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UNIVERSITI SAINS MALAYSIA

Final Examination  
2015/2016 Academic Session

May/June 2016

**JIM 212 – Statistical Methods**  
*[Kaedah Statistik]*

Duration : 3 hours  
*[Masa: 3 jam]*

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Please ensure that this examination paper contains **TWENTY SIX** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH ENAM** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

*Baca arahan dengan teliti sebelum anda menjawab soalan.*

*Setiap soalan diperuntukkan 100 markah.*

*Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan.]*

1. A company employs three job cost engineers. They have to estimate the cost of four jobs. The company wanted to compare the mean level of the engineers' estimates on the same four jobs. The data (in hundreds of thousands of RM) are shown in the table below.

		Job				Treatment Means (A)
		I	II	III	IV	
Engineer	X	4.6	6.2	5.0	6.6	5.60
	Y	4.9	6.3	5.4	6.8	5.85
	Z	4.4	5.9	5.4	6.3	5.50
Block Means (B)		4.63	6.13	5.27	6.57	

$$SS_T = 7.21, SS_A = 0.26, SS_B = 6.763$$

- (a) Construct an ANOVA table for the data. (40 marks)
- (b) Using  $\alpha = 0.05$ , test to determine whether there is sufficient evidence to indicate differences among
- (i) treatment means,
  - (ii) block means.

(60 marks)

2. The manager of a bank wanted to know whether the four tellers serve, on average, the same number of customers per hour. A research officer of the bank observed each of the four tellers for a certain number of hours. The following table gives the number of customers served by the four tellers during each of the observed hours.

Teller I	Teller II	Teller III	Teller IV
20	18	13	18
26	16	17	24
21	13	13	26
19	21	14	21
24	14	16	19
	11	14	
$\bar{x}_1 = 22$	$\bar{x}_2 = 15.5$	$\bar{x}_3 = 14.5$	$\bar{x}_4 = 21.6$
$s_1^2 = 8.5$	$s_2^2 = 13.1$	$s_3^2 = 2.7$	$s_4^2 = 11.3$
$\bar{X}_{GM} = 18.091$			

By using  $\alpha = 0.05$ ,

- (a) Test for homogeneity of variances. (40 marks)
  - (b) Test the hypothesis that the mean number of customers served per hour by each of these four tellers is the same. (30 marks)
  - (c) If the null hypothesis is rejected, test all possible pair wise comparisons for the mean number of customers served. (30 marks)
3. Information on income,  $x$  (RM), and food expenditures,  $y$  (RM), for the past month was collected from a sample of seven households. The information obtained is given below.

$$\sum x = 21,200 \quad \sum y = 6,400$$

$$SS_{xx} = 8,014,285.71$$

$$SS_{xy} = 2,117,142.86$$

$$SS_{yy} = 608,571.43$$

- (a) Compute the value of the correlation coefficient. (20 marks)
- (b) Test the significance of the correlation coefficient equal to 0 at  $\alpha = 0.05$ . (20 marks)
- (c) Find the equation of the regression line. (20 marks)
- (d) Test at  $\alpha = 0.01$  whether the slope of the regression line is positive. (20 marks)
- (e) Find a 99% prediction interval for the predicted food expenditure for a randomly selected household with a monthly income of RM3,500. (20 marks)

4. (a) Give brief descriptions of the following terms:

- (i) Experimental design
- (ii) Completely randomized design
- (iii) Latin square design.
- (iv) Nonparametric statistics
- (v) Two-way analysis of variance

(40 marks)

(b) A medical researcher wishes to test the effects of two different diets and two different exercise programs on the glucose level (mg/dl) in a person's blood. Three subjects are randomly assigned to each group. Analyze the data shown here, using a two-way ANOVA with  $\alpha = 0.05$ .

Exercise program	Diet	
	A	B
I	66, 64, 62	62, 58, 53
II	72, 68, 65	91, 85, 83

Below is the ANOVA table.

Source	SS	d.f.	MS	F
Exercise	816.750	?	?	?
Diet	102.083	?	?	?
Interaction	444.083	?	?	?
Within	?	?	?	
Total	1470.916	?		

- (i) What are the names of the two variables?
- (ii) How many levels does each variable contain?
- (iii) Complete the ANOVA table.
- (iv) State the hypotheses for the study.
- (v) What are your conclusions for the study?

