First Semester Examination
Academic Session 2019/2020

December 2019/January 2020

EAS661 – Advanced Structural Mechanics

Duration : 2 hours

Please check that this examination paper consists of **NINE (9)** pages of printed material before you begin the examination.

Instructions : This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions.

All questions must be answered in English.

Each question **MUST BE** answered on a new page.

(1). (a). **Figure 1** shows an infinitesimal volume taken from an interior of an elastic body under the action of external load. Show all the stress components (in Cartesian coordinate system) acting on the infinitesimal volume.

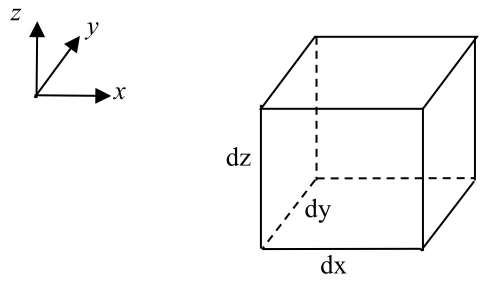

Figure 1

Explain why there are only **SIX (6)** independent stress components.

Derive the displacement function *u* for a prismatic bar with cross-sectional area *A* subjected to a uniformly distributed load *w* as shown in **Figure 2**. It is given that the material of the bar is linearly elastic with modulus of elasticity *E.*

[15 marks]

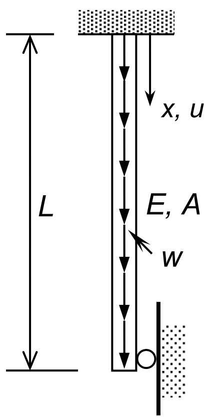

Figure 2

(b). **Figure 3** shows a thin wall structure loaded with a uniformly distributed load *w* in y-direction. Justify why this problem can be solved as a plane stress problem.

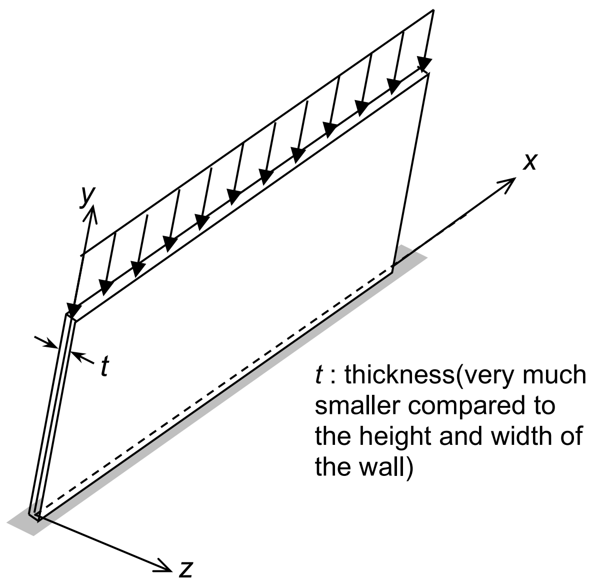

Figure 3

Using a diagram of an infinitesimal volume taken from the interior of the wall in **Figure 3**, show clearly the non-zero stress components.

[10 marks]

(2). (a). The strain energy U_p stored in an elastic body can be obtained using the following equation:

$$U_P = \int_{Vol} v_p dvol$$

where v_p is strain energy density and vol is volume of the elastic body. Using the above equation, derive the general expression for strain energy stored in a linearly elastic bar of length L with arbitrary variation of cross-section $A(x)$ where x represents the longitudinal axis of the bar.

[10 marks]

(b). **Figure 4** shows a cantilever column with height *H* subjected to a uniformly distributed load *w*. One linear spring with spring constant *k* is located at the mid height of the column. The following expression for lateral displacement field *v* has been suggested:

$$v = A(1 - \cos(\pi x/2H))$$

where *A* is a constant. Show that the above displacement field is kinematically admissible. Next, solve for the constant *A* by applying the principle of minimum potential energy (PMPE). Flexural rigidity of the column is *EI*.

