Find $f''(1.5)$ using centered-difference formula of $O(h^2)$ with step size $h = 0.5$.

(ii) Repeat question (i) by using step size $h = 0.25$.

(iii) Use the results in (i) and (ii) to find an improved approximation using Richardson extrapolation.

(iv) The exact value is given by $f''(1.5) = -0.77436$. Compare the results obtained in (i), (ii) and (iii) by finding the percent relative error. Give one comment regarding the comparison.

(45 marks)

2. (a) Given the function $f(x)$ at the following values:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.5403	0.4536	0.3624	0.2675	0.1700	0.0707	–0.0292

Approximate $\int_0^{0.6} f(x)\ dx$ using composite Simpson's rule.

(20 marks)

(b) Use Romberg integration to compute $R_{3,3}$ for the integral

$$\int_0^1 e^{-x^2/2}\ dx.$$

Give your answer in five decimal places.

(40 marks)

(c) Approximate the integral

$$\int_1^2 \ln x\ dx$$

using Gaussian quadrature with $n = 3$.

(40 marks)

3. (a) Use Euler's method to approximate the solution for the following initial value problem

$$y' = 2(t+1)y, \qquad y(0) = 1,$$

in the interval $0 \le t \le 1.0$ with $h = 0.25$.

(30 marks)

(b) The solutions of an initial value problem

$$y' = t + ty$$

from $t = 0$ to $t = 1.5$ are given in the table below:

t	0	0.5	1.0	1.5
y	1	1.2662	2.2974	5.1604

Approximate $y(2.0)$ by using

(i) the Runge-Kutta method of order four,

(40 marks)

(ii) the Adams fourth-order predictor-corrector method.

(30 marks)

4. (a) Solve the following linear system

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

using

(i) LU decomposition technique,

(35 marks)

(ii) Gauss-Seidel iterative method, starting with $\mathbf{x}^{(0)} = (0.5, -0.75, 1.5)^t$ and iterating until $\varepsilon = \left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty = 0.05$.

(35 marks)

(b) Given a matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}.$$

Find the largest eigenvalue and the corresponding eigenvector of the matrix A with tolerance $\varepsilon = 10^{-2}$. Assume the initial eigenvector is $\mathbf{x}^{(0)} = (1,0,1)^t$.

(30 marks)

1. (a) Diberi persamaan

$$\sin x - x + 5 = 0 .$$

(i) Tunjukkan bahawa persamaan di atas mempunyai sekurang-kurangnya satu punca di dalam selang (4, 4.5).

(ii) Cari batasan bilangan lelaran yang diperlukan untuk mencapai penyelesaian hampiran dengan kejituan 10^{-2}. Selesaikan persamaan di atas dengan kejituan tersebut dengan menggunakan kaedah pembahagian dua sama.

(iii) Nyatakan satu kebaikan dan satu kelemahan kaedah pembahagian dua sama.

(40 markah)

(b) Binakan jadual beza berbahagi dan kemudian anggarkan nilai $f(0.5)$ jika $f(0.2) = 0.7328$, $f(0.4) = 1.7902$, $f(0.6) = 3.2798$ dan $f(0.8) = 5.3413$.

(15 markah)

(c) Pertimbangkan jadual data berikut:

x	1.00	1.25	1.50	1.75	2.00
$f(x)$	1.20935	1.23549	1.22063	1.15776	1.04463

(i) Cari $f''(1.5)$ dengan menggunakan rumus beza tengah $O(h^2)$ dengan saiz langkah $h = 0.5$.

(ii) Ulangi soalan (i) dengan menggunakan saiz langkah $h = 0.25$.

(iii) Guna jawapan daripada (i) dan (ii) untuk mendapatkan penghampiran yang lebih baik dengan menggunakan kaedah ekstrapolasi Richardson.

(iv) Diberi penyelesaian tepat ialah $f''(1.5) = -0.77436$. Bandingkan jawapan yang diperolehi daripada (i), (ii) dan (iii) dengan mencari peratus ralat relatif. Beri satu komen tentang perbandingan tersebut.

(45 markah)

2. (a) Diberi fungsi $f(x)$ pada nilai berikut:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.5403	0.4536	0.3624	0.2675	0.1700	0.0707	–0.0292

Anggarkan $\int_0^{0.6} f(x)\ dx$ dengan menggunakan petua gubahan Simpson.

(20 markah)

(b) Gunakan pengamiran Romberg untuk mengira $R_{3,3}$ bagi kamiran

$$\int_0^1 e^{-x^2/2}\ dx.$$

Beri jawapan anda dalam lima titik perpuluhan.

(40 markah)

(c) Anggarkan kamiran

$$\int_1^2 \ln x\ dx$$

menggunakan kuadratur Gaussian dengan $n = 3$.

