UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2002/2003

Februari/Mac 2003

JIM 312/4 – Teori Kebarangkalian

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH SATU** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan yang disediakan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah dan markah subsoalan diperlihatkan di penghujung subsoalan itu.

1. (a) Ruang sampel yang terhasil daripada eksperimen melemparkan 4 syiling adil dipamerkan seperti berikut.

KKKK	KBKK	BKKK	BBKK
KKKB	KBKB	BKKB	BBKB
KKBK	KBBK	BKBK	BBBK
KKBB	KBBB	BKBB	BBBB

Anggapkan setiap ahli ruang sampel ini mempunyai kebarangkalian kemunculan yang sama. Andaikan A_i menandai peristiwa tepat i kepala yang muncul dan B_i pula menandai sekurang-kurangnya i kepala yang muncul, $i = 0, 1, 2, 3, 4$. Senaraikan titik sampel dan hitungkan kebarangkalian di dalam peristiwa-peristiwa berikut:

(i) A_0.
(ii) A_1.
(iii) B_3.
(iv) B_4.
(v) A_4.

(50 markah)

(b) Di dalam suatu permainan loteri, seseorang itu boleh mencapai kemenangan jika dia memilih 6 nombor yang berlainan daripada $\{1, 2, ..., 36\}$ dan nombor-nombor tersebut bersepadan dengan nombor-nombor yang dipilih oleh penganjur loteri. Apakah kebarangkalian kemenangan?

(20 markah)

(c) Suatu syarikat insurans mengkelaskan pemandu-pemandu di dalam 3 kelas: kelas A (risiko yang rendah), kelas B (risiko yang sederhana) dan kelas C (risiko yang tinggi). Peratus pemandu di dalam setiap kelas, masing-masing, adalah 20%, 65% dan 15%. Kebarangkalian seorang pemandu di dalam setiap kelas mengalami kemalangan jalanraya semasa memandu di dalam tempoh setahun masing-masing adalah 0.01, 0.02 dan 0.03. Seorang pemandu mengalami kemalangan jalanraya semasa memandu selepas membeli polisi insurans daripada syarikat ini. Cari kebarangkalian yang pemandu tersebut berisiko kelas:

(i) A.
(ii) B.
(iii) C.

(30 markah)

2. (a) Suatu pembolehubah rawak X tertabur secara N(60, 25). Hitungkan

   (i) $P(X < 50)$.
   (ii) Nilai c supaya $P(|X - 60| < c) = 0.95$.
   (iii) Nilai c supaya $P(X < c) = 0.01$.

(50 markah)

(b) Pembolehubah rawak X mempunyai fungsi jisim kebarangkalian $p(x) = \frac{2x}{n(n+1)}$, $x = 1, 2, \ldots, n$. Hitungkan $E(X)$.

(20 markah)

(c) Andaikan X adalah pembolehubah rawak seragam $[0, 2\pi]$. Hitungkan

   (i) jangkaan dan varians $g(X) = \cos X$.
   (ii) jangkaan $h(X) = |\cos X|$.

(30 markah)

3. (a) (X, Y) mempunyai fungsi jisim kebarangkalian tercantum $p(x, y)$ yang diberikan oleh jadual berikut:

	Y			
X	2	3	4	5
0	1/24	3/24	1/24	1/24
1	1/12	1/12	3/12	1/12
2	1/12	1/24	1/12	1/24

   (i) Hitungkan $P(X \leq 1, Y \leq 3)$.
   (ii) Dapatkan fungsi-fungsi jisim kebarangkalian sut daripada taburan tercantum ini.
   (iii) Buktikan atau sangkalkan pernyataan X dan Y tak bersandar.

(50 markah)

(b) Diberikan X dan Y tak bersandar. $E(X) = 2$, $\text{Var}(X) = 9$, $E(Y) = -3$ dan $\text{Var}(Y) = 16$. Andaikan $W = 3X - 2Y$. Cari $E(W)$ dan $\text{Var}(W)$.

(20 markah)

(c) X dan Y mempunyai taburan normal bivariat berparameterkan $\mu_X = 2$, $\mu_Y = 1$, $\sigma_X^2 = 9$, $\sigma_Y^2 = 9$ dan $\rho = 3/4$. Hitungkan

(i) $P(Y < 1)$.
(ii) $P(Y < 1 \mid X = 0)$.
(iii) $E(Y \mid X = 0)$.

