

**IMPROVING TORSIONAL BEHAVIOUR OF  
REINFORCED CONCRETE BEAM  
STRENGTHENED WITH ULTRA HIGH  
PERFORMANCE FIBRE REINFORCED  
CONCRETE**

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BEAM STRENGTHENED WITH ULTRA HIGH PERFORMANCE FIBRE  
REINFORCED CONCRETE**

**by**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

{يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ}

صَدَقَ اللَّهُ الْعَظِيمِ

المجادلة (11)

## **DEDICATION**

*To the soul of my mother who had dreamt to witness these moments...*

*To my kind-hearted father for his unlimited love, inspires, supports, protections, sacrifices, and prayers...*

*To all my brothers for their supports, help, and encouragement...*

*To my wife and my kids for their endless love, patience, encouragement, and supports...*

*I dedicated this work hoping that I made all of them proud...*

***Thaer Jasim Mohammed***

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## LIST OF ABBREVIATIONS

ACI	American concrete institute
ASTM	American society for testing and materials
CARDIFRC	Ultra-high-performance fibre cementitious composite
DFRCC	Ductile fibre-reinforced cementitious composite
ECC	Engineered cementitious composite
FRC	Fibre reinforced cement
FRP	Fibre reinforced polymer
GFRP	Glass-fibre-reinforced polymer
HPC	High performance concrete
HPFRC	High performance fibre-reinforced concrete
HPFRCC	High performance fibre-reinforced cementitious composites
NC	Normal concrete
OPC	Ordinary Portland cement
RC	Reinforced concrete
RPC	Reactive powder concrete
SCC	Self compacting concrete
SFRC	Steel fibre reinforced concrete
UHPC	Ultra high performance concrete
UHPdC	Ultra high performance dura concrete
UHPFC	Ultra high performance fibre concrete

## LIST OF SYMBOLS

$A_c$	Gross area of the concrete section
$A_{sl}$	Total area of longitudinal reinforcement to resist torsion
$A_{st}$	Area of one leg of a closed stirrup resisting torsion
$D$	Diameter of the specimen
$d_f$	Diameter of steel fibre
$e$	A horizontal distance between load cell to the centre of the beam
$E_c$	Modulus of elasticity of concrete.
$E_s$	Modulus of elasticity of steel reinforcement.
$\mathcal{E}$	Strain of concrete
$\mathcal{E}_o$	Strain at ultimate compressive strength
$f$	Apply load
$f'_c$	Compressive strength of concrete
$f_t$	Splitting tensile strength
$f_y$	Specified yield strength of steel reinforcement.
$f_{yt}$	Specified yield strength of the transverse reinforcement.
$k$	Initial torsional stiffness
$k_{cr}$	Cracked torsional stiffness
$L$	Length of the specimen
$l_f$	Length of steel fibre
$P$	Maximum applied load
$P_h$	Perimeter of the steel stirrup
$S$	Spacing of steel stirrups

STCC	Shear transfer coefficient for a closed crack
STOC	Shear transfer coefficient for a opened crack
$T$	Torque
$T_{cr}$	Cracking torque
$T_u$	Ultimate torque
$\Delta_y$	Displacement of the beam in the y-direction
$\theta$	Angle of twist
$\nu$	Poisson's ratio
$\rho_l$	Longitudinal rebar ratio (%)
$\rho_{st}$	Stirrup ratio (%)