UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

ZCT 205/3 - Quantum Mechanics
[Mekanik Kuantum]

Duration: 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **FIVE** printed pages before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all **SIX** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab kesemua **ENAM** soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. (a) Explain in details the photoelectric effect and how this can be explained successfully using Planck's postulate.
[Terangkan secara teliti kesan fotoelektrik dan bagaimana fenomenon itu dapat diterangkan dengan jayanya melalui postulat Planck.]

(60/100)

(b) Describe briefly three other experiments which demonstrate the failure of classical physics and how the quantum concept can explain the results.
[Terangkan secara ringkas tiga eksperimen lain yang menunjukkan kegagalan fizik klasik dan memerlukan konsep kuantum untuk menjelaskan keputusan-keputusan.]

(40/100)

2. (a) State precisely Heisenberg uncertainty principle.
[Nyatakan dengan tepat prinsip ketakpastian Heisenberg.]

(10/100)

(b) (i) The position of a 10 KeV electron can be determined to a precision of 10^{-11}m. What is the uncertainty of its linear momentum, Δp_x?
Calculate $\frac{\Delta p_x}{p_x}$.
[Posisi sesuatu elektron bertenaga 10 KeV ditentukan dengan kepersisan 10^{-11}m. Beberapakah ketakpastian momentum linearnya, Δp_x? Hitungkan $\frac{\Delta p_x}{p_x}$.]

(10/100)

(ii) The position of billard ball of 10 grams moving with a velocity of 20 cm/sec can be determined to a precision of 10^{-6}m. What is the uncertainty in its linear momentum, Δp_x? Calculate $\frac{\Delta p_x}{p_x}$.
*[Posisi sesuatu bola billard yang beratnya 10 gm dan bergerak dengan halaju 20 cm/sec ditentukan dengan kepersisan 10^{-6}m. Beberapakah ketakpastian momentum linearnya, Δp_x?
Hitungkan $\frac{\Delta p_x}{p_x}$]*

(10/100)

(iii) Discuss the results of (i) and (ii) above.
Bincangkan keputusan-keputusan di bahagian (i) dan (ii) di atas.]

(20/100)

(c) Starting from the relationship, $[\hat{p}_x, \hat{x}] = i\hbar$, derive from first principles, the exact expression for Heisenberg Uncertainty Principle.
[Mulai daripada perhubungan $[\hat{p}_x, \hat{x}] = i\hbar$, terbitkan ekpresi tepat bagi prinsip ketakpastian Heisenberg melalui prinsip pertama.]

(50/100)

3. (a) In Quantum Mechanics, $|\psi^2|$ is defined as the probability density. Using the concept of conservation of probability, derive the expression for probability density current, S, in one-dimension. Generalise the expression obtained to three-dimension.
[Dalam mekanik kuantum, $|\psi^2|$ ditakrifkan sebagai ketumpatan kebarangkalian. Dengan menggunakan konsep keabadian kebarangkalian, terbitkan ekspresi bagi arus ketumpatan kebarangkalian, S, dalam bentuk dimensi-satu. Dapatkan ekspresi dalam bentuk dimensi-tiga.]

(50/100)

(b) Derive from first principles the time-dependent Schrodinger Equation and the time-independent Schrodinger Equation. What are the necessary conditions?
[Terbitkan melalui prinsip pertama persamaan Schrodinger bersandar masa dan persamaan Schrodinger tak-bersandar masa. Nyatakan syarat-syarat yang digunakan.]

(50/100)

4. (a) Discuss the concept of measurements in a quantum system when the system is in (i) a pure state and (ii) a mixed state.
[Bincangkan konsep pengukuran di dalam suatu sistem kuantum bila sistem itu berada di dalam (i) keadaan tulin, dan (ii) keadaan campuran.]