(30 markah)

4. (a) X_1, X_2 dan X_3 adalah sampel rawak daripada populasi bertaburan N(50, 20). Andaikan $W = X_1 - 2X_2 + 2X_3$. Hitungkan

(i) Var(W).
(ii) $P(|W - 50| \leq 25)$.
(iii) kuantil ke-90 taburan W.

(50 markah)

(b) Buktikan pernyataan ini. Jika $Z_1, Z_2, \ldots, Z_n$ adalah pembolehubah-pembolehubah tak bersandar yang tertabur secara secaman N(0,1), maka $Y = \sum_{i=1}^{n} Z_i^2$ tertabur secara χ_{n-1}^2.

(20 markah)

(c) Andaikan suatu sampel rawak bersaiz 16 diambil daripada suatu taburan normal, $\sigma^2 = 5$. Hitungkan kebarangkalian sisihan piawai sampel berada di antara 1.5 dan 2.9.

(30 markah)

5. (a) Cari fungsi taburan longgokan di kalangan fungsi-fungsi berikut. Dapatkan fungsi ketumpatan yang sepadan jika boleh.

(i) $F(x) = \begin{cases} 0, & x \leq 0 \\ \dfrac{x}{1+x}, & x > 0. \end{cases}$

(ii) $F(x) = \begin{cases} 0, & x \leq 2 \\ 1 - \dfrac{4}{x^2}, & x > 2. \end{cases}$

(iii) $F(x) = \begin{cases} 0, & x \leq -\pi/2 \\ \sin x, & -\pi/2 < x < \pi/2 \\ 1, & x \geq \pi/2. \end{cases}$

(25 markah)

(b) Nyatakan sama ada pernyataan-pernyataan berikut benar atau palsu. Jika palsu berikan contoh lawan.

(i) $E(X^2) = (E(X))^2$.
(ii) $E(1/X) = 1/E(X)$.
(iii) $E(X) = 0 \Rightarrow X = 0$.

(25 markah)

(c) (i) Tunjukkan $\mathrm{Cov}(aX, cY) = ac\mathrm{Cov}(X, Y)$.
(ii) Andaikan X, Y dan W sebagai pembolehubah-pembolehubah rawak. Tunjukkan $\mathrm{Cov}(X + Y, W) = \mathrm{Cov}(X, W) + \mathrm{Cov}(Y, W)$.

(25 markah)

(d) Buktikan pernyataan berikut. Jika t_n mempunyai taburan t dengan darjah kebebasan n, maka t_n^2 tertabur secara $F_{1,n}$.

(25 markah)

Rumus-Rumus

Modul 1

Pelajaran 1

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

3. $P(\bar{A}) = 1 - P(A)$

4. ${}^{n}p_{r} = \dfrac{n!}{(n-r)!}$

5. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

6. $N = \dfrac{n!}{n_1!\, n_2! \ldots n_k!}$

Pelajaran 2

1. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

2. $P(A \cap B) = P(A)P(B)$

3. $P(A) = P(A \mid B)\, P(B) + P(A \mid \bar{B})\, P(\bar{B})$

4. $P(B_i \mid A) = \dfrac{P(A \cap B_i)}{\sum_{j=1}^{n} P(A \mid B_j)\, P(B_j)}$