(60 marks)

5. (a) (i) Explain briefly the difference between fixed effects model and random effects model in the one-way analysis of variance.
- (ii) Give the hypothesis of random effects model in the one-way analysis of variance.

(20 marks)

- (b) Eight music videos were ranked by teenagers and their parents on style and clarity, with 1 being the highest ranking. At  $\alpha = 0.05$ , is there a relationship between the rankings?

Music Videos	1	2	3	4	5	6	7	8
Teenagers	4	6	2	8	1	7	3	5
Parents	1	7	5	4	3	8	2	6

(40 marks)

- (c) A researcher wishes to determine if the number of viewers for 10 returning television shows has not changed since last year. The data are given in millions of viewers. Using Paired-Sample Sign Test, test the claim that the number of viewers has not changed at  $\alpha = 0.01$ .

Show	1	2	3	4	5	6	7	8	9	10
Last year	21.0	19.2	28.9	26.4	20.8	25.0	15.3	16.8	18.8	13.7
This year	18.9	17.8	26.6	20.5	20.8	19.1	15.8	16.0	16.7	16.8

(40 marks)

1. Sebuah syarikat menggaji tiga orang jurutera penilai kos pekerjaan. Mereka perlu menganggar kos empat jenis pekerjaan. Syarikat tersebut ingin membandingkan min paras anggaran yang dibuat oleh jurutera-jurutera tersebut ke atas keempat-empat pekerjaan itu. Data (dalam ribu RM) ditunjukkan di bawah.

		Pekerjaan				Min
		I	II	III	IV	Olahan (A)
Jurutera	X	4.6	6.2	5.0	6.6	5.60
	Y	4.9	6.3	5.4	6.8	5.85
	Z	4.4	5.9	5.4	6.3	5.50
Min Blok	(B)	4.63	6.13	5.27	6.57	

$$SS_T = 7.21, SS_A = 0.26, SS_B = 6.763$$

- (a) Bina jadual ANOVA bagi data ini. (40 markah)
- (b) Dengan menggunakan  $\alpha = 0.05$ , uji sama ada terdapat cukup bukti yang menunjukkan terdapat perbezaan antara,
- (i) min olahan.
  - (ii) min blok. (60 markah)

2. Pengurus sebuah bank ingin mengetahui sama ada keempat-empat juruwang secara purata melayan bilangan pelanggan yang sama bagi setiap jam tertentu. Seorang penyelidik di bank tersebut telah memerhati keempat-empat juruwang pada beberapa jam yang tertentu. Jadual yang berikut menunjukkan bilangan pelanggan yang dilayan oleh keempat-empat juruwang pada setiap jam yang diperhatikan.

Juruwang I	Juruwang II	Juruwang III	Juruwang IV
20	18	13	18
26	16	17	24
21	13	13	26
19	21	14	21
24	14	16	19
	11	14	
$\bar{x}_1 = 22$	$\bar{x}_2 = 15.5$	$\bar{x}_3 = 14.5$	$\bar{x}_4 = 21.6$
$s_1^2 = 8.5$	$s_2^2 = 13.1$	$s_3^2 = 2.7$	$s_4^2 = 11.3$
$\bar{X}_{GM} = 18.091$			

Pada  $\alpha = 0.05$ ,

- (a) Jalankan ujian kesamaan varians. (40 markah)
- (b) Uji hipotesis bahawa min bilangan pelanggan yang dilayan oleh keempat-empat juruwang per jam adalah sama. (30 markah)
- (c) Jika hipotesis nol ditolak, jalankan ujian berpasangan untuk min bilangan pelanggan yang dilayan. (30 markah)
3. Sampel mengenai pendapatan,  $x$  (RM), dan perbelanjaan makanan,  $y$  (RM), untuk sebulan diambil dari tujuh isi rumah. Maklumat yang diperolehi diberikan di bawah.

$$\begin{aligned} \sum x &= 21,200 & \sum y &= 6,400 \\ SS_{xx} &= 8,014,285.71 \\ SS_{xy} &= 2,117,142.86 \\ SS_{yy} &= 608,571.43 \end{aligned}$$

- (a) Hitung nilai pekali korelasi. (20 markah)
- (b) Ujikan keertian pekali korelasi sama dengan 0 pada  $\alpha = 0.05$ . (20 markah)

