

**NEW EVOLVING FUZZY SYSTEM  
ALGORITHMS USING DYNAMIC CONSTRAINT**

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**NEW EVOLVING FUZZY SYSTEM ALGORITHMS USING DYNAMIC  
CONSTRAINT**

**by**

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## LIST OF ABBREVIATIONS

Acc.	Accuracy
ANN	Artificial Neural Network
ASTM	American Society for Testing and Materials
AUC	Area Under Curve
APE	Average Percentage Error
BNR	Balanced Number of Rule
CLIP	Categorical Learning-Induced Partitioning
CI	Computational Intelligence
ECSFS	Evolving-