Pelajaran 3

1. $P(a \le X \le b) = \int_{o}^{b} f(x)\, dx$

2. $P(a < X < b) = \sum_{a<x<b} p(x)$

3. $F(t) = P(X \le t)$

4. $P(a < X \le b) = F(b) - F(a)$

5. $\frac{d}{dt} F(t) = f(t)$

6. $F_Y(t) = F_X (g^{-1}(t))$

7. $F_Y(t) = 1 - F_X(g^{-1}(t))$

8. $f_Y(t) = f_X(g^{-1}(t)) \mid J \mid$

9. $J = \frac{dg^{-1}(t)}{dt}$

10. $f_Y(t) = \sum_{i=1}^{k} f_X \; (g_i^{-1}(t)) \mid J_i \mid)$

11. $J_i = \frac{d}{dt} g_i^{-1} (t)$

12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2

Pelajaran 1

1. $E(X) = \sum_{x \in \text{Julat } X} xp(x)$

2. $1 + x + x^2 + ... + x^n + ... = \frac{1}{1-x}, \mid x \mid < 1$

3. $1 + 2x + ... + n x^{n-1} + ... = \frac{1}{(1-x)^2}, \mid x \mid < 1$

4. $E(X) = \int_{-\infty}^{\infty} x f(x) \; dx$

5. $E(X) = \int_{0}^{\infty} [1 - f(x)] \; dx - \int_{-\infty}^{0} F(x) \; dx$

6. $E[G(X)] = \sum_{x \in \text{Julat } X} G(x) \; p(x)$

7. $E[G(X)] = \int_{-\infty}^{\infty} G(x)\, f(x)\, dx$

8. $E[c] = c$

9. $E[cX] = c\, E[X]$

10. $E[X + c] = E[X] + c$

11. $\text{Var}(X) = E[X - E[X]]^2$

12. $\text{Var}(X) = E[X^2] - \mu_X^2$

13. $\text{Var}(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$

14. $\text{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu_X^2$

15. $\text{Var}(a) = 0$

16. $\text{Var}(aX + b) = a^2\, \text{Var}(X)$

17. $F_X(t_k) = k, \; 0 < k < 1$

Pelajaran 2

1. $m_k = E[X^k]$

2. $m_k = \sum_{x \in \text{Julat } X} x^k\, p(x)$

3. $m_k = \int_{-\infty}^{\infty} x^k f(x)\, dx$

4. $\mu_k = E[(X - \mu_X)^k$

5. $\gamma_1 = \mu_3 / \sigma_X^3$

6. $\gamma_2 = \dfrac{\mu_4}{\sigma_X^4} - 3.$

7. $\mu_{[k]} = E[X(X-1)(X-2) \ldots (X-k+1)]$

8. $m(t) = E[e^{tX}]$

9. $m(t) = \sum_{x \in \text{Julat } X} e^{tx} p(x)$

10. $m(t) = \int_{-\infty}^{\infty} e^{tx} f(x) \, dx$

11. $m_Y(t) = E[e^{tg(X)}]$

12. $m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)} p(x)$

13. $m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)} f(x) \, dx$

14. $m_Y(t) = e^{bt} m_X(at)$

15. $m^{(i)}(0) = m_i$

16. $k(t) = \ell n \, m(t)$

17. $\psi(t) = E[t^X]$

18. $f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$

19. $\psi^{(i)}(0) = i! \, p(i)$

20. $P(|X| \geq a) < \frac{1}{a^2} E[X^2]$

21. $P(|X - \mu| \geq a\sigma) \leq \frac{1}{a^2}$

22. $P(|X - \mu| < a\sigma) \geq 1 - \frac{1}{a^2}$

23. $P(X \geq a) \leq \frac{E[X]}{a}$

24. $E[X^n] = \int_0^{\infty} nx^{n-1} (1 - F(x)) \, dx$

Pelajaran 3

1. (i) $p(x) = \begin{cases} q, & x = 0 \\ p, & x=1 \\ 0, & \text{ditempat lain} \end{cases}$ $X \sim$ Bernoulli (p)

   (ii) $E[X] = p$

   (iii) $Var(X) = pq$

   (iv) $m(t) = q + pe^t$

2. (i) $p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x=0, 1, 2,..., n \\ 0 & , \text{di tempat lain} \end{cases}$ $X \sim$ Binomial (n, p)

   (ii) $E[X] = np$

   (iii) $Var(X) = npq$

   (iv) $m(t) = (q + pe^t)^n$

3. (i) $p(x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, & x=0, 1, 2,..., n \\ 0 & , \text{di tempat lain} \end{cases}$ $X \sim$ hipergeometri (N, k, n)

   (ii) $E[X] = \dfrac{nK}{N}$

   (iii) $Var(X) = \dfrac{nK(N-K)(N-n)}{N^2(N-1)}$

4. $(a + b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

5. (i) $p(x) = \begin{cases} q^{x-1}p \text{ , } x = 1, 2, 3, \ldots \\ 0 \quad , \text{ di tempat lain} \end{cases}$ X ~ geometri (p)

