IMPROVED RESIDUAL DISTRIBUTION SCHEMES FOR THE MAXWELL'S EQUATIONS

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IMPROVED RESIDUAL DISTRIBUTION SCHEMES FOR THE MAXWELL'S EQUATIONS

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LIST OF ABBREVIATIONS

CEM	computational electromagnetics
CFD	computational fluid dynamics
FDTD	finite-difference time domain
FE(M)	finite-element (method)
FVTD	finite-volume time-domain
RD	residual distribution
RD-LW	Lax-Wendroff residual distribution
RD-LDA	low diffusion A of residual distribution
RD-N scheme	narrow scheme of residual distribution
1^{st} **Maxwell's**	first-order system of Maxwell's equations
2^{nd} **Maxwell's**	second-order scalar Maxwell's equation
2D	two dimensions or two-dimensional
3D	three dimensions or three-dimensional
TM	transverse magnetic
TE	transverse electric
PEC	perfect electrical conductor
BC	boundary condition
CFL	Courant-Friedrichs-Levy

LIST OF SYMBOLS

for electrodynamics

μ	magnetic permeability
ε	electric permittivity
c	speed of the wave
Z	intrinsic impedance of a medium
ρ	charge density
$\mathbf{J}$	current density
$\mathbf{E}$	electric field
$\mathbf{H}$	magnetic field
x, y, z	spatial Cartesian coordinates
ρ, ϕ, z	spatial cylindrical coordinates
r, θ, ϕ	spatial spherical coordinates
$J_\nu(\beta\rho)$	cylindrical Bessel function of the first kind
$Y_\nu(\beta\rho)$	cylindrical Bessel function of the second kind
$H_\nu^{(2)}(\beta\rho)$	cylindrical Hankel function of second kind
ω	angular frequency
β_m	propagation mode coefficient
$p_{\nu m}$	m^{th}-root of first kind Bessel function $J_\nu(\beta\rho)$
$\kappa_m, \kappa_n, \kappa_{mn}, \kappa_{\nu m}$	number of bounded standing waves (wave number)

for numerical methods

u	scalar conserved variable
$\mathbf{U}$	set of conserved variables
$\mathbf{F}(\mathbf{U})$	general form of spatial flux for hyperbolic system
$\mathscr{A}$	Jacobian or characteristic matrix of Maxwell's equations
A_x, A_y, A_z	components of Jacobian matrix for Maxwell's equations
β_j^T	distribution coefficient for scalar advection equation
B_j^T	distribution matrix for hyperbolic system of equations
ϕ^T	local flux residual for scalar advection equation
$\mathbf{\Phi}^T$	local flux residual for hyperbolic system of equations
α^T	local unsteady residual for hyperbolic system of equations
M^T	local mass-matrix for element T
Q^T	boundary integral matrix for element T
K^T	local stiffness matrix for element T
m_{ij}^T	local mass-matrix components for scalar advection
M_{ij}^T	local mass-matrix components for hyperbolic system
$\mathbf{e}_j^T$	outwardly scaled normal for triangular elements
$\mathbf{n}_j^T$	inwardly scaled normal for triangular elements
$\mathbf{a}_j^T$	outwardly scaled area vector for tetrahedral elements
$\mathbf{s}_j^T$	inwardly scaled area vector for tetrahedral elements

τ_j^T	edge normal of median dual cell
η_{jx}, η_{jy}	unit vector components for $\mathbf{n}_j^T$
$\zeta_{jx}, \zeta_{jy}, \zeta_{jz}$	unit vector components for $\mathbf{s}_j^T$
S_T	triangular element area
S_i	median dual cell area
V_T	tetrahedral element volume
V_i	median dual cell volume
k_j^T	local inflow parameter for scalar advection equation
K_j^T	local inflow matrix for hyperbolic system of equations
K_j^+	positive inflow matrix for hyperbolic equations
K_j^-	negative inflow matrix for hyperbolic equations
Λ	diagonal matrix of eigenvalues $\{\lambda\}$
R	right-eigenvectors set
R^{-1}	left-eigenvectors set
$\psi_j^T(\mathbf{x})$	local Lagrange basis function
$A_j^T(\mathbf{x})$	simplex coordinate or barycentric coordinate for triangular element
$\omega_j^T(\mathbf{x})$	Petrov-Galerkin type of weight function
$\mathbf{X}$	displacement vectors set from neighboring nodes j to node i
t^n	time level
Δt	time step