- (c) Dapatkan persamaan garis regresi. (20 markah)
- (d) Uji pada  $\alpha = 0.01$  sama ada kecerunan garis regresi adalah positif. (20 markah)
- (e) Dapatkan selang ramalan 99% bagi perbelanjaan makanan untuk isi rumah berpendapatan bulanan sebanyak RM3,500 yang dipilih secara rawak. (20 markah)
4. (a) Berikan penerangan ringkas bagi istilah-istilah yang berikut: (40 markah)
- (i) Rekabentuk ujikaji.
  - (ii) Rekabentuk rawak lengkap.
  - (iii) Rekabentuk segiempat sama Latin.
  - (iv) Statistik tak berparameter.
  - (v) Analisis varians dua-hala.
- (b) Seorang penyelidik perubatan ingin menguji kesan dua jenis diet dan dua jenis program senaman ke atas paras glukos (mg/dl) dalam darah seseorang. Tiga orang diagihkan secara rawak kepada setiap kumpulan. Dengan menggunakan ANOVA ujikaji dua faktor, jalankan analisis pada  $\alpha = 0.05$ .

Program senaman	Diet	
	A	B
I	66, 64, 62	62, 58, 53
II	72, 68, 65	91, 85, 83

Di bawah adalah jadual ANOVA.

Sumber	SS	d.k.	MS	F
Senaman	816.750	?	?	?
Diet	102.083	?	?	?
Interraksi	444.083	?	?	?
Ralat	?	?	?	
Jumlah	1470.916	?		

- (i) Apakah nama dua pembolehubah tersebut?
- (ii) Berapakah paras bagi setiap pembolehubah?
- (iii) Lengkapkan jadual ANOVA.
- (iv) Nyatakan hipotesis-hipotesis kajian.
- (v) Apakah kesimpulan-kesimpulan anda?

(60 markah)

5. (a) (i) Terangkan secara ringkas perbezaan antara model kesan tetap dan model kesan rawak dalam analisis varians sehala.
- (ii) Berikan hipotesis bagi model kesan rawak dalam analisis varians sehala.

(20 markah)

- (b) Lapan video muzik dinilai dari segi gaya dan kejelasan oleh remaja dan ibubapa mereka, dengan 1 sebagai pangkat yang tertinggi. Pada  $\alpha = 0.05$ , adakah perkaitan di antara kedua-dua pangkat tersebut?

Video Muzik	1	2	3	4	5	6	7	8
Remaja	4	6	2	8	1	7	3	5
Ibubapa	1	7	5	4	3	8	2	6

(40 markah)

- (c) Seorang penyelidik ingin menentukan sama ada 10 rancangan televisyen yang masih disiarkan mempunyai bilangan penonton yang sama seperti tahun lepas. Data menunjukkan bilangan penonton dalam juta. Dengan menggunakan Ujian Tanda Berpasangan, uji dakwaan bahawa bilangan penonton tidak berubah pada  $\alpha = 0.01$ .

Rancangan	1	2	3	4	5	6	7	8	9	10
Tahun lepas	21.0	19.2	28.9	26.4	20.8	25.0	15.3	16.8	18.8	13.7
Tahun ini	18.9	17.8	26.6	20.5	20.8	19.1	15.8	16.0	16.7	16.8

(40 markah)

**FORMULAS**

$$1. \chi^2 = \sum \frac{(O-E)^2}{E}$$

$$2. r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$3. t = r\sqrt{\frac{n-2}{1-r^2}}$$

$$4. a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$5. b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$6. s_{est} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

$$7. y' \pm t_{\alpha/2} s_{est} \sqrt{1 + \frac{1}{n} + \frac{n(x-\bar{x})^2}{n\sum x^2 - (\sum x)^2}}$$

$$8. s_B^2 = \frac{\sum n_i (\bar{x}_i - \bar{X}_{GM})^2}{k-1}$$

$$9. s_W^2 = \frac{\sum (n_i - 1) s_i^2}{\sum (n_i - 1)}$$

$$10. F_s = \frac{(\bar{x}_i - \bar{x}_j)^2}{s_W^2 \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}$$

$$11. \quad q = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{s_w^2 / n}}$$

$$12. \quad (a) \quad SS_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n}$$

$$(b) \quad SS_{yy} = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - \frac{\left(\sum_{i=1}^n y_i\right)^2}{n}$$

$$(c) \quad SS_{xy} = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n x_i y_i - \frac{\left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)}{n}$$

$$13. \quad s_{est} = \sqrt{\frac{\sum (y - y')^2}{n - 2}} = \sqrt{\frac{SSE}{n - 2}} = \sqrt{\frac{SS_{yy} - bSS_{xy}}{n - 2}}$$

