
UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIM 105 – Basic Mathematics
[Matematik Asas]

Duration: 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains **EIGHT** printed pages before you begin the examination.

Answer **ALL** questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan].

1. (a) Construct a 3×3 matrix whose entries is given by $a_{ij} = 2i - j$.

(20 marks)

- (b) Find the determinant of matrix

$$A = \begin{pmatrix} -4 & -2 & 1 \\ 2 & 6 & -1 \\ 5 & 3 & -7 \end{pmatrix}.$$

(30 marks)

- (c) Solve the following system of linear equations

$$2x - y + z = 3$$

$$2y - z = 2$$

$$-x + y = -1$$

using

- (i) Cramer's rule,
- (ii) matrix inverse method, and
- (iii) Gauss-Jordan elimination method.

(50 marks)

2. (a) Given $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$, state the dimension of A and B .

Then find AB if exists.

(30 marks)

- (b) If $A = \begin{pmatrix} -1 & 2 & 5 \\ 2 & 1 & 2 \\ 3 & 6 & 4 \end{pmatrix}$, find a reduced row echelon form of A .

(30 marks)

- (c) By using elementary row operations, find the inverse of the matrix

$$B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix}.$$

(40 marks)

3. (a) Evaluate the following limits if exist

(i) $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

(ii) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

(iii) $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$

(iv) $\lim_{x \rightarrow \infty} \frac{4x^2 - 2x + 1}{2x^2 + x - 1}$

(40 marks)

(b) Given $f(x) = \begin{cases} x+1 & \text{if } x \leq 2 \\ x^2 - 1 & \text{if } x > 2, \end{cases}$

determine if f is continuous at $x = 2$. Give your reason.

(30 marks)

(c) Use the definition of derivative to compute $f'(x)$ where $f(x) = x^2 + 1$.

(30 marks)

4. (a) Find $\frac{dy}{dx}$ if

(i) $y = (x^2 - 4)(e^{2x} + 4)$

(ii) $y = (x^5 - 2)^4$

(iii) $y = \cos 2x^3$

(iv) $y = x^{\sin x}$

(v) $3xy^2 - 4x = 10y$

(50 marks)

- (b) Given the function $f(x) = x^3 - 3x^2 - 9x$.
- (i) Find all critical numbers.
 - (ii) Determine all the local maximum and minimum.
 - (iii) Determine the intervals where the graph of f concave up and concave down.
 - (iv) Find the inflection point if exists.
 - (v) Sketch the graph of f .

(50 marks)

5. (a) Evaluate

- (i) $\int (3x^2 - 1) dx$
- (ii) $\int (x-1)(x+2) dx$
- (iii) $\int \frac{x-1}{x^2-2x-3} dx$
- (iv) $\int \cos x \sin x dx$

(40 marks)

- (b) Compute

- (i) $\int_0^\pi x \sin x dx$
- (ii) $\int_1^2 e^{2x} dx$
- (iii) $\int_1^e \frac{\ln x}{x} dx$

(30 marks)

- (c) Find the area bounded by the graph of $y = 4 - x^2$ from $x = 0$ and $x = 4$.

(30 marks)

1. (a) Binakan satu matriks 3×3 di mana pemasukannya diberi oleh $a_{ij} = 2i - j$.
(20 markah)

- (b) Cari penentu bagi matriks

$$A = \begin{pmatrix} -4 & -2 & 1 \\ 2 & 6 & -1 \\ 5 & 3 & -7 \end{pmatrix}.$$

(30 markah)

- (c) Selesaikan sistem persamaan linear berikut

$$2x - y + z = 3$$

$$2y - z = 2$$

$$-x + y = -1$$

dengan menggunakan

- (i) petua Cramer,
- (ii) kaedah songsangan matriks, dan
- (iii) kaedah penghapusan Gauss-Jordan.

(50 markah)

2. (a) Diberi $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$ dan $B = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$, nyatakan dimensi bagi A dan B .

Seterusnya cari AB jika wujud.

(30 markah)

- (b) Jika $A = \begin{pmatrix} -1 & 2 & 5 \\ 2 & 1 & 2 \\ 3 & 6 & 4 \end{pmatrix}$, dapatkan suatu bentuk baris terturun bagi A .

(30 markah)

- (c) Dengan menggunakan operasi baris permulaan, cari songsangan bagi matriks

$$B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix}.$$

(40 markah)

3. (a) Nilai kan had-had berikut jika wujud

(i) $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

(ii) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

(iii) $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$

(iv) $\lim_{x \rightarrow \infty} \frac{4x^2 - 2x + 1}{2x^2 + x - 1}$

(40 markah)

(b) Diberi $f(x) = \begin{cases} x+1 & \text{jika } x \leq 2 \\ x^2 - 1 & \text{jika } x > 2, \end{cases}$

tentukan jika f selanjar pada $x = 2$. Berikan alasan anda.

(30 markah)

(c) Gunakan takrif terbitan untuk mendapatkan $f'(x)$ di mana $f(x) = x^2 + 1$.

(30 markah)

4. (a) Cari $\frac{dy}{dx}$ jika

(i) $y = (x^2 - 4)(e^{2x} + 4)$

(ii) $y = (x^5 - 2)^4$

(iii) $y = \cos 2x^3$

(iv) $y = x^{\sin x}$

(v) $3xy^2 - 4x = 10y$

(50 markah)

- (b) Diberi fungsi $f(x) = x^3 - 3x^2 - 9x$.
- (i) cari semua titik genting.
 - (ii) tentukan semua maksimum tempatan dan minimum tempatan.
 - (iii) tentukan selang di mana graf bagi f cekung ke atas dan cekung ke bawah.
 - (iv) cari titik lengkok balas jika wujud.
 - (v) lakarkan graf bagi f .

(50 markah)

5. (a) Nilaikan

- (i) $\int (3x^2 - 1) dx$
- (ii) $\int (x-1)(x+2) dx$
- (iii) $\int \frac{x-1}{x^2-2x-3} dx$
- (iv) $\int \cos x \sin x dx$

(40 markah)

(b) Hitungkan

- (i) $\int_0^\pi x \sin x dx$
- (ii) $\int_1^2 e^{2x} dx$
- (iii) $\int_1^e \frac{\ln x}{x} dx$

(30 markah)

(c) Cari luas rantau yang dibatasi oleh graf bagi $y = 4 - x^2$ dari $x = 0$ dan $x = 4$.

(30 markah)

Formula for JIM105**(A) Table of derivatives:**

$\frac{d}{dx}[c] = 0$, c is a constant	$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$
$\frac{d}{dx}[u^n] = nu^{n-1} \cdot \frac{du}{dx}$	$\frac{d}{dx}[\sin u] = \cos u \cdot \frac{du}{dx}$
$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$	$\frac{d}{dx}[\cos u] = -\sin u \cdot \frac{du}{dx}$
$\frac{d}{dx}[a^u] = a^u \cdot \ln a \cdot \frac{du}{dx}$	$\frac{d}{dx}[\tan u] = \sec^2 u \cdot \frac{du}{dx}$

(B) Table of integration

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ for $n \neq -1$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
$\int \sin x dx = -\cos x + c$	$\int \sec^2 x dx = \tan x + c$
$\int \csc^2 x dx = -\cot x + c$	$\int e^x dx = e^x + c$
$\int e^{-x} dx = -e^{-x} + c$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$	$\int u dv = uv - \int v du$