[15 marks]

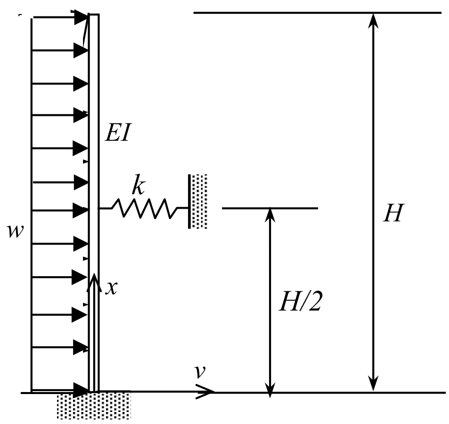

Figure 4

(3). **Figure 5** shows a stepped bar which is fixed at one end. Length of portion AB and BC of the bar is L and $0.5L$, respectively. Elastic modulus of the bar is E and cross-sectional area of portion AB and BC is $2a$ and a, respectively. The bar is subjected to a uniformly distributed load $2w$ and w per unit length for portion AB and BC, respectively. Apart from that, a concentrated load $2wL$ acts at the lower end C of the bar. Using piece-wise Rayleigh-Ritz method, derive the expression for axial displacement u. Divide the bar into two portions and assume linear displacement field for each portion. Express u in terms of point displacements at the ends of each portion. Show clearly all the steps involved in the derivation.

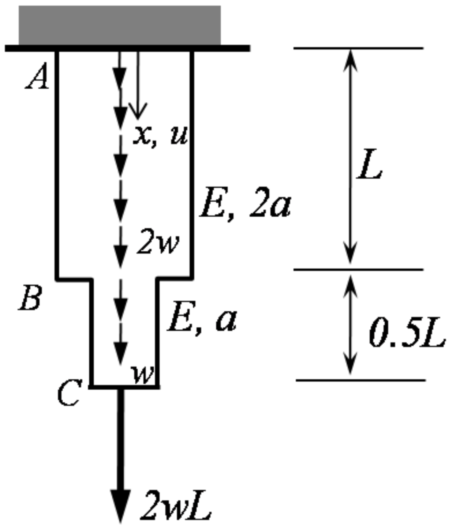

Figure 5

[25 marks]

(4). (a). Derive the element stiffness matrices and global matrix in terms of *E*, *A* and *L* for the three bars assembly which is loaded with force *P*, and constrained at the two ends as shown in **Figure 6**.

[5 marks]

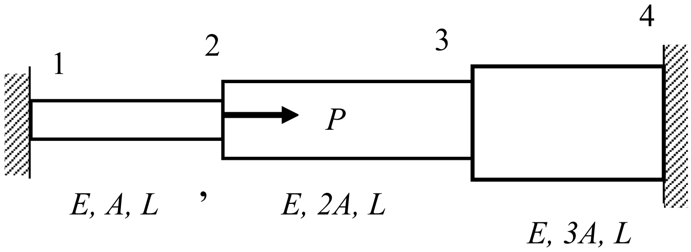

Figure 6

(b). Derive the element stiffness matrices and global matrix for the two bars assembly which is loaded with force 10*P* at node 2 as shown in **Figure 7**. End bars are constrained at end B and free at end A with a gap of Δ at end A. Given the value of P = 60 kN, E = 20 kN/mm^2, L = 200 mm, A= 250 mm^2 and Δ = 1.2 mm, determine:

(i). the displacements at nodes 1, 2 and 3
(ii). the support reaction force at A

[20 marks]

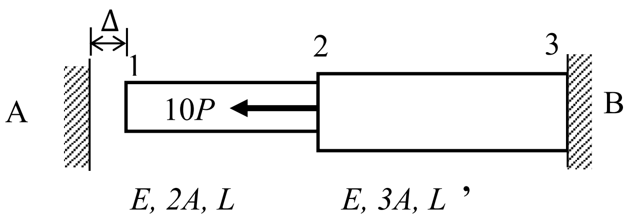

Figure 7

(5). (a). Two plates shown in **Figure 8** and **Figure 9**, shall be analysed as a plane strain problem. Both plates are divided into 9 elements. Each node has been labelled accordingly. Calculate the bandwidth, B = (R+1)×NDOF for the plate assuming two degrees of freedom at each node.

[5 marks]

(b). Rearrange the node labeling in such a way that a minimum value of R is obtained.

[5 marks]

