

**HYDRODYNAMIC EFFECT ON LOW FIELD
GRADIENT MAGNETOPHORESIS OF
SUPERPARAMAGNETIC NANOPARTICLES**

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**HYDRODYNAMIC EFFECT ON LOW FIELD GRADIENT
MAGNETOPHORESIS OF SUPERPARAMAGNETIC NANOPARTICLES**

by

LEONG SIM SIONG

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- Figure A3 (a) The illustration for infinitesimal surface element on MNP collection plane for $0 < r \leq R$ (see Figure 4.25 for the top view of MNP collection plane). (b) The illustration for infinitesimal surface element for $R < r \leq \sqrt{2}R$. (c) The zoom in image for a quadrant in (b). This figure serves as an illustration to the derivation of Equation (A28).
- Figure A4 (a) Side view of the cuvette bottom wall. (b) Top view of the cuvette bottom wall. (c) Illustration of MNP collection plane (red bolded curve) in order to calculate average displacement of the given plane from magnet pole.
- Figure A5 The plot of fractional error against γ . The dotted line is the polynomial fit (degree of 3) into the calculated data, which gives R^2 value of 0.9999.

LIST OF ABBREVIATIONS

CFD	Computational fluid dynamic
DLS	Dynamic light scattering
FEM	Finite element method
HGMS	High gradient magnetic separation
HeLa	Human ovarian cancer
HIV	Human immunodeficiency virus
LGMS	Low gradient magnetic separation
MB	Methylene Blue
MNP	Magnetic nanoparticle
MP	Magnetic particle
MS	Magnetic separation
PDE	Partial differential equation
PEG	Polyethylene glycol
TEM	Transmission electron microscopy
UV-vis	Ultraviolet-visible light
VSM	Vibrating sample magnetometer