$\Delta\tau$	fictitious time step
$\iint_T \cdot d\Omega$	area integration over element T
$\oint_{\partial T} \cdot d\vec{\ell}$	contour integration along the perimeter of T
$\iiint_T \cdot dV$	volume integration over tetrahedron T
$j \in T$	$\{$ set of local vertices within element T $\}$
$T \in \cup\Delta_i$	$\{$ set of elements sharing node i $\}$
$i \in \Omega$	$\{$ set of nodes in the whole computational domain Ω $\}$
$T_h \in \Omega$	$\{$ set of elements in the whole computational domain Ω $\}$
$i \in \partial\Omega$	$\{$ set of nodes that fall on the outer boundary $\partial\Omega$ $\}$
$T_h \in \partial\Omega$	$\{$ set of elements abutted on the outer boundary $\partial\Omega$ $\}$

PENAMBAHBAIKAN KAEDAH PENGEDARAN SISA UNTUK PERSAMAAN MAXWELL

ABSTRAK

Daya elektromagnet mempunyai pelbagai aplikasi, salah satu daripadanya ialah pengesanan objek asing yang terbenam, sebagai contoh dalam tubuh badan melalui pembiasan gelombang. Gelombang telekomunikasi memerlukan pancaran elektromagnet, dan panduan gelombang optik yang membolehkan penghantaran isyarat pada halaju cahaya. Tujuan penyelidikan ini ialah penggunaan teknik pengedaran sisa yang berasas pada bucu segi tiga, salah satu kaedah yang tak tersirat dengan ketepatan orde kedua. Pengkomputeran elektromagnet tidak menetap pada suatu topologi kerangka tertentu, dan ini akan melembapkan kemajuan dalam teknik pengkomputeran. Salah satu skema pengedaran sisa yang terkenal dengan pemeliharaan ketepatan orde kedua ialah kaedah pengedaran sisa (RD) Lax-Wendroff (LW). Selain itu, kaedah pengedaran sisa ini terunggul dengan skema berdasarkan hilir yang mampat, misalnya skema RD-LDA (resapan rendah A), tetapi wujud sebagai kaedah tersirat bagi masalah bendalir yang bersandar kepada waktu. Pembaharuan yang pertama dalam kerja ini ialah memperolehi kaedah RD-LDA yang tak tersirat sementara memelihara ketepatan orde kedua. Di samping itu, skema RD-Galerkin yang jarang ditemui akan dicadangkan dalam kerja ini. Sumbangan yang kedua dalam kerja ini menyetelkan kaedah unsur terhingga (FEM) Galerkin untuk persamaan Maxwell orde kedua yang bersandar kepada waktu, dan juga mereka skema pengedaran sisa yang setara bagi persamaan Maxwell orde kedua ini, iaitu atur cara kecerunan sisa. Kedua-dua kaedah berangka yang lebih cekap ini memerlukan penurunan persamaan Maxwell dari-

pada orde pertama kepada orde kedua. Kaedah unsur terhingga Galerkin adalah kaedah berangka yang amat jitu, tetapi kurang berkesan dengan syarat sempadan berbanding dengan atur cara kecerunan sisa yang diperkenalkan dalam kerja ini. Pembaharuan dalam kerja ini ialah perkenalan kaedah pengedaran sisa (RD) untuk persamaan Maxwell orde pertama, dan mengarang kaedah tersebut untuk persamaan Maxwell orde kedua. Pengujian atur cara dalam kerja ini merangkumi tiga fenomena electromagnetik, iaitu penyebaran dalam panduan gelombang, pemancaran gelombang dan pembiasan gelombang. Masalah dalam tiga dimensi juga dikaji demi mengesahkan kesesuaian kaedah-kaedah ini dalam aplikasi sebenar. Keputusan daripada kaedah berangka yang diubahsuai atau direka dalam kerja ini tidak menunjukkan isu kemantapan. Penggumpalan matriks bagi skema RD-LDA tak tersirat menyusutkan tempoh pengkomputeran sebanyak 50 kali, walaupun jangka masa ini masih 4 hingga 6 kali lebih tiggi daripada kaedah RD-LW. Keseluruhannya, kaedah yang berpusat pada ruang seperti RD-LW, RD-Galerkin, Galerkin lemah FEM dan atur cara kecerunan sisa menawarkan ketepatan antara 1.4212 dengan 2.43871. Di sebaliknya, kaedah RD-LDA yang berpandu kepada hilir hanya mencapai ketepatan antara 0.7825 dengan 0.9335.