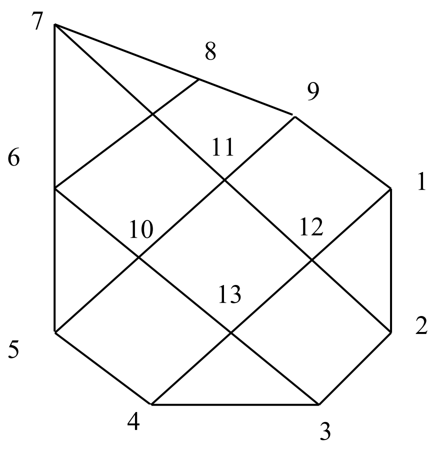

Figure 8

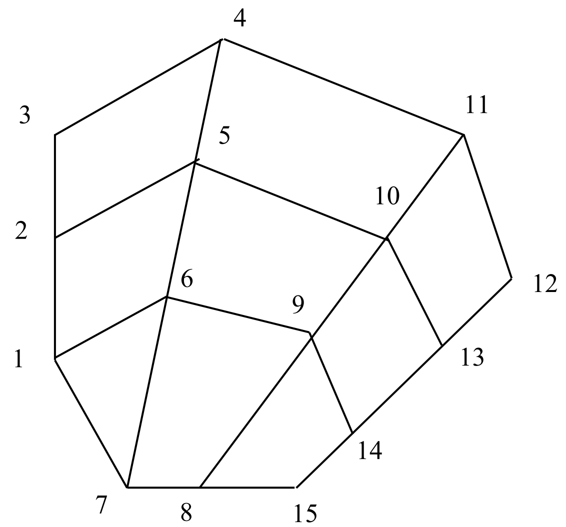

Figure 9

(c). **Figure 10** shows a frame structure labeled as nodes 1, 2 and 3 which are subjected to a nodal force of P = 20 kN at node 2 and uniformly distributed load of 6 kN/m. The frame is fixed at node 1 and pinned at node 3. Given the value of E = 207 GPa, I = 3×10^{-5} m^4 and A = 0.005 m^2.

(i). Derive the global stiffness matrix for the frame

(ii). Determine the deflection u_2, v_2, θ_2 and u_3, v_3, θ_3 in unit metre and rad, respectively

[15 marks]

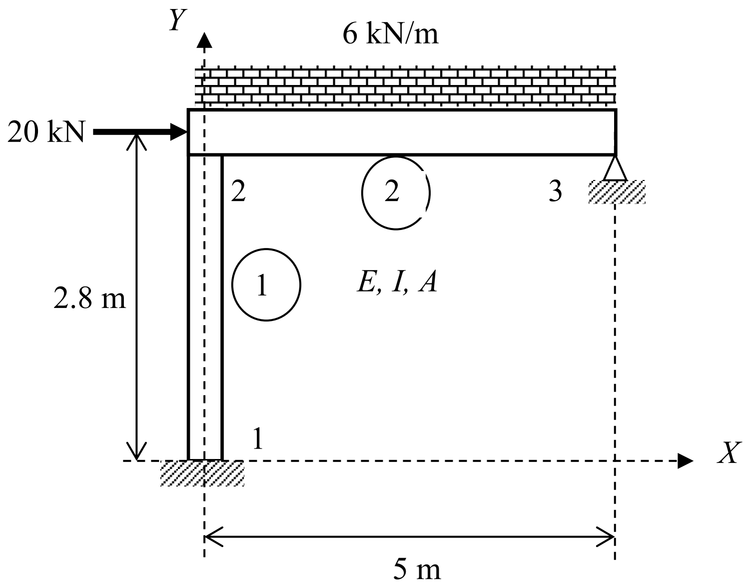

Figure 10

Given the stiffness of the beam and spring elements in dimensional space:

$$k = \frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \text{ for beam element}$$

$$k = \begin{array}{c} \begin{array}{cc} u_i & u_j \end{array} \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array} \text{ for spring element}$$

(6). (a). Explain the importance of model validity and accuracy of the following factors in the modeling procedures for Finite Element Method.

(i). Geometry

(ii). Material properties

(iii). Loading conditions

[6 marks]

(b). Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in **Figure 11**. Hence calculate the displacements at node 3 and horizontal displacement at node 2. Given E = 10^3 kN/m^2, ν = 0.3 and t = 0.05 m.

[19 marks]

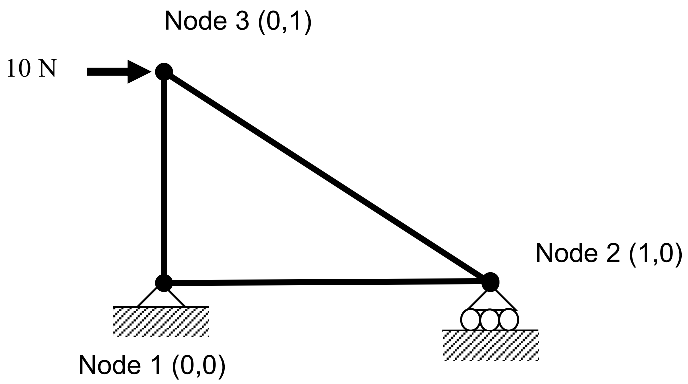

Figure 11

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