

**CONSTRAINT ESTIMATION SCHEMES FOR
SEMI-ELLIPTICAL SURFACE CRACKS UNDER
TENSILE AND BENDING LOADS**

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by

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LIST OF ABBREVIATIONS

ASME	American Society of Mechanical Engineers
BLF	Boundary Layer Formulation
CCP	Center cracked tension panel
CT	Compact Tension
HRR	Hutchinson, Rice & Rosengren
EDI	Equivalent Domain Integral
LEFM	Linear Elastic Fracture Mechanics
MBLF	Modified Boundary Layer Formulation
SCP	Surface Cracked Plate
SEN	Single Edge Notched
SENB	Single Edge Notched Bend Bar
SSY	Small Scale Yielding

LIST OF SYMBOLS

A	Area
a	Crack depth
a_{eff}	Effective crack length, considering the plastic zone
A_2	Constraint parameter
A_c	Area of cylinder V for the EDI method
A_ε	Area of cylinder V_ε for the EDI method
A_n	Non-dimensional constants for T-stress equations
B_n	Non-dimensional constants for T-stress equations
B	SCP specimen width
c	Crack width
c_{cr}	Critical dimension of the cracked body
D_θ	Length between pairs of outermost nodes
d_1	Non-dimensional constant
E	Young's Modulus
E_{cf}	Number of layers of elements along the crack front.
E_r	Number of radial elements in the Crack Tip Region
E_T	Number of elements in the Transition Region
E_θ	Number of elements in the angular direction
e_θ	Coordinate system direction
e_r	Coordinate system direction
e_{cf}	Integer varying between 0 and N_{layer}
$F\left(\frac{a}{t}, \frac{a}{c}, \frac{c}{W}, \phi\right)$	Geometric function for K in semi-elliptical surface cracks

$f\left(\frac{a}{w}\right)$	Geometric function for K in straight cracks
$f_{ij}(\theta)$	Angular stress function
f_w	Dimensionless function for K in semi-elliptical surface cracks
f_ϕ	Dimensionless function for K in semi-elliptical surface cracks
G	Shear Modulus
G_1	Dimensionless function for K in semi-elliptical surface cracks
G_2	Dimensionless function for K in semi-elliptical surface cracks
g	Dimensionless function for K in semi-elliptical surface cracks
H	SCP specimen length
H_s	Dimensionless function for K in semi-elliptical surface cracks
H_1	Dimensionless function for K in semi-elliptical surface cracks
H_2	Dimensionless function for K in semi-elliptical surface cracks
h_1	Fully plastic factor
$I(s)$	Interaction integral
I_n	Integration constant for HRR fields
J	J-integral
J_D	Domain Integral
J_e	Elastic component of the J-integral
J_p	Plastic component of the J-integral
J_{local}	Local J-integral along the crack front
$J_{\phi=90}$	Local J-integral at $\phi = 90$ in semi-elliptical surface cracks
J_{max}	Maximum J-integral along the semi-elliptical crack front
J_f	J-integral determined via the Equivalent Domain Integral
k	Yield stress in shear

K	Stress Intensity Factor
K_{far}	Far field Stress Intensity Factor
K_I	Stress intensity factor under mode I loading
K_{eff}	Effective K , considering the plastic zone
L_r	Proximity to plastic collapse
M	Global bending moment
M_L	Limit load in Nm
M_o	Limit bending moment
M_1	Dimensionless function for K in semi-elliptical surface cracks
M_2	Dimensionless function for K in semi-elliptical surface cracks
M_3	Dimensionless function for K in semi-elliptical surface cracks
N_L	Limit load in N
$N(s)$	Membrane Force
N_{layer}	Number of layers of nodes along the semi-elliptical crack front
N_{edge}	Number of Edge nodes along the angular direction
$N_{mid-side}$	Number of Mid-side nodes along the angular direction
n_{corner}	Node label for Corner nodes
n_{Middle}	Node label for Middle nodes
$n_{Mid-side}$	Node label for Mid-side nodes
$n_{\theta M}$	Integer varying between 0 and $N_{mid-side}$
$n_{\theta E}$	Integer varying between 0 and N_{edge}
n	Strain hardening exponent
P	Applied load
P_o	Limit load

p	Dimensionless function for K in semi-elliptical surface cracks
Q	Constraint parameter
Q_s	Dimensionless constant
Q_T	Modified Q term used in the J - Q_T - T_z approach
r	Radial distance ahead of the crack tip
r_i	i^{th} layer of nodes from the crack tip
$r_{initial}$	Size of the element at the crack tip
r_p	Plastic zone size
r_i^{TR}	i^{th} layer of nodes in the Transition Region
r_{final}	Radius of Crack Tip Region
RF	Reaction Force
S	Span between load and boundary condition
$S1$	Term in HRR field
S_b	Far field bending stress
S_t	Far field tensile stress
S_{EDI}	Arbitrary function for EDI method
t	Thickness of SCP specimen
t_N	Calibration factor for T-stress in SEN specimens
t_M	Calibration factor for T-stress in SEN specimens
T	T-stress
T_z	Triaxiality parameter
$u_i (i = 1,2,3)$	Displacement components in (x_1, x_2, x_3)
V	Cylinder for EDI method
V_ε	Cylinder for EDI method

ν	Poisson's ratio
X_i	Constants used for T-stress calculations, ($i = 1,2,3 \dots$)
W	SCP specimen width
w	Straight crack width
z	Distance along the semi-elliptical crack front
α	Material constant
α_s	Slip line
β	Biaxiality ratio
β^{thin}	Biaxiality for thin plates
β^{2D}	Two-dimensional biaxiality
β_s	Slip line
β_{fs}	Corner singularity constant
σ	Stress
σ_{app}	Applied stress
σ_o	Yield stress
$\sigma_{ij}(i, j = 1,2,3)$	Stress components in (x_1, x_2, x_3) directions
$\sigma_{ij}(i, j = r, \theta, z)$	Stress components in (r, θ, z)
$\sigma_e, \bar{\sigma}$	Von Mises stress
σ_m	Mean stress
ε	Strain
$\varepsilon_{ij}(i, j = 1,2,3)$	Strain components in (x_1, x_2, x_3) directions
ε_o	Yield strain
$\bar{\varepsilon}^p$	Equivalent plastic strain
ε_p	Plastic strain

μ	Level of plastic deformation
θ	Angular notation about the crack tip
θ_o	Angular interval between nodes
ϕ	Angular position along the semi-elliptical crack front
ϕ_{Jmax}	Position along semi-elliptical crack front with the highest J
Π	Potential energy
Γ	Arbitrary contour about the crack tip
ρ	Triaxiality parameter
$\gamma_{ij}(i, j = 1,2,3)$	Shear strains in (x_1, x_2, x_3) directions
γ_r	Non-dimensional number for J - $\Delta\sigma$ approach
γ_{rt}	Non-dimensional number for semi-elliptical J - $\Delta\sigma$ approach
γ_{rb}	Non-dimensional number for semi-elliptical J - $\Delta\sigma$ approach