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UNIVERSITI SAINS MALAYSIA

Final Examination  
2015/2016 Academic Session

May/June 2016

**JIM 101 – Calculus**  
*[Kalkulus]*

Duration: 3 hours  
*[Masa: 3 jam]*

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Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

*Jawab **SEMUA** soalan.*

*Baca arahan dengan teliti sebelum anda menjawab soalan.*

*Setiap soalan diperuntukkan 100 markah.*

*Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan.]*

1. (a) Find the following limits if they exist:

(i)  $\lim_{x \rightarrow -2} \frac{x^3}{x-2}$ .

(ii)  $\lim_{x \rightarrow 2} \frac{x-2}{(\sqrt{x^2+5})-3}$ .

(iii)  $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+5}}{4-x}$ .

(40 marks)

(b) Given

$$f(x) = \begin{cases} \frac{x^2-9}{x^2-5x+6}, & x \neq 3, \\ kx+5, & x = 3. \end{cases}$$

Find the value of  $k$  so that the function  $f(x)$  is continuous.

(15 marks)

(c) Find  $\frac{dy}{dx}$  for each of the following:

(i)  $y = 2\pi^2 - 5x$ .

(ii)  $y = \frac{\ln x}{x^2}$ .

(iii)  $x^2y^2 - \sin xy + 4x = 0$ .

(45 marks)

2. (a) Find the values of  $n$  if  $y = Ae^{nx}$ , where  $A$  is a constant, satisfying the equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0.$$

(20 marks)

(b)  $A = 4\pi r^2$  is the surface area of a sphere where  $r$  is the radius that decreases at the rate of  $2\text{ cm s}^{-1}$ . How fast is the change in the area of the sphere,  $\frac{dA}{dt}$  when the radius is 4 cm ?

(20 marks)

- (c) Given the function  $f(x) = x^3 - 12x + 6$ .
- (i) Find all critical points.
  - (ii) Determine the interval where  $f$  is increasing or decreasing, then find the local extreme values.
  - (iii) Determine the interval where  $f$  is concave up or concave down, then find the point of inflection.

(60 marks)

3. (a) Evaluate the following integrals.

(i)  $\int \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) dx.$

(ii)  $\int x^8 + x^{-8} dx.$

(iii)  $\int_1^2 \frac{2x^5 - x + 3}{x^2} dx.$

(50 marks)

(b) Evaluate  $\int_0^1 x^5 \sqrt{x^3 + 1} dx.$

(20 marks)

- (c) (i) Determine the area of the region bounded by  $y = 2x^2 + 10$  and  $y = 4x + 16$ .

- (ii) Determine the volume of the solid obtained by rotating the region bounded by  $y = x^2 - 2x$  and  $y = x$  and about the line  $y = 4$ .

(30 marks)

4. (a) (i) Find an equation of the ellipse with foci  $(2, -2)$ ,  $(4, -2)$  and vertices  $(1, -2)$ ,  $(5, -2)$ .

- (ii) Sketch the conic  $9x^2 - 4y^2 - 72x + 8y + 176 = 0$  and find its foci.

(50 marks)

- (b) Graph  $r = 5 - 5\sin\theta$ .

(20 marks)

- (c) Determine the area of the region outside  $r = 3 + 2\sin\theta$  and inside  $r = 2$ .

(30 marks)

5. (a) Evaluate the following integrals:

(i)  $\int dx.$

(ii)  $\int_{-2}^3 5x^6 - 10x + \frac{1}{x} dx.$

(25 marks)

(b) Evaluate  $\int \frac{\cos x}{(5 + \sin x)^2} dx.$

(25 marks)

(c) Evaluate  $\int \frac{x^3 + 10x^2 + 3x + 36}{(x-1)(x^2+4)^2} dx.$

(25 marks)

(d) For the point with polar coordinates  $\left(2, \frac{\pi}{7}\right)$  determine three different sets of coordinates for the same point all of which have angles different from  $\frac{\pi}{7}$  and are in the range of  $-2\pi \leq \theta \leq 2\pi.$

(25 marks)

1. (a) Cari had berikut jika wujud:

(i)  $\lim_{x \rightarrow -2} \frac{x^3}{x-2}$ .