(ii) E[X] = 1/p

(iii) Var (X) = q/p^2

(iv) $m(t) = \dfrac{pe^t}{1 - qe^t}$

6. (i) $p(x) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, \; x=r, r+1, r+2 \\ \qquad\qquad r=2, 3, 4, \ldots \\ 0 \quad , \text{ di tempat lain} \end{cases}$ X ~ negatif binomial (r, p)

(ii) E[X] = r/p

(iii) Var (X) = rq/p^2

(iv) $m(t) = \left[\dfrac{pe^t}{1-qe^t}\right]^r$

7. (i) $p(x) = \begin{cases} e^{-\lambda} \dfrac{\lambda^x}{x!} \text{ , } x = 0, 1, 2, \ldots \\ 0 \quad \text{, di tempat lain} \end{cases}$ X ~ Poisson (λ)

(ii) E[X] = λ

(iii) Var (X) = λ

(iv) $m(t) = e^{\lambda(e^t - 1)}$

8. $\underset{x \to 0}{\text{had}} \, (1 + x)^{1/x} = e$

9. $\underset{x \to \infty}{\text{had}} \left(1 + \dfrac{1}{x}\right)^x = e$

10. $\underset{x \to 0}{\text{had}} \, (1 + ax)^{1/x} = e^a$

Pelajaran 4

1. (i) $f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{di tempat lain} \end{cases}$ $\qquad X \sim$ seragam (a, b)

   (ii) $E[X] = \frac{a+b}{2}$

   (iii) $Var(X) = \frac{(b-a)^2}{12}$

   (iv) $m(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$

2. (i) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$ $\qquad X \sim N(\mu, \sigma^2)$

   (ii) $E[X] = \mu$

   (iii) $Var(X) = \sigma^2$

   (iv) $m(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

3. $\lim\limits_{n\to\infty} P\left[a \le \frac{S_n - np}{\sqrt{npq}} \le b\right] \to P(Z \ge a) - P(Z > b)$ (had)

4. $\lim\limits_{\lambda\to\infty} P\left[a \le \frac{X-\lambda}{\sqrt{\lambda}} < b\right] \to P(Z > a) - P(Z \ge b)$ (had)

5. (i) $f(x) = \begin{cases} \lambda e^{-\lambda}, & x \ge 0 \\ 0, & \text{di tempat lain} \end{cases}$ $\qquad X \sim$ eksponen (λ)

   (ii) $E[X] = 1/\lambda$

   (iii) $Var(X) = 1/\lambda^2$

   (iv) $m(t) = \frac{\lambda}{\lambda - t}$

6. $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} \, dx$

7. $\Gamma(n) = (n-1)\,\Gamma(n-1)$

8. $\Gamma(n) = (n-1)!$

9. (i) $f(x) = \begin{cases} \dfrac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0 & , \text{ di tempat lain} \end{cases}$ $\qquad X \sim \text{Gamma}(n, \lambda)$

   (ii) $E[X] = n/\lambda$

   (iii) $\text{Var}(X) = n/\lambda^2$

   (iv) $m(t) = \left(\dfrac{\lambda}{\lambda - t}\right)^n$

10. (i) $f(x) = \begin{cases} \dfrac{1}{2^{\upsilon/2}\Gamma\left(\dfrac{\upsilon}{2}\right)} x^{\upsilon/2-1} e^{-x/2} & , x > 0 \\ 0 \quad , & \text{di tempat lain} \end{cases}$ $\qquad X \sim \chi^2_\upsilon$

   (ii) $E[X] = \upsilon$

   (iii) $\text{Var}(X) = 2\upsilon$

   (iv) $m(t) = \left(\dfrac{1}{1-2t}\right)^{\upsilon/2}$

11. $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} \, dt$

12. $B(x, y) = \int_0^\infty \dfrac{t^{x-1}}{(1+t)^{x+y}} \, dt$

13. $B(x, y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

14. (i) $f(x) = \begin{cases} \dfrac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} & , 0 < x < 1 \\ 0 \ , & \text{di tempat lain} \end{cases}$ $\quad X \sim \text{Beta}(a, b)$

