UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIF 318 – Quantum Mechanics
[Mekanik Kuantum]

Time : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **ELEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in English or in Bahasa Malaysia.

Read the instructions carefully before answering.

In the event of any discrepancies in the exam questions, the English version shall be used.

Sila pastikan kertas peperiksaan ini mengandungi ***SEBELAS*** *muka surat yang bercetak sebelum anda menjawab sebarang soalan.*

Jawab ***SEMUA*** *soalan. Anda dibenarkan menjawab soalan* **sama ada** *dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca setiap arahan dengan teliti sebelum menjawab.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Answer **ALL** questions.

1. (a) Neils Bohr used the concept of quantum physics proposed by Max Planck to explain the arrangement of orbital electrons around a nucleus. However, latter physicists claimed that Bohr's postulates have some weaknesses. Elaborate on these weaknesses.

Neils Bohr telah menngunakan konsep fizik kuantum yang diutarakan oleh Max Planck untuk menjelaskan susun-atur elektron orbit di sekeliling suatu nukleus. Bagaimanapun, ahli-ahli fizik terkemudian telah mengatakan bahawa terdapat beberapa kelemahan dalam postulat-postulat Bohr itu. Huraikan kelemahan-kelemahan tersebut.

(10 marks/*markah*)

(b) Calculate the de Broglie wavelength of a proton (mc^2 = 938 MeV) with

Hitung panjang gelombang satu proton (mc^2 = 938 MeV) yang mempunyai

(i) a kinetic energy of 0.1 MeV

tenaga kinetik 0.1 MeV

(ii) a total energy of 3 GeV.

jumlah tenaga 3 GeV.

(10 marks/*markah*)

2. (a) Evaluate all of the following, and express the final answers in the form of $a + bi$:

Nilaikan semua yang berikut, dan ungkapkan semua jawapan akhir anda dalam bentuk $a + bi$:

(i) $i(2-3i)(3+5i)$

(ii) $\dfrac{i}{(i-1)}$

(iii) $(1+i)^{30}$

(10 marks/*markah*)

(b) A complex number is given as $z = 1 + e^{i\theta}$. Calculate

Suatu nombor kompleks diberikan sebagai $z = 1 + e^{i\theta}$. *Hitung*

(i) z^*

(ii) z^2

(iii) $|z|^2$.

(10 marks/*markah*)

3. (a) Define the probability density of a particle in space.

Takrifkan ketumpatan kebarangkalian bagi suatu zarah dalam ruang.

(6 marks/*markah*)

(b) A particle of mass *m* is moving in a 1-D potential *V*(*x*, *t*). The wave function for the particle is

Suatu zarah berjisim m bergerak dalam suatu keupayaan 1-D V(x, t). Fungsi gelombang bagi zarah ini ialah

$$\Psi(x,t) = Axe^{-\left(\sqrt{km}/2\hbar\right)x^2} e^{-\sqrt{k/m}(3/2)t}$$

for $-\infty < x < +\infty$, where *k* and *A* are constants.

bagi $-\infty < x < +\infty$ *, di sini k dan A adalah pemalar.*

(i) Normalise the wave function.

Normalkan fungsi gelombang ini.

(ii) Calculate the expectation value of position $\langle x \rangle$.

Hitung nilai jangkaan kedudukan $\langle x \rangle$.

(iii) Calculate the expectation value of momentum $\langle p \rangle$.

Hitung nilai jangkaan momentum $\langle p \rangle$.

(14 marks/*markah*)

4. (a) Explain the following terms:
Jelaskan sebutan-sebutan berikut:

(i) linear operator.
operator linear

(ii) Hermitian operator.
operator Hermitian

(iii) adjoint operator.
operator adjoin

(iv) commutator.
komutator

(v) Dirac notation.
tatatanda Dirac.

(10 marks/*markah*)

(b) The operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ are all Hermitian with $\left[\hat{A}, \hat{B}\right] = \hat{C}$. Show that $\hat{C} = 0$.

Operator-operator $\hat{A}$, $\hat{B}$, *dan* $\hat{C}$ *adalah semuanya Hermitian dengan* $\left[\hat{A}, \hat{B}\right] = \hat{C}$. *Tunjukkan bahawa* $\hat{C} = 0$.

(5 marks/*markah*)

(c) The operators $\hat{A}$ and $\hat{B}$ are both Hermitian with $\left[\hat{A}, \hat{B}\right] = i\hbar$. Determine whether $\hat{A}\hat{B}$ is a Hermitian operator.