$$14. \quad SSR = B(SS_{xy}) = B^2(SS_{xx})$$

$$15. \quad t = \frac{b - B}{s_{est} / \sqrt{SS_{xx}}}$$

$$16. \quad B = 2.3026 \frac{Q}{h}$$

$$Q = (n - a) \log S_p^2 - \sum_{i=1}^a (n_i - 1) \log s_i^2$$

$$S_p^2 = \frac{1}{n - a} \sum_{i=1}^a (n_i - 1) s_i^2$$

$$h = 1 + \frac{1}{3(a - 1)} \left( \sum_{i=1}^a \frac{1}{n_i - 1} - \frac{1}{n - a} \right)$$

17. Two-Way ANOVA

$$y_{i..} = \sum_{j=1}^b \sum_{k=1}^n y_{ijk} \quad \bar{y}_{i..} = \frac{y_{i..}}{bn} \quad i = 1, 2, \dots, a$$

$$y_{.j.} = \sum_{i=1}^a \sum_{k=1}^n y_{ijk} \quad \bar{y}_{.j.} = \frac{y_{.j.}}{an} \quad j = 1, 2, \dots, b$$

$$y_{ij.} = \sum_{k=1}^n y_{ijk} \quad \bar{y}_{ij.} = \frac{y_{ij.}}{n} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \end{cases}$$

$$y_{...} = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk} \quad \bar{y}_{...} = \frac{y_{...}}{abn}$$

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^a \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^b \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij.}^2}{n} - \sum_{i=1}^a \frac{y_{i..}^2}{bn} - \sum_{j=1}^b \frac{y_{.j.}^2}{an} + \frac{y_{...}^2}{abn}$$

18. Randomized Complete Block Designs

$$y_{i.} = \sum_{j=1}^b y_{ij} \quad , \quad i = 1, 2, \dots, a$$

$$y_{.j} = \sum_{i=1}^a y_{ij} \quad , \quad j = 1, 2, \dots, b$$

$$y_{..} = \sum_{i=1}^a \sum_{j=1}^b y_{ij} = \sum_{i=1}^a y_{i.} = \sum_{j=1}^b y_{.j}$$

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b y_{ij}^2 - \frac{y_{..}^2}{ab}$$

$$SS_A = \sum_{i=1}^a \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{ab}$$

$$SS_B = \sum_{j=1}^b \frac{y_{.j}^2}{a} - \frac{y_{..}^2}{ab}$$

$$19. z = \frac{(X + 0.5) - (n/2)}{\sqrt{n/2}}$$

$$20. z = \frac{R - \mu_R}{\sigma_R}$$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$21. z = \frac{w_s - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

$$22. H = \frac{12}{N(N+1)} \left( \frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \dots + \frac{R_k^2}{n_k} \right) - 3(N+1)$$

$$23. r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Appendix

Table E The Standard Normal Distribution										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: Use 0.4999 for z values above 3.09.

Source: Frederick Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, Table A-1 (Reading, Mass.: Addison-Wesley, 1973). Reprinted with permission of the copyright owners.

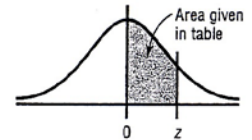
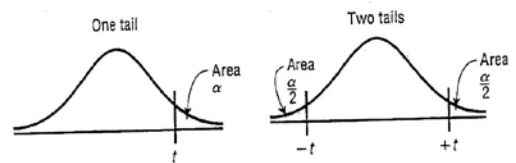


Table F The <i>t</i> Distribution							
d.f.	Confidence intervals	50%	80%	90%	95%	98%	99%
	One tail, $\alpha$	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, $\alpha$	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
4		.741	1.533	2.132	2.776	3.747	4.604
5		.727	1.476	2.015	2.571	3.365	4.032
6		.718	1.440	1.943	2.447	3.143	3.707
7		.711	1.415	1.895	2.365	2.998	3.499
8		.706	1.397	1.860	2.306	2.896	3.355
9		.703	1.383	1.833	2.262	2.821	3.250
10		.700	1.372	1.812	2.228	2.764	3.169
11		.697	1.363	1.796	2.201	2.718	3.106
12		.695	1.356	1.782	2.179	2.681	3.055
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
17		.689	1.333	1.740	2.110	2.567	2.898
18		.688	1.330	1.734	2.101	2.552	2.878
19		.688	1.328	1.729	2.093	2.539	2.861
20		.687	1.325	1.725	2.086	2.528	2.845
21		.686	1.323	1.721	2.080	2.518	2.831
22		.686	1.321	1.717	2.074	2.508	2.819
23		.685	1.319	1.714	2.069	2.500	2.807
24		.685	1.318	1.711	2.064	2.492	2.797
25		.684	1.316	1.708	2.060	2.485	2.787
26		.684	1.315	1.706	2.056	2.479	2.779
27		.684	1.314	1.703	2.052	2.473	2.771
28		.683	1.313	1.701	2.048	2.467	2.763
(z) $\infty$		.674	1.282 <sup>a</sup>	1.645 <sup>b</sup>	1.960	2.326 <sup>c</sup>	2.576 <sup>d</sup>