(ii) $F_X(p) = \sum_{x=a}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

(iii) $E[X] = \dfrac{a}{a+b}$

(iv) $\text{Var}(X) = \dfrac{ab}{(a+b+1)(a+b)^2}$

Modul 3

Pelajaran 1

1. $P(X \le x,\ Y \le y) = \sum_{t_1 \le x} \sum_{t_2 \le y} p(t_1,\ t_2)$

2. $P(X \le x,\ Y \le y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(t_1, t_2)\, dt_1\, dt_2$

3. $F(x, y) = P(X \le x,\ Y \le y)$

4. $f(x, y) = \dfrac{\partial^2 F(x, y)}{dxdy}$

Pelajaran 2

1. $p(x) = \sum_{y} p(x, y)$

2. $p(y) = \sum_{x} p(x, y)$

3. $f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$

4. $f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$

5. $F(x) = F(x, \infty)$

6. $F(y) = F(\infty, y)$

7. $f(x) = \dfrac{\partial F(x, \infty)}{\partial x}$

8. $f(y) = \dfrac{\partial F(\infty, y)}{\partial y}$

9. $p(x \mid y) = \dfrac{p(x, y)}{p(y)}$

10. $f(x \mid y) = \dfrac{f(x, y)}{f(y)}$

11. $p(x, y) = p(x)\, p(y)$

12. $f(x, y) = f(x)\, f(y)$

Pelajaran 3

1. $E[g(X, Y)] = \sum_{x} \sum_{y} g(x, y)\, p(x, y)$

2. $E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y) dx\, dy$

3. $E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$

4. $E[h_1(X)\, h_2(Y)] = E[h_1(X)]\, E[h_2(Y)]$

5. (i) $\text{Cov}(X, Y) = E[X - \mu_X)(Y - \mu_Y)]$

   (ii) $\text{Cov}(X, Y) = E[XY] - \mu_X\mu_Y$

6. $\text{Cov}(aX, bY) = ab\, \text{Cov}(X, Y)$

7. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\, \text{Cov}(X, Y)$

8. $\text{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \text{Var}(X_i) + 2\sum_{i<j}\sum \text{Cov}(X, Y)$

9. $\rho(X, Y) = \dfrac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

10. $E[g(X, Y) \mid Y = y] = \sum_{x} g(x, y)\, p(x \mid y)$

11. $E[g(X, Y) \mid Y = y] = \int_{-\infty}^{\infty} g(x,y) f(x|y)\, dx$

12. $E[E[X \mid Y = y]] = E[X]$

13. $E[E[Y \mid X = x]] = E[Y]$

14. $E[E[g(X) \mid Y = y]] = E[g(X)]$

15. $E[E[g(Y) \mid X = x]] = E[g(Y)]$

16. $\text{Var}(X \mid Y = y) = E[X^2 \mid Y = y] - (E[X \mid Y = y)^2$

17. $m(t_1, t_2) = E\left[e^{t_1X_1 + t_2X_2}\right]$

18. $m(t_1, t_2, \ldots, t_n) = E\left[e^{\sum_{i=1}^{n} t_i X_i}\right]$

19. $m(t_1) = \lim_{t_2 \to 0} m(t_1, t_2)$

20. $m(t_1, t_2, \ldots, t_n) = m(t_1)\, m(t_2) \ldots m(t_n)$

Pelajaran 4

1. (i) $p(x_1, x_2, \ldots, x_k) = \dfrac{n!}{x_1!\, x_2! \ldots x_k!}\, p_1^{x_1}\, p_2^{x_2} \ldots p_k^{x_k}$

   (ii) $p(x_i) = \binom{n}{x_i} p_i^{x_i} (1 - p_i)^{n - x_i}$

   (iii) $p(x_i, x_j) = \dfrac{n!}{x_i! x_j! (n - x_i - x_j)!}\, p_i^{x_i}\, p_j^{x_j} (1 - p_i - p_j)^{n - x_i - x_j}$

   (iv) $E[X_i X_j] = n(n-1)\, p_i p_j$

   (v) $\text{Cov}(X_i, X_j) = -np_i p_j$

2. (i) $$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 -2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)+\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]\right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