Operator-operator $\hat{A}$ *dan* $\hat{B}$ *adalah kedua-duanya Hermitian dengan* $\left[\hat{A}, \hat{B}\right] = i\hbar$. *Tentukan sama ada* $\hat{A}\hat{B}$ *adalah suatu operator Hermitian.*

(5 marks/*markah*)

5. (a) What is meant by degeneracy of an energy level? Give examples.
Apakah yang dimaksudkan dengan terdegenerat? Berikan contoh-contoh.

(4 marks/*markah*)

…5/-

(b)

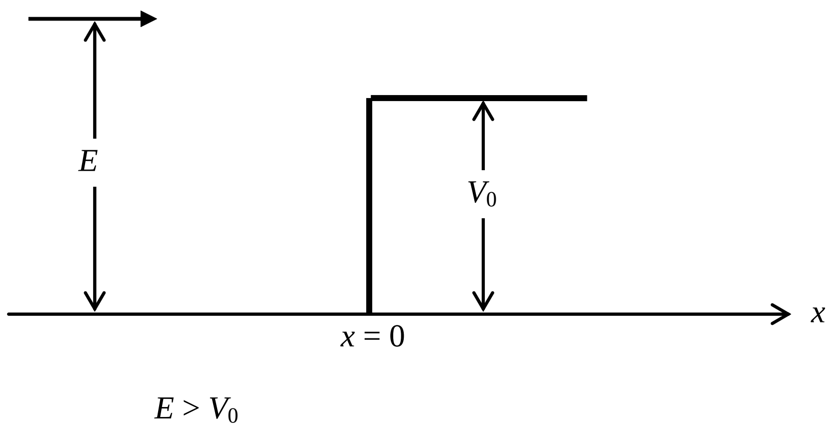

Figure 1/*Rajah 1*

Figure 1 shows an entity with energy E moving in the positive x direction towards a step potential V_0. Given that $E > V_0$. Obtain
Rajah 1 menunjukkan suatu entiti dengan tenaga E bergerak dalam arah x positif menuju suatu keupayaan bertangga V_0. Diberikan $E > V_0$. Dapatkan

(i) the reflection coefficient R
pekali pantulan R

(ii) the transmission coefficient T.
pekali transmissi T.

(iii) Discuss the results obtained with those expected from classical physics.
Bincangkan keputusan yang diperolehi dengan keputusan yang dijangkakan dari fizik klasik.

(16 marks/*markah*)

Constants:

Speed of light $c = 3.0 \times 10^{8}$ m s^{-1}

Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

Planck constant $h = 6.63 \times 10^{-34}$ J s

Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}

Permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1}

Permeability of free space $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

Basic charge $e = 1.6 \times 10^{-19}$ C

Electron rest-mass $m_e = 9.1 \times 10^{-31}$ kg

Proton rest-mass $m_p = 1.6725 \times 10^{-27}$ kg ≡ 1.0072766 u

Neutron rest-mass $m_n = 1.6748 \times 10^{-27}$ kg ≡ 1.0086654 u

Bohr's radius $a = 5.3 \times 10^{-11}$ m

1 eV = 1.6×10^{-19} J

1 u ≡ 931 MeV c^{-2}

1 barn = 10^{-28} m^{2}

1 fm = 10^{-15} m

1 Ci = 3.7×10^{10} s^{-1}

USEFUL MATHEMATICS IN QUANTUM MECHANICS

Exponential series

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \cdots + \frac{x^{n}}{n!}$$

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

Trigonometric series

$$\sin x = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{7}}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{6}}{6!} + \cdots$$

$$\tan x = x + \frac{x^{3}}{3} + \frac{2x^{5}}{15} + \cdots$$

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Differentiation and integration (Standard forms)

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$ $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
$\frac{d}{dx}\log x = \frac{1}{x}$ $\frac{d}{dx}\log(ax+b) = \frac{a}{ax+b}$	$\int \frac{dx}{x} = \log x + c$ $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b) + c$
$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}e^{mx} = me^{mx}$	$\int e^x dx = e^x + c$ $\int e^{mx} dx = \frac{e^{mx}}{m} + c$
$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\sin mx = m\cos mx$	$\int \cos x\, dx = \sin x + c$ $\int \cos mx\, dx = \frac{\sin mx}{m} + c$
$\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\cos mx = -m\sin mx$	$\int \sin x\, dx = -\cos x + c$ $\int \sin mx\, dx = -\frac{\cos mx}{m} + c$
$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\tan mx = m\sec^2 mx$	$\int \sec^2 x\, dx = \tan x + c$ $\int \sec^2 mx\, dx = \frac{\tan mx}{m} + c$
$\frac{d}{dx}\cot x = -\mathrm{cosec}^2 x$ $\frac{d}{dx}\cot mx = -m\,\mathrm{cosec}^2 mx$	$\int \mathrm{cosec}^2 x\, dx = -\cot x + c$ $\int \mathrm{cosec}^2 mx\, dx = -\frac{\cot mx}{m} + c$
$\frac{d}{dx}\sinh x = \cosh x$	$\int \cosh x\, dx = \sinh x + c$
$\frac{d}{dx}\cosh x = \sinh x$	$\int \sinh x\, dx = \cosh x + c$

Integration by substitution

Some common types of substitutions:

(i) Integrals containing a term of the form $(ax+b)^n \rightarrow$ let $ax+b=z$.