<sup>a</sup>This value has been rounded to 1.28 in the textbook.  
<sup>b</sup>This value has been rounded to 1.65 in the textbook.  
<sup>c</sup>This value has been rounded to 2.33 in the textbook.  
<sup>d</sup>This value has been rounded to 2.58 in the textbook.

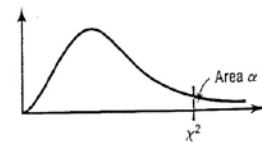
Source: Adapted from W. H. Beyer, *Handbook of Tables for Probability and Statistics*, 2nd ed., CRC Press, Boca Raton, Fla., 1986. Reprinted with permission.



**Table G** The Chi-Square Distribution

Degrees of freedom	$\alpha$									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Source: Donald B. Owen, *Handbook of Statistics Tables*. The Chi-Square Distribution Table. © 1962 by Addison-Wesley Publishing Company, Inc. Copyright renewal © 1990. Reprinted by permission of Pearson Education, Inc.



**Table H**  
The F Distribution

d.f.N.: degrees of freedom, numerator  
d.f.D.: degrees of freedom, denominator  
 $\alpha = 0.005$

	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091	24,224	24,426	24,630	24,836	24,940	25,044	25,148	25,253	25,359	25,465
2	198.5	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83
3	55.55	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32
4	31.33	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.14
5	22.78	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88
6	18.63	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.65	7.53	7.42	7.31	7.19	7.08
7	16.24	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95
8	14.69	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19
9	13.61	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.64
10	12.81	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.24	5.05	4.86	4.76	4.65	4.55	4.44	4.34	4.23
11	12.23	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90
12	11.75	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65
13	11.37	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.44
14	11.06	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.79	3.69	3.58	3.48	3.37	3.26
15	10.80	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11
16	10.58	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98
17	10.38	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87
18	10.22	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78
19	10.07	6.99	5.82	5.17	4.76	4.47	4.25	4.09	3.96	3.85	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69
20	9.94	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77	3.60	3.43	3.24	3.15	3.05	2.95	2.84	2.73	2.61
21	9.83	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.54	3.36	3.18	3.08	2.98	2.88	2.77	2.66	2.55
22	9.73	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.47	3.30	3.12	3.02	2.92	2.82	2.71	2.60	2.48
23	9.63	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.42	3.25	3.06	2.97	2.87	2.77	2.66	2.55	2.43
24	9.55	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.37	3.20	3.01	2.92	2.82	2.72	2.61	2.50	2.38
25	9.48	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.33	3.15	2.97	2.87	2.77	2.67	2.56	2.45	2.33
26	9.41	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.28	3.11	2.93	2.83	2.73	2.63	2.52	2.41	2.29
27	9.34	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.25	3.07	2.89	2.79	2.69	2.59	2.48	2.37	2.25
28	9.28	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.21	3.04	2.86	2.76	2.66	2.56	2.45	2.33	2.22
29	9.23	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.18	3.01	2.82	2.73	2.63	2.52	2.42	2.30	2.18
30	9.18	6.31	5.20	4.58	4.19	3.91	3.70	3.54	3.41	3.30	3.14	2.97	2.78	2.69	2.59	2.48	2.38	2.26	2.14
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	2.95	2.78	2.60	2.50	2.40	2.30	2.18	2.06	1.93
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	2.29	2.19	2.08	1.96	1.83	1.69
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.54	2.37	2.19	2.09	1.98	1.87	1.75	1.61	1.43
$\infty$	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.90	1.79	1.67	1.53	1.36	1.00

