UNIVERSITI SAINS MALAYSIA

Final Examination
2015/2016 Academic Session

May/June 2016

JIF 315 – Mathematical Methods
[Kaedah Matematik]

Time : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question carries 100 marks.

In the event of any discrepancies in the exam questions, the English version shall be used.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Table of Laplace Transform

[Jadual transformasi Laplace]

$f(t)$	$L\{f(t)\} = F(s)$
a	$\frac{a}{s}$
$t^n , \quad n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(at)$	$\frac{a}{s^2 + a^2}$
$\cos(at)$	$\frac{s}{s^2 + a^2}$
$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
$e^{at} f(t)$	$F(s-a)$

Legendra Polynomial Function

[Jadual fungsi Legendra Polynomial]

$P_0(x) = 1,$ $\quad$ $P_1(x) = x,$

$P_2(x) = \frac{1}{2}(3x^2 - 1),$ $\quad$ $P_3(x) = \frac{1}{2}(5x^3 - 3x),$

$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3),$ $\quad$ $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

Answer ALL questions.

1. (a) Find the Laplace transform of the following function,

(i)

$$f(x)=\begin{cases}2, & for\ 0<\text{t}<2\\ 0, & otherwise\end{cases}$$

(ii) $f(t)=1-2e^{-2t}$

(65 marks)

(b) Find the inverse Laplace transform of the following function,

$$f(s)=\frac{2}{s^2+3s+2}$$

(35 marks)

2. (a) Consider the following form of differential equation

$$y''+\frac{1-\frac{1}{2}a}{x}y'+\left[b^2c^2x^{2c-1}+\frac{a^2-n^2c^2}{x^2}\right]y=0$$

$$y=x^a z_n(bx^c)$$

Find the general solutions of the following equations in terms of Bessel functions,

(i) $y''+\frac{1}{9}xy=0$

(ii) $y''-\frac{3}{x}y'+25xy=0$

(50 marks)

[JIF 315]

(b) The Legendre's Equation is given as follows,

$$(1-x^2)y''-2xy'+n(n+1)y=0$$

where n is a constant. Express the following equations in terms of the Legendre's Polynomial:

(i) $2x^3 - 5x^2 + 6x + 1$

(ii) $-x^3 - 4x^2 - 5x + 5$

(50 marks)

3. Find all the eigenvalues, λ_n and the corresponding eigenfunction, φ_n of the following Sturm-Liouville problem. Consider all cases for λ.

$$y''+\lambda y=0,\ y(0)-y(\pi)=0,\ y'(0)-y'(\pi)=0$$

(100 marks)

4. Let $f(x)$ be a function of period 2π such that

$$f(x)=\frac{x}{2},\ \text{for } 0 < x < 2\pi$$

(a) Find the Fourier series for $f(x)$.

(55 marks)

(b) Using the result in (a), show that

$$\frac{x}{2}-[\sin x+\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x+...]$$

(25 marks)

(c) Sketch the graph of the function $f(x)$ in the interval $0 < x < 4\pi$

(20 marks)

5. (a) Consider the following form of differential equation.

$$f(x)=\begin{cases}\sin(x), & \text{for } 0<x<\pi \\ 0, & \text{otherwise}\end{cases}$$

Determine the Fourier transform of $f(x)$.

(50 marks)

(b)

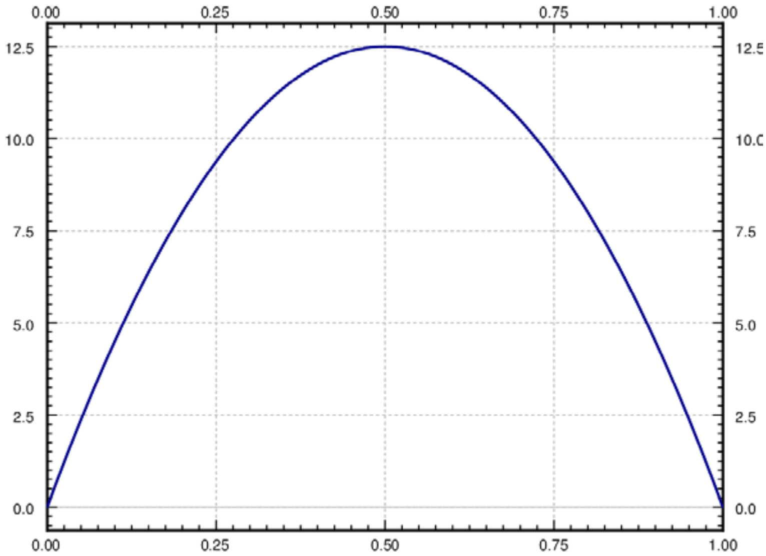


Figure 1: Initial distribution of temperature in the wire.