(ii) Integrals containing a term in the form of $\sqrt{a^2-x^2}$ or $\left(\sqrt{a^2-x^2}\right)^n \rightarrow$ let $x=a\sin\theta$.

(iii) As in (ii) but with $\sqrt{a^2-x^2}$ replaced by $\sqrt{a^2+x^2} \rightarrow$ let $x=a\sinh\theta$ or $x=a\tan\theta$.

(iv) As in (ii) but with $\sqrt{a^2-x^2}$ replaced by $\sqrt{x^2-a^2} \rightarrow$ let $x=a\cosh\theta$.

(v) Integrals of *odd powers of sine or cosine* $\rightarrow$ let $\cos x=c$ or $\sin x=s$ respectively.

(vi) Integrals of the form $\int \frac{dx}{a+b\cos x}$ or $\int \frac{dx}{a+b\sin x} \rightarrow$ let $\tan\frac{x}{2}=t$.

(vii) Integrals of the form $\int \frac{dx}{x\sqrt{ax^2+bx+c}} \rightarrow$ let $x=\frac{1}{z}$;

or $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}} \rightarrow$ let $px+q=\frac{1}{z}$.

Integration by parts

$$\int u\upsilon\, dx = u\int \upsilon\, dx - \int\left\{\int \upsilon\, dx\right\}\frac{du}{dx}\,dx$$

Integration common in Quantum Mechanics

$$f(x) = \int_0^{\infty} x^n e^{-ax^2}\, dx$$

n	$f(n)$	n	$f(n)$
0	$\frac{1}{2}\sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4}\sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8}\sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16}\sqrt{\frac{\pi}{a^7}}$	7	$\frac{3}{a^4}$

If n is even, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\, dx = 2f(x)$

If n is odd, $\int_{-\infty}^{\infty} x^n e^{-ax^2}\, dx = 0$

Other standard integrals

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

$$\int_0^{\infty} \frac{x}{(e^x - 1)}\, dx = \frac{\pi^2}{6}$$

$$\int_0^{\infty} \frac{x^3}{(e^x - 1)}\, dx = \frac{\pi^4}{15}$$

Reciprocal identities

$\sin u = \frac{1}{\csc u}$ $\qquad$ $\cos u = \frac{1}{\sec u}$

$\tan u = \frac{1}{\cot u}$ $\qquad$ $\cot u = \frac{1}{\tan u}$

$\csc u = \frac{1}{\sin u}$ $\qquad$ $\sec u = \frac{1}{\cos u}$

Pythagorean identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

Quotient identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Co-function identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

Parity identities (even & odd)

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \qquad \cot(-u) = -\cot u$$
$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u$$

Sum & difference formulas

$$\sin(u \pm \upsilon) = \sin u \cos \upsilon \pm \cos u \sin \upsilon$$
$$\cos(u \pm \upsilon) = \cos u \cos \upsilon \mp \sin u \sin \upsilon$$
$$\tan(u \pm \upsilon) = \frac{\tan u \pm \tan \upsilon}{1 \mp \tan u \tan \upsilon}$$

Double angle formulas

$$\sin(2u) = 2\sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

Power reducing/half angle formulas

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$
$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$
$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Sum-to-product formulas

$$\sin u + \sin \upsilon = 2\sin\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\sin u - \sin \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u + \cos \upsilon = 2\cos\left(\frac{u+\upsilon}{2}\right)\cos\left(\frac{u-\upsilon}{2}\right)$$
$$\cos u - \cos \upsilon = -2\sin\left(\frac{u+\upsilon}{2}\right)\sin\left(\frac{u-\upsilon}{2}\right)$$

Product-to-sum formulas

$$\sin u \sin \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) - \cos(u+\upsilon)\right]$$
$$\cos u \cos \upsilon = \frac{1}{2}\left[\cos(u-\upsilon) + \cos(u+\upsilon)\right]$$
$$\sin u \cos \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) + \sin(u-\upsilon)\right]$$
$$\cos u \sin \upsilon = \frac{1}{2}\left[\sin(u+\upsilon) - \sin(u-\upsilon)\right]$$

- oooOooo -