**Table H** (continued)

d.f.D.: degrees of freedom, denominator	d.f.N.: degrees of freedom, numerator																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366	
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	
9	10.56	8.02	7.06	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00	
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.66	
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.70	2.61	2.52	2.42	
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.25	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.50	2.40	2.31	2.21	
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06	
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03	
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00	

**Table H**  
(continued)

d.f.D.: degrees of freedom, denominator	d.f.N.: degrees of freedom, numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.58	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

$\alpha = 0.025$

**Table H**  
*(continued)*

$\alpha = 0.05$

d.f.D.: degrees of freedom, denominator	d.f.N.: degrees of freedom, numerator																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.3	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.78	2.74	2.70	2.66	2.62	2.58	2.54	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.39	2.35	2.30	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.20	2.15	2.11	2.06	2.01	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.54	2.49	2.44	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.20	2.15	2.11	2.06	2.02	1.97	1.92	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.13	2.08	2.04	1.99	1.95	1.90	1.84	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

**Table I** Critical Values for PPMC

Reject  $H_0: \rho = 0$  if the absolute value of  $r$  is greater than the value given in the table. The values are for a two-tailed test; d.f. =  $n - 2$ .

d.f	$\alpha = 0.05$	$\alpha = 0.01$
1	0.999	0.999
2	0.950	0.999
3	0.878	0.959
4	0.811	0.917
5	0.754	0.875
6	0.707	0.834
7	0.666	0.798
8	0.632	0.765
9	0.602	0.735
10	0.576	0.708
11	0.553	0.684
12	0.532	0.661
13	0.514	0.641
14	0.497	0.623
15	0.482	0.606
16	0.468	0.590
17	0.456	0.575
18	0.444	0.561
19	0.433	0.549
20	0.423	0.537
25	0.381	0.487
30	0.349	0.449
35	0.325	0.418
40	0.304	0.393
45	0.288	0.372
50	0.273	0.354
60	0.250	0.325
70	0.232	0.302
80	0.217	0.283
90	0.205	0.267
100	0.195	0.254

Source: From *Biometrika Tables for Statisticians*, vol. 1 (1962), p. 138. Reprinted with permission.

**Table J** Critical Values for the Sign Test

Reject the null hypothesis if the smaller number of positive or negative signs is less than or equal to the value in the table.

n	One-tailed, $\alpha = 0.005$	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
	Two-tailed, $\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	3	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	6	7

Note: Table J is for one-tailed or two-tailed tests. The term  $n$  represents the total number of positive and negative signs. The test value is the number of less frequent signs.

Source: From *Journal of American Statistical Association*, vol. 41 (1946), pp. 557-66. W. J. Dixon and A. M. Mood.

**Table K** Critical Values for the Wilcoxon Signed-Rank Test

Reject the null hypothesis if the test value is less than or equal to the value given in the table.

n	One-tailed, $\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
	Two-tailed, $\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

Source: From *Some Rapid Approximate Statistical Procedures*, Copyright 1949, 1964 Lederle Laboratories, American Cyanamid Co., Wayne, N.J. Reprinted with permission.

**Table L** Critical Values for the Rank Correlation Coefficient

Reject  $H_0: \rho = 0$  if the absolute value of  $r_s$  is greater than the value given in the table.

n	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
5	0.900	—	—	—
6	0.829	0.886	0.943	—
7	0.714	0.786	0.893	0.929
8	0.643	0.738	0.833	0.881
9	0.600	0.700	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.536	0.618	0.709	0.818
12	0.497	0.591	0.703	0.780
13	0.475	0.566	0.673	0.745
14	0.457	0.545	0.646	0.716
15	0.441	0.525	0.623	0.689
16	0.425	0.507	0.601	0.666
17	0.412	0.490	0.582	0.645
18	0.399	0.476	0.564	0.625
19	0.388	0.462	0.549	0.608
20	0.377	0.450	0.534	0.591
21	0.368	0.438	0.521	0.576
22	0.359	0.428	0.508	0.562
23	0.351	0.418	0.496	0.549
24	0.343	0.409	0.485	0.537
25	0.336	0.400	0.475	0.526
26	0.329	0.392	0.465	0.515
27	0.323	0.385	0.456	0.505
28	0.317	0.377	0.448	0.496
29	0.311	0.370	0.440	0.487
30	0.305	0.364	0.432	0.478

Source: From N. L. Johnson and F. C. Leone, *Statistical and Experimental Design*, vol. I (1964), p. 412. Reprinted with permission from the Institute of Mathematical Statistics.