(30/100)

(b) Consider two variables represented by the operators, $\hat{P}$ and $\hat{Q}$, and ϕ_i and χ_i are the eigenfunctions of $\hat{P}$ and $\hat{Q}$ respectively, i.e.
[Mempertimbangkan dua pembolehubah yang diwakili oleh operator, $\hat{P}$ dan $\hat{Q}$, dan fungsieigen adalah ϕ_i dan χ_i berturut-turut bagi $\hat{P}$ dan $\hat{Q}$ respectively, iaitu]

$$\hat{P}\phi_i = p_i\phi_i$$
$$\hat{Q}\chi_i = q_i\chi_i$$

An entity is represented by the wave function ψ, where
[Suatu entiti diwakili oleh suatu fungsi-gelombang ψ, di mana]

$$\psi = \phi_1 + 2\phi_2 + 5\phi_3$$
$$= \chi_{10}.$$

Explain the results that shall be obtained with measurements represented by:
Terangkan keputusan yang akan didapati dengan pengukuran yang diwakili oleh:]

(i) $\hat{P}\psi$, and *[dan]*

(ii) $\hat{Q}\psi$.

What are the results if these measurements are made on an ensemble of this system?
[Berikan keputusan bila pengukuran dilakukan ke atas ensembel sistem itu.]

(70/100)

5. (a) State the postulates of Quantum Mechanics. Explain why only Hermitian operators are used in Quantum Mechanics and prove it.
[Nyatakan postulat-postulat Mekanik Kuantum. Terangkan mengapa operator Hermitian hanya digunakan di dlam bidang Mekanik Kuantum dan membuktinya.]

(40/100)

(b) (i) An entity is confined to a two-dimensional box defined by:
[Suatu entiti dikurungkan di dalam suatu kotak berdimensi-dua yang ditakrifkan sebagai:]

$V = o$ for *[bagi]* $o \le x \le a$ and *[dan]* $o \le y \le b$
$V = \infty$ otherwise *[di tempat lain]*.

Obtain the wavefunction and energy of this confined entity.
[Dapatkan fungsi-gelombang dan tenaga bagi entiti yang terkurung ini.]

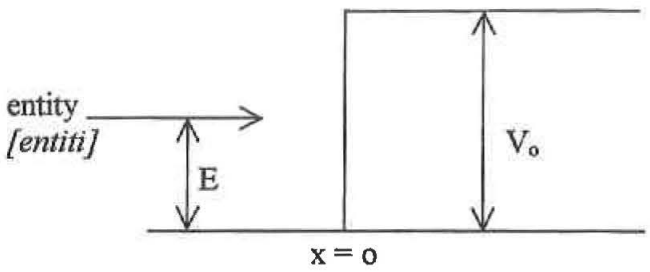
(40/100)

(ii) Discuss the energy degeneracy of this system when a = b by drawing the energy diagram. What is the energy required for this entity to jump from the ground state to the 3rd excited state?
[Bincangkan kedegeneratan paras tenaga bagi sistem itu bila a = b dengan melukiskan suatu rajah tenaga. Beberapakah tenaga yang diperlukan untuk entiti itu supaya dapat melompat dari keadaan asas ke keadaan teruja ketiga?]

(20/100)

6. (a) Obtained the reflection and transmission factors, R and T respectively, for the quantum system shown below:
[Dapatkan pekali pembalikan, R, dan pekali penghantaran, T, bagi sistem kuantum yang ditunjukkan di bawah:]

An entity of energy E moves in a positive-x direction and encounters a step potential V_o at $x = o$, $(E < V_o)$
[Suatu entiti bertenaga E bergerak ke arah positif-x dan menghadapai suatu potential tangga pada $x = o$, $(E < V_o)$]



Discuss the results obtained in this quantum system with the classical physics case.
[Bincangkan keputusan yang didapati dalam sistem kuantum itu dengan kes fizik klasik.]

(40/100)

(b) Using the results obtained above, discuss the concept of Tunnel Effect. Give three examples of the Tunnel Effect.
[Dengan menggunakan keputusan di atas, bincangkan konsep kesan penerowongan. Beri tiga contoh Kesan Penerowongan.]

(20/100)

(c) The time-independent Schrodinger Equation for an isotropic 3-D harmonic oscillator is:
[Persamaan Schrodinger tak-bersandar masa bagi suatu osilator harmonik 3D isotropik adalah:]

$$\left[\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}kr^2\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Obtain $\psi(\vec{r})$ and E.
[Dapatkan $\psi(\vec{r})$ dan E.]

Discuss the results by comparing with the classical harmonic oscillator.
[Bincangkan keputusan-keputusan dengan membandingkan kes osilator harmonik klasik.]

(40/100)