Suppose that we have an insulated wire of length 1 m, such that the ends of the wire are embedded in ice (temperature 0 ℃). Let k = 0.003, then suppose that initial heat distribution in wire (See Figure 1) is $u(x,0)=50x(1-x)$. Find the temperature function *u (x, t)*.

(50 marks)

Jawab semua soalan.

1. *(a)* *Carikan tranformasi Laplace bagi fungsi yang berikut,*

(i) $f\left(x\right)=\begin{cases}2, & untuk\ \ 0<\mathrm{t}<2\\ 0, & Lain-lain\end{cases}$

(ii) $f\left(t\right)=1-2e^{-2t}$

(65 markah)

(b) *Carikan tranformasi Laplace songsang bagi fungsi berikut,*

$$f\left(s\right)=\frac{2}{s^{2}+3s+2}$$

(35 markah)

2. (a) *Pertimbangkan persamaan pembezaan berikut,*

$$y''+\frac{1-\frac{1}{2}a}{x}y'+\left[b^{2}c^{2}x^{2c-1}+\frac{a^{2}-n^{2}c^{2}}{x^{2}}\right]y=0$$

$$y=x^{a}z_{n}(bx^{c})$$

Carikan penyelesaian am bagi persamaan berikut dalam sebutan fungsi Bessel,

(i) $y''+\frac{1}{9}xy=0$

(ii) $y''-\frac{3}{x}y'+25xy=0$

(50 markah)

(b) *Persamaan Legendre adalah seperti berikut,*

$$(1-x^2)y''-2xy'+n(n+1)y=0$$

di mana n adalah pemalar. Ungkapkan persamaan berikut dalam bentuk polinomial Legendre :

(i) $2x^3 - 5x^2 + 6x + 1$

(ii) $-x^3 - 4x^2 - 5x + 5$

(50 markah)

3. *Cari semua nilai-eigen , λ_n dan fungsi-eigen yang sepadan, φ_n bagi masalah Sturm - Liouville berikut. Pertimbangkan semua kes bagi λ .*

$$y''+\lambda y=0,\ y(0)-y(\pi)=0,\ y'(0)-y'(\pi)=0$$

(100 markah)

4. *Andaikan f(x) adalah fungsi tempoh 2π seperti*

$$f(x)=\frac{x}{2},\ \textit{untuk } 0 < x < 2\pi$$

(a) Carikan siri Fourier bagi $f(x)$.

(55 markah)

(b) Dengan menggunakan keputusan di (a), tunjukkan bahawa

$$\frac{x}{2}-[\sin x+\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x+...]$$

(25 markah)

(c) Lakarkan graf bagi fungsi $f(x)$ dalam lingkungan $0 < x < 4\pi$

(20 markah)

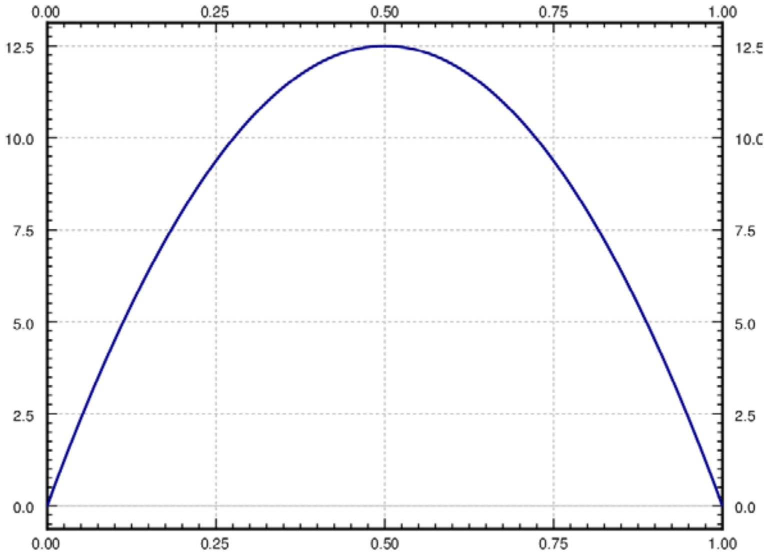
5. *(a) Persamaan Perbezaan ialah seperti berikut*

$$f(x) = \begin{cases} \sin(x), & untuk\ 0 < x < \pi \\ 0, & Lain - lain \end{cases}$$

Tentukan transformasi Fourier bagi $f(x)$.

(50 markah)

(b)



Rajah 1: *Permulaan pengagihan haba dalam wayar.*

Andaikan bahawa kita mempunyai wayar bertebat dengan panjang 1 m, kedua-dua hujung dawai ditanam dalam ais (bersuhu 0 °C). Andaikan k = 0.003, kemudian anggapkan bahawa permulaan pengagihan haba dalam wayar (Lihat Rajah 1) ialah $u(x,0) = 50x(1-x)$. Carikan fungsi suhu u (x, t).

(50 markah)

-oooOooo -