**Table M** Critical Values for the Number of Runs

This table gives the critical values at  $\alpha = 0.05$  for a two-tailed test. Reject the null hypothesis if the number of runs is less than or equal to the smaller value or greater than or equal to the larger value.

Value of $n_1$	Value of $n_2$																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2
3	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
4	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3
5	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
6	1	1	1	2	2	2	3	3	3	3	3	3	3	3	3	4	4	4	4	4
7	6	8	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
8	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5
9	6	8	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	12	12
10	1	2	2	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6
11	6	8	9	10	11	12	12	13	13	13	13	13	14	14	14	14	14	14	14	14
12	1	2	2	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	5	6
13	6	8	10	11	12	13	13	14	14	14	14	14	15	15	15	15	16	16	16	16
14	1	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7
15	6	8	10	11	12	13	14	14	14	15	15	16	16	16	16	16	17	17	17	17
16	1	2	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7
17	6	8	10	12	13	14	14	15	15	16	16	16	17	17	17	17	18	18	18	18
18	1	2	3	3	4	4	4	5	5	5	5	6	6	6	6	7	7	7	8	8
19	6	8	10	12	13	14	15	15	16	16	16	17	17	17	18	18	18	18	18	18
20	1	2	3	3	4	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
21	6	8	10	12	13	14	15	16	16	17	17	17	18	18	18	18	19	19	19	19
22	1	2	3	4	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	9
23	6	8	10	12	13	14	15	16	16	17	17	18	18	18	18	19	19	19	20	20
24	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
25	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
26	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
27	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
28	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
29	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
30	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
31	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
32	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
33	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
34	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
35	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
36	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
37	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
38	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
39	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
40	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
41	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
42	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
43	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
44	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
45	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
46	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
47	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
48	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
49	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
50	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
51	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
52	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
53	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
54	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
55	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
56	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
57	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
58	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
59	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
60	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
61	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
62	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
63	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
64	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
65	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
66	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
67	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
68	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
69	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
70	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
71	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
72	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
73	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
74	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
75	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
76	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
77	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
78	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
79	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
80	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
81	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
82	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
83	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
84	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
85	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20	20
86	1	2	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	9	9
87																				

**Table N** Critical Values for the Tukey Test  $\alpha = 0.01$

$k$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	90.03	135.0	164.3	185.6	202.2	215.8	227.2	237.0	245.6	253.2	260.0	266.2	271.8	277.0	281.8	286.3	290.4	294.3	298.0
2	14.04	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69	32.59	33.40	34.13	34.81	35.43	36.00	36.53	37.03	37.50	37.95
3	8.26	10.62	12.17	13.33	14.24	15.00	15.64	16.20	16.69	17.13	17.53	17.89	18.22	18.52	18.81	19.07	19.32	19.55	19.77
4	6.51	8.12	9.17	9.96	10.58	11.10	11.55	11.93	12.27	12.57	12.84	13.09	13.32	13.53	13.73	13.91	14.08	14.24	14.40
5	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48	10.70	10.89	11.08	11.24	11.40	11.55	11.68	11.81	11.93
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.48	9.65	9.81	9.95	10.08	10.21	10.32	10.43	10.54
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.65
8	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.03
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49	7.65	7.78	7.91	8.03	8.13	8.23	8.33	8.41	8.49	8.57
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.49	7.60	7.71	7.81	7.91	7.99	8.08	8.15	8.23
11	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95
12	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19	7.27	7.35	7.42	7.48	7.55
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05	7.13	7.20	7.27	7.33	7.39
15	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26
16	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48	6.57	6.66	6.73	6.81	6.87	6.94	7.00	7.05
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65	6.73	6.79	6.85	6.91	6.97
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.28	6.37	6.45	6.52	6.59	6.65	6.71	6.77	6.82
24	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41
40	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60	5.69	5.76	5.83	5.90	5.96	6.02	6.07	6.12	6.16	6.21
60	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.78	5.84	5.89	5.93	5.97	6.01
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.37	5.44	5.50	5.56	5.61	5.66	5.71	5.75	5.79	5.83
$\infty$	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65

**Table N**  
(continued)

$\alpha = 0.05$

$k/p$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07	50.59	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83	59.56
2	6.08	8.33	9.80	10.88	11.74	12.44	13.03	13.54	13.99	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.37	16.57	16.77
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.15	10.35	10.53	10.69	10.84	10.98	11.11	11.24
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
$\infty$	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

