UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 201 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Consider the following matrices :

$$M = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix} \text{ and } (NM)^{T} = \begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}.$$

Find the matrix N.

(40 marks)

(b) If $\begin{vmatrix} -3g+2i & 2h & -i \\ -3d+2f & 2e & -f \\ -3a+2c & 2b & -c \end{vmatrix} = 12$, find $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$.

(40 marks)

(c) Determine the type of solution for the homogenous system of linear equations below:

$$\begin{aligned} 2x + 2z &= 0 \\ x + y - 3z &= 0 \\ 3x - y + 4z &= 0 \end{aligned}$$

(20 marks)

2. Consider the following system of linear equations:

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 3 \\ 3x + 3y + az &= a^2 - 6 \end{aligned}$$

(a) Determine all values of a so that the system above will

(i) have a unique solution;

(ii) have no solution;

(iii) have infinitely many solutions.

(60 marks)

(b) if $a = 0$, solve x using Cramer's rule.

(40 marks)

3. (a) Solve the system of linear equations by using Gauss-Jordan elimination method.

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 + 15x_6 &= 0 \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

(40 marks)

(b) Solve the following systems of linear equations using the inverse.

(i)
$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ 2x_1 + 4x_2 &= 0 \\ 6x_2 - 5x_3 &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 4x_2 &= -2 \\ 6x_2 - 5x_3 &= 4 \end{aligned}$$

(60 marks)

4. (a) Let W be a subspace spanned by u_1 = (1,2,3,0), u_2 = (1,2,0,0) and u_3 = (1,0,0,1). Find

(i) an orthogonal basis of W

(ii) an orthonormal basis of W.

(50 marks)

(b) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$

(i) Find elementary matrices E_1, E_2, E_3, such that $E_3E_2E_1A = U$ where U is an upper triangular matrix.

(ii) Determine the inverses of E_1, E_2, E_3, and set $L = E_1^{-1}$, E_2^{-1}, E_3^{-1}. What type of matrix is L? Verify that $A = LU$.

(50 marks)

5. (a) Consider the matrix

$$A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

(i) Find the eigenvalues and corresponding eigenvectors of A.

(ii) Is A diagonalizable? Give a reason for your answer.

(50 marks)

(b) Let $B = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. Find

(i) the nullspace of B,

(ii) a basis for row space of B,

(iii) a basis for column space of B,

(iv) nullity of B,

(v) rank of B.

(50 marks)

1. (a) Pertimbangkan matriks berikut :

$$M = \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix} \text{ dan } (NM)^{\mathrm{T}} = \begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}.$$

Cari matriks *N*.

(40 markah)

(b) Jika $\begin{vmatrix} -3g+2i & 2h & -i \\ -3d+2f & 2e & -f \\ -3a+2c & 2b & -c \end{vmatrix} = 12$, cari $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$.

(40 markah)

(c) Tentukan jenis penyelesaian kepada system persamaan linear homogen berikut:

$$2x + 2z = 0$$
$$x + y - 3z = 0$$
$$3x - y + 4z = 0$$

(20 markah)

2. Pertimbangkan sistem persamaan linear yang berikut:

$$x + y + z = 1$$
$$2x + 3y + 2z = 3$$
$$3x + 3y + az = a^2 - 6$$

(a) Nyatakan semua nilai untuk **a** supaya sistem di atas akan

(i) mempunyai penyelesaian yang unik;

(ii) tiada penyelesaian;

(iii) mempunyai penyelesaian yang tidak terhingga.

(60 markah)

(b) Jika $= 0$, selesikan *x* menggunakan petua Cramer.

(40 markah)

3. (a) Selesaikan persamaan linear berikut menggunakan kaedah penghapusan Gauss-Jordan.

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 + 15x_6 &= 0 \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

(40 markah)

(b) Selesaikan persamaan linear berikut menggunakan kaedah songsang.

(i)
$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ 2x_1 + 4x_2 &= 0 \\ 6x_2 - 5x_3 &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 4x_2 &= -2 \\ 6x_2 - 5x_3 &= 4 \end{aligned}$$

(60 markah)

4. (a) Katakankan W subruang direntangi oleh u_1 = (1,2,3,0), u_2 = (1,2,0,0) and u_3 = (1,0,0,1). Cari

(i) asas ortogon W

(ii) asas orthonormal W.

(50 markah)

(b) Diberi $A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}$

(i) Cari operasi baris permulaan E_1, E_2, E_3, seperti $E_3E_2E_1A = U$ di mana U adalah matriks segitiga atas.

(ii) Tentukan penyongsangan ke atas E_1, E_2, E_3, dan set $L = E_1^{-1}, E_2^{-1}, E_3^{-1}$. Apakah jenis matriks L? Tentukan bahawa $A = LU$.

(50 marks)

5. (a) Pertimbangkan matrik

$$A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

(i) Cari nilai eigen dan vector eigen yang sepadan bagi matrik *A*.

(ii) Adakah *A* terpepenjurukan? Berikan sebab jawapan anda.

(50 markah)

(b) Andai $B = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. Cari

(i) ruang nol *B*,

(ii) suatu asas bagi ruang baris *B*,

(iii) suatu asas bagi ruang lajur *B*,

(iv) nol *B*,

(v) pangkat *B*.

(50 markah)

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