(40 markah)

3. (a) Gunakan kaedah Euler untuk menganggarkan penyelesaian bagi masalah nilai awal

$$y' = 2(t+1)y, \qquad y(0) = 1,$$

dalam selang $0 \le t \le 1.0$ dengan $h = 0.25$.

(30 markah)

(b) Penyelesaian bagi masalah nilai awal

$$y' = t + ty$$

dari $t = 0$ ke $t = 1.5$ diberikan dalam jadual di bawah:

t	0	0.5	1.0	1.5
y	1	1.2662	2.2974	5.1604

Anggarkan $y(2.0)$ dengan menggunakan

(i) kaedah Runge-Kutta berperingkat empat,

(40 markah)

(ii) kaedah Adams peramal-pembetul berperingkat empat.

(30 markah)

4. (a) Selesaikan sistem linear berikut

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

dengan menggunakan

(i) teknik penghuraian *LU*,

(35 markah)

(ii) kaedah lelaran Gauss-Seidel, bermula dengan $\mathbf{x}^{(0)} = (0.5, -0.75, 1.5)^t$ dan mengulangi sehingga $\varepsilon = \left\| \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} \right\|_\infty = 0.05$.

(35 markah)

(b) Diberi suatu matriks

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix}.$$

Dapatkan nilai eigen dominan dan vektor eigen yang sepadan bagi matriks *A* tepat kepada $\varepsilon = 10^{-2}$. Andaikan vektor eigen awal ialah $\mathbf{x}^{(0)} = (1,0,1)^t$.

(30 markah)

List of formula:

1. $$f'(x_0) \approx \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]$$

$$f'(x_0) \approx \frac{1}{2h}\left[f(x_0-2h) - 4f(x_0-h) + 3f(x_0)\right]$$

$$f'(x_0) \approx \frac{1}{2h}\left[f(x_0-h) + f(x_0+h)\right]$$

2. $$f''(x_0) \approx \frac{1}{h^2}\left[f(x_0) - 2f(x_0+h) + f(x_0+2h)\right]$$

$$f''(x_0) \approx \frac{1}{h^2}\left[f(x_0) - 2f(x_0-h) + f(x_0-2h)\right]$$

$$f''(x_0) \approx \frac{1}{h^2}\left[f(x_0-h) - 2f(x_0) + f(x_0+h)\right]$$

3. $$N_j(h) = N_{j-1}\left(\frac{h}{2}\right) + \frac{N_{j-1}\left(\frac{h}{2}\right) - N_{j-1}(h)}{4^{j-1}-1} \quad \text{for } j = 2,3,\ldots$$

4. $$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$$

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

5. $$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n)\right]$$

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + f(x_n)\right]$$

6. $$h_k = \frac{(b-a)}{2^{k-1}} \quad \text{for } k = 1,2,3,\ldots$$

$$R_{1,1} = \frac{h_1}{2}\left[f(a) + f(b)\right]$$

$$R_{k,1} = \frac{1}{2}\left[R_{k-1,1} + h_{k-1}\sum_{i=1}^{2^{k-2}} f(a + (2i-1)h_k)\right] \quad \text{for } k = 2,3,\ldots$$

$$R_{k,j} = R_{k,j-1} + \frac{1}{4^{j-1}-1}\left(R_{k,j-1} - R_{k-1,j-1}\right) \quad \text{for } k = j, j+1,\ldots$$

7. $x = \frac{1}{2}\left[(b-a)t + (a+b)\right]$

$\int_{-1}^{1} f(x) \quad dx \approx f\left(0.5773503\right) + f\left(-0.5773503\right)$

$\int_{-1}^{1} f(x) \quad dx \approx 0.5555556 f\left(0.7745967\right) + 0.8888889 f\left(0\right) + 0.5555556 f(-0.7745967)$

8. $y_{i+1} = y_i + h\, f(t_i, y_i)$

9. $y_{i+1}^{0} = y_i + h\, f(t_i, y_i)$

$y_{i+1} = y_i + \frac{h}{2}\left[f(t_i, y_i) + f(t_{i+1}, y_{i+1}^{0})\right]$

10. $k_1 = h\, f(t_i, y_i)$

$k_2 = h\, f\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_1\right)$

$k_3 = h\, f\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_2\right)$

$k_4 = h\, f\left(t_{i+1}, y_i + k_3\right)$

$y_{i+1} = y_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

11. $y_{i+1} = y_i + \frac{h}{24}\left[55 f(t_i, y_i) - 59 f(t_{i-1}, y_{i-1}) + 37 f(t_{i-2}, y_{i-2}) - 9 f(t_{i-3}, y_{i-3})\right]$

12. $y_{i+1} = y_i + \frac{h}{24}\left[9 f(t_{i+1}, y_{i+1}) + 19 f(t_i, y_i) - 5 f(t_{i-1}, y_{i-1}) + f(t_{i-2}, y_{i-2})\right]$

- oooOooo -