(ii) $$f(x|y)=\frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}}\exp\left\{-\frac{1}{2(1-\rho^2)\sigma_X^2}\left[x-\mu_X-\rho\frac{\sigma_X}{\sigma_Y}(y-\mu_Y)\right]^2\right\}$$

$$-\infty < x < \infty$$

(iii) $$m(t_1,t_2)=\exp\left[t_1\mu_X+t_2\mu_Y+\frac{1}{2}\left(t_1^2\sigma_X^2+2\rho t_1t_2\sigma_X\sigma_Y+t_2^2\sigma_Y^2\right)\right]$$

(iv) $E[XY] = \mu_X\mu_Y + \rho\,\sigma_X\sigma_Y$

(v) $Cov(X, Y) = \rho\,\sigma_X\sigma_Y$

Modul 4

Pelajaran 1

1. $M_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

2. $E[M_k] = m_k$

3. $Var(M_k) = \frac{1}{n}[m_{2k} - m_k)^2]$

4. $E[\bar{X}] = \mu$

5. $Var(\bar{X}) = \frac{1}{n}\sigma^2$

6. $S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(X_i - \bar{X})^2$

7. $E[S^2] = \sigma^2$

8. $\text{Var}(S^2) = \frac{1}{n}\left(\mu_4 - \frac{(n-3)}{(n-1)}\sigma^4\right)$

9. $\sum_{i=1}^{n}(X_i - \mu)^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$

10. $\bar{X} - \mu = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)$

Pelajaran 2

1. $p(u, v) = p_{X,Y}\left(g_1^{-1}(u, v),\ g_2^{-1}(u, v)\right)$

2. $f(u, v) = f_{X,Y}\left(g_1^{-1}(u, v), g_2^{-1}(u, v)\right) \mid J \mid$

3. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

4. $f(u,v) = \sum_{i=1}^{m} |J_i| f_{X,Y}\left(g_i^{-1}(u,v), h_i^{-1}(u,v)\right)$

5. $J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u,v)}{\partial u} & \frac{\partial g_i^{-1}(u,v)}{\partial v} \\ \frac{\partial h_i^{-1}(u,v)}{\partial u} & \frac{\partial h_i^{-1}(u,v)}{\partial v} \end{vmatrix}$

6. $m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 g(x,y) + t_2 h(x,y)} f(x,y)\,dx\,dy$

7. $m_u(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{tg(x,y)} f(x,y)\,dx\,dy$

8. (i) $f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, u-x)\, dx$

(ii) $f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u-y, y)\, dy$

9. (i) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-u)\, dx$

(ii) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u+y, y)\, dy$

10. (i) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X,Y}(x, u/x)\, dx$

(ii) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}(u/y, y)\, dy$

11. $f_{u=X/Y}(u) = \int_{-\infty}^{\infty} |y|\, f_{X,Y}(uy, y)\, dy$

Pelajaran 3

1. (i) $f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}} \left(1+\frac{x^2}{n}\right)^{-(n+1)/2}, -\infty < x < \infty$ $\quad X \sim t_n$

(ii) $T = \frac{Z}{\sqrt{V/n}}$

(iii) $E[X] = 0$

(iv) $Var\,[X] = \frac{n}{n-2}$

2. (i) $$f(x) = \begin{cases} \dfrac{\Gamma[m+n)/2}{\Gamma(m/2)\Gamma(n/2)} \left(\dfrac{m}{n}\right)^{m/2} \dfrac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}}, x>0 \\ 0 \quad , \text{ di tempat lain} \end{cases} \qquad X \sim F_{m,n}$$

(ii) $F = \dfrac{U/m}{V/m}$

(iii) $E[X] = \dfrac{n}{n-2}$

(iv) $\text{Var}(X) = \dfrac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$

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Senarai Rumus Tambahan

1. $\sum_{x=1}^{N} x = \frac{N(N+1)}{2}$

2. $\sum_{x=1}^{N} x^2 = \frac{N(N+1)(2N+1)}{6}$

3. Diberikan $S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$. Jika $X_1, X_2, \ldots, X_n$ adalah sampel rawak daripada taburan sebarang normal, maka $\frac{(n-1)S^2}{\sigma^2}$ tertabur secara χ^2_{n-1}.

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