**Table N**  
(concluded)

$k \backslash p$	$\alpha = 0.10$																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	8.93	13.44	16.36	18.49	20.15	21.51	22.64	23.62	24.48	25.24	25.92	26.54	27.10	27.62	28.10	28.54	28.96	29.35	29.71	
2	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.72	10.01	10.26	10.49	10.70	10.89	11.07	11.24	11.39	11.54	11.68	
3	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29	7.49	7.67	7.83	7.98	8.12	8.25	8.37	8.48	8.58	8.68	
4	3.01	3.98	4.59	5.03	5.39	5.68	5.93	6.14	6.33	6.49	6.65	6.78	6.91	7.02	7.13	7.23	7.33	7.41	7.50	
5	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82	5.97	6.10	6.22	6.34	6.44	6.54	6.63	6.71	6.79	6.86	
6	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50	5.64	5.76	5.87	5.98	6.07	6.16	6.25	6.32	6.40	6.47	
7	2.68	3.45	3.93	4.28	4.55	4.78	4.97	5.14	5.28	5.41	5.53	5.64	5.74	5.83	5.91	5.99	6.06	6.13	6.19	
8	2.63	3.37	3.83	4.17	4.43	4.65	4.83	4.99	5.13	5.25	5.36	5.46	5.56	5.64	5.72	5.80	5.87	5.93	6.00	
9	2.59	3.32	3.76	4.08	4.34	4.54	4.72	4.87	5.01	5.13	5.23	5.33	5.42	5.51	5.58	5.66	5.72	5.79	5.85	
10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91	5.03	5.13	5.23	5.32	5.40	5.47	5.54	5.61	5.67	5.73	
11	2.54	3.23	3.66	3.96	4.20	4.40	4.57	4.71	4.84	4.95	5.05	5.15	5.23	5.31	5.38	5.45	5.51	5.57	5.63	
12	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.65	4.78	4.89	4.99	5.08	5.16	5.24	5.31	5.37	5.44	5.49	5.55	
13	2.50	3.18	3.59	3.88	4.12	4.30	4.46	4.60	4.72	4.83	4.93	5.02	5.10	5.18	5.25	5.31	5.37	5.43	5.48	
14	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68	4.79	4.88	4.97	5.05	5.12	5.19	5.26	5.32	5.37	5.43	
15	2.48	3.14	3.54	3.83	4.05	4.23	4.39	4.52	4.64	4.75	4.84	4.93	5.01	5.08	5.15	5.21	5.27	5.32	5.38	
16	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61	4.71	4.81	4.89	4.97	5.04	5.11	5.17	5.23	5.28	5.33	
17	2.46	3.11	3.50	3.78	4.00	4.18	4.33	4.46	4.58	4.68	4.77	4.86	4.93	5.01	5.07	5.13	5.19	5.24	5.30	
18	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55	4.65	4.75	4.83	4.90	4.98	5.04	5.10	5.16	5.21	5.26	
19	2.45	3.09	3.47	3.75	3.97	4.14	4.29	4.42	4.53	4.63	4.72	4.80	4.88	4.95	5.01	5.07	5.13	5.18	5.23	
20	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51	4.61	4.70	4.78	4.85	4.92	4.99	5.05	5.10	5.16	5.20	
24	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.34	4.44	4.54	4.63	4.71	4.78	4.85	4.91	4.97	5.02	5.07	5.12	
30	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38	4.47	4.56	4.64	4.71	4.77	4.83	4.89	4.94	4.99	5.03	
40	2.38	2.99	3.35	3.60	3.80	3.96	4.10	4.21	4.32	4.41	4.49	4.56	4.63	4.69	4.75	4.81	4.86	4.90	4.95	
60	2.36	2.96	3.31	3.56	3.75	3.91	4.04	4.16	4.25	4.34	4.42	4.49	4.56	4.62	4.67	4.73	4.78	4.82	4.86	
120	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.10	4.19	4.28	4.35	4.42	4.48	4.54	4.60	4.65	4.69	4.74	4.78	
$\infty$	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13	4.21	4.28	4.35	4.41	4.47	4.52	4.57	4.61	4.65	4.69	

Source: "Tables of Range and Studentized Range", *Annals of Mathematical Statistics*, vol. 31, no. 4, Reprinted with permission of the Institute of Mathematical Sciences.