(ii)  $\lim_{x \rightarrow 2} \frac{x-2}{(\sqrt{x^2+5})-3}$ .

(iii)  $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+5}}{4-x}$ .

(40 markah)

(b) Diberi

$$f(x) = \begin{cases} \frac{x^2-9}{x^2-5x+6}, & x \neq 3, \\ kx+5, & x = 3. \end{cases}$$

Cari nilai  $k$  supaya fungsi  $f(x)$  adalah selanjar.

(15 markah)

(c) Dapatkan  $\frac{dy}{dx}$  bagi setiap yang berikut:

(i)  $y = 2\pi^2 - 5x$ .

(ii)  $y = \frac{\ln x}{x^2}$ .

(iii)  $x^2y^2 - \sin xy + 4x = 0$ .

(45 markah)

2. (a) Cari nilai-nilai  $n$  jika  $y = Ae^{nx}$ , di mana  $A$  ialah pemalar, memenuhi persamaan

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0.$$

(20 markah)

(b)  $A = 4\pi r^2$  ialah luas permukaan sebuah sfera dengan jejari  $r$  menyusut dengan kadar  $2 \text{ ms}^{-1}$ . Berapakah kadar perubahan sfera,  $\frac{dA}{dt}$  apabila jejari ialah 4 sm?

(20 markah)

- (c) Diberi suatu fungsi  $f(x) = x^3 - 12x + 6$ .
- (i) Cari semua titik kritikal.
  - (ii) Tentukan selang di mana  $f$  menokok atau menyusut, kemudian cari nilai ekstremum setempat.
  - (iii) Tentukan selang di mana  $f$  cekung ke atas atau cekung ke bawah, kemudian cari titik lengkok balas.

(60 markah)

3. (a) Nilaikan kamiran-kamiran berikut:

(i)  $\int \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) dx.$

(ii)  $\int x^8 + x^{-8} dx.$

(iii)  $\int_1^2 \frac{2x^5 - x + 3}{x^2} dx.$

(50 markah)

(b) Nilaikan  $\int_0^1 x^5 \sqrt{x^3 + 1} dx.$

(20 markah)

- (c) (i) Tentukan luas kawasan yang dibatasi oleh  $y = 2x^2 + 10$  dan  $y = 4x + 16$ .

- (ii) Tentukan isipadu pepejal yang diperolehi daripada putaran rantau yang dibatasi oleh  $y = x^2 - 2x$  dan  $y = x$  sekitar garis  $y = 4$ .

(30 markah)

4. (a) (i) Dapatkan persamaan elips berfokus pada  $(2, -2)$ ,  $(4, -2)$  dan berbucu pada  $(1, -2)$ ,  $(5, -2)$ .

- (ii) Lakar keratan kon  $9x^2 - 4y^2 - 72x + 8y + 176 = 0$  dan cari fokus-fokus keratan kon ini.

(50 markah)

- (b) Grafkan  $r = 5 - 5\sin\theta$ .

(20 markah)

- (c) Tentukan luas kawasan di luar  $r = 3 + 2\sin\theta$  dan di dalam  $r = 2$ .

(30 markah)

5. (a) Nilai kamiran-kamiran berikut:

(i)  $\int dx.$

(ii)  $\int_{-2}^3 5x^6 - 10x + \frac{1}{x} dx.$

(25 markah)

(b) Nilai kamiran  $\int \frac{\cos x}{(5 + \sin x)^2} dx.$

(25 markah)

(c) Nilai kamiran  $\int \frac{x^3 + 10x^2 + 3x + 36}{(x-1)(x^2+4)^2} dx.$

(25 markah)

(d) Bagi titik yang mempunyai koordinat kutub  $\left(2, \frac{\pi}{7}\right)$  tentukan tiga set koordinat yang mempunyai sudut berbeza daripada  $\frac{\pi}{7}$  dan di dalam julat  $-2\pi \leq \theta \leq 2\pi.$

(25 markah)