First Semester Examination
Academic Session 2019/2020

December 2019/January 2020

EAS663 – Dynamics and Stability of Structures

Duration : 2 hours

Please check that this examination paper consists of **NINE (9)** pages of printed material including appendix before you begin the examination.

Instructions : This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions.

All questions must be answered in English.

Each question **MUST BE** answered on a new page.

(1). Beam-columns are structural members which combine the beam function of transmitting transverse forces or moments with the compression (or tension) member function of transmitting axial forces. Beam-columns may act as if isolated, as in the case of eccentrically loaded compression members with simple end connections, or they may form part of a rigid frame. Discuss the in-plane behaviour of isolated beam-columns by using graph and suitable sketches.

[25 marks]

(2). (a). Prove that a single storey building can be modelled using a mass-spring-damper system in mechanical vibration.

[4 marks]

(b). Sketch the frequency-response curve for a single degree of freedom system subjected to harmonic excitation. Explain the effect of frequency ratio on the deformation response of the system.

[6 marks]

(c). A single degree of freedom system is excited by a simple harmonic force as shown in **Figure 1**. Assume the girder is rigid whereas the columns are flexible to the lateral deformation but rigid in vertical direction. Using E = 30 GPa and I = 15(10^6) mm^4, 5 % damping and neglecting the mass of the columns, determine

(i) The natural period of building
(ii) The steady state amplitude of vibration, and
(iii) The maximum shear force and bending moment in the column

Refer **Appendix A** for the general solution for equations of motion.

[15 marks]

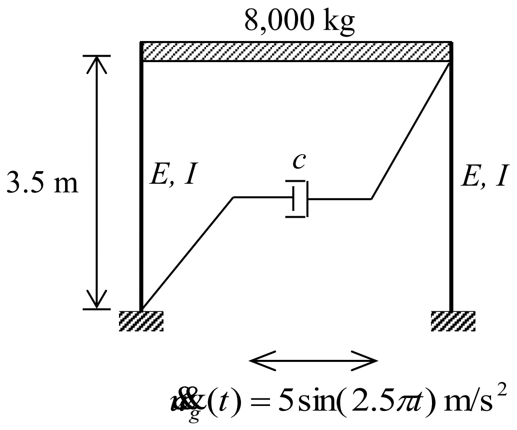

Figure 1

(3). A double degree of freedom system is excited by an external dynamic load shown in **Figure 2**. Formulate the equations of motion. Determine the natural frequencies and vibration mode shapes from the eigenvalue problem. Solve the equations of motion and determine the steady state response of the system.

[25 marks]

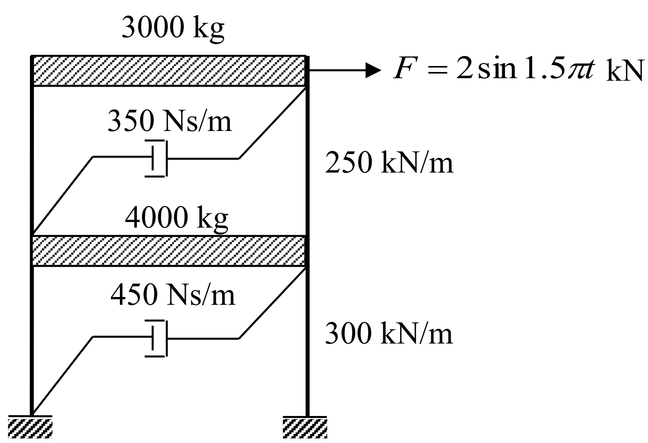

Figure 2

(4). (a). Using method of neutral equilibrium, show that the critical load of the column shown in **Figure 3** is $\pi^2 EI/L^2$. The lower end of the column is fixed and the upper end is prevented from rotating but free to translate laterally. Given flexural rigidity of the column is *EI*.

[10 marks]

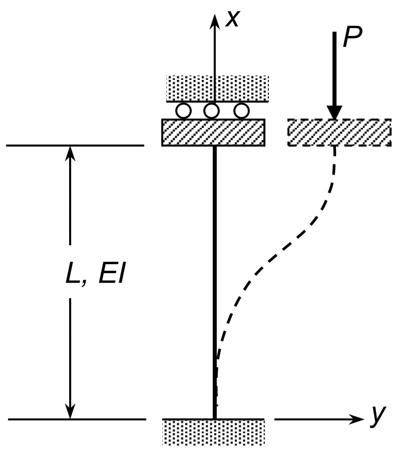

Figure 3

(b). Explain the meaning of a perfect column. The second order differential equation for the imperfect column shown in **Figure 4** is given as follows:

$$y'' + k^2 y = -k^2 a \sin\frac{\pi x}{L}$$

where $k^2 = P/EI$ and *a* is the amplitude of the initial deformation at mid-height of the column.

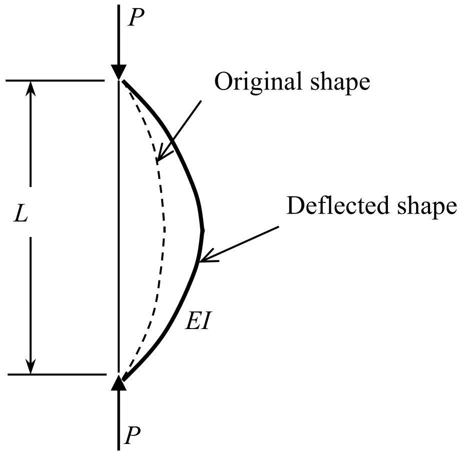

Figure 4

Using the above equation, derive the following relation:

$$\delta = \frac{a}{1 - P/P_E}$$

where δ : the mid-height deflection and P_E : Euler buckling load (=$\pi^2 EI/L^2$).

Using the above derived equation and a plot of P/P_E vs δ, explain the behavior of an imperfect column.

[15 marks]

(5). (a). Obtain the critical load of the cantilever column in **Figure 5** by using Rayleigh-Ritz method. Assume the following two cases of deflections:

Case I : $y = Ax^2$

Case II : $y = A\left[1 - \cos\frac{\pi x}{2L}\right]$

where A : amplitude of lateral displacement of the column at the free-end. Provide possible reason affecting the accuracy of the solutions for Case I and Case II in comparison with the exact solution of $P_{cr,\ exact}$ = 2.467 EI/L^2.

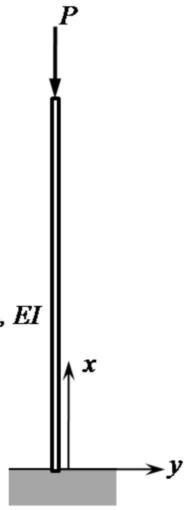

Figure 5

[20 marks]

(b). Mid-span deflection of the simply supported beam subjected to a lateral load *W* at mid-span and axial load *P* as shown in **Figure 6** is given by the following equation:

$$\delta = \Delta \frac{1}{1-(P/P_{cr})}$$

where Δ : the mid-span deflection in the absence of axial load and P_{cr} : critical axial load of the beam.

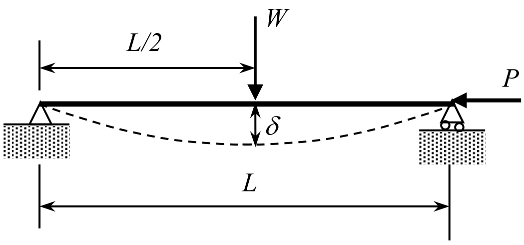

Figure 6

Using the above equation and suitable sketch, explain the effect *P* on the mid-span deflection of the beam.

[5 marks]

(6). (a). Using slope-deflection method, determine the effective length of column AB for the frame shown in **Figure 7**. Refer **Appendix B** for the basic equations used in slope deflection method.

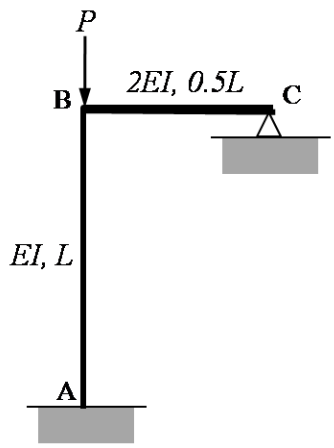

Figure 7

[20 marks]

(b). For the braced and unbraced frames shown in **Figure 8**, sketch the buckling mode corresponding to the lowest buckling load.

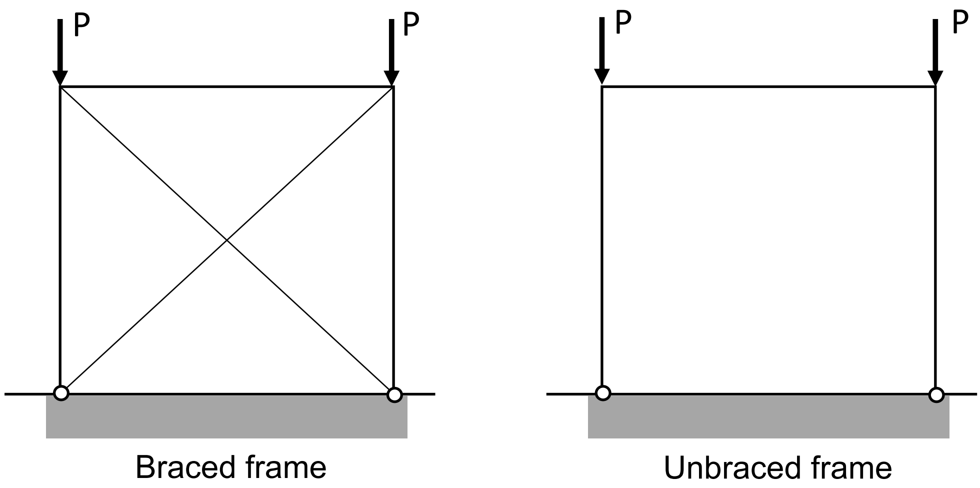

Figure 8

[5 marks]

APPENDIX A

General Solution for Equations of Motion

A) Response under free vibration:

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}\left[A\cos\omega_D t + B\sin\omega_D t\right]$$

B) Response to harmonic vibration, $F(t) = F_0 \sin \omega t$

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}\frac{1}{1-(\omega/\omega_n)^2}\sin\omega t$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}(A\cos\omega_D t + B\sin\omega_D t) + C\sin\omega t + D\cos\omega t$$

$$C = \frac{F_0}{k}\frac{1-(\omega/\omega_n)^2}{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}$$

$$D = \frac{F_0}{k}\frac{-2\xi(\omega/\omega_n)}{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}$$

(C) Response to a block pulse

i) Undamped SDOF system

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}$$

ii) Damped SDOF system

$$u(t) = e^{-\xi\omega_n t}(A\cos\omega_D t + B\sin\omega_D t) + F_0/k$$

APPENDIX B

Slope deflection equations for a beam-column are given as follows:

$$M_A = \frac{EI}{L}\left(\alpha_n\theta_A + \alpha_f\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(\alpha_f\theta_A + \alpha_n\theta_B\right)$$

where α_n and α_f are given as follows:

$$\alpha_n = \frac{\phi_n}{\phi_n^2 - \phi_f^2};\ \alpha_f = \frac{\phi_f}{\phi_n^2 - \phi_f^2}$$

and

$$\phi_n = \frac{1}{(kL)^2}\left(1 - kL\cot kL\right);\quad \phi_f = \frac{1}{(kL)^2}\left(kL\csc kL - 1\right)$$

M_A, M_B, θ_A and θ_B are as shown in Fig.A1.

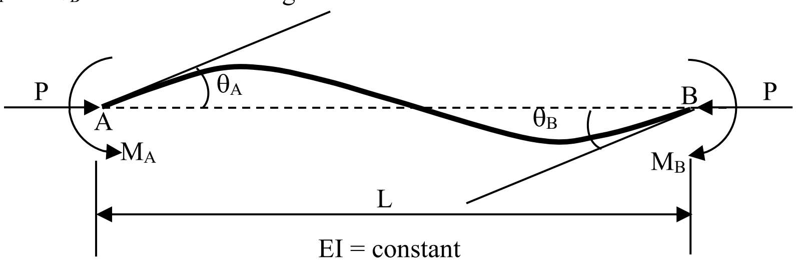

Fig.A1

Stiffness matrix for a beam column member

$$[k] = EI\begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} - P\begin{bmatrix} \frac{6}{5L} & -\frac{1}{10} & -\frac{6}{5L} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2L}{15} & \frac{1}{10} & -\frac{1}{30} \\ -\frac{6}{5L} & \frac{1}{10} & \frac{6}{5L} & \frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{30} & \frac{1}{10} & \frac{2L}{15} \end{bmatrix}$$

where EI: flexural rigidity of member; L:length of member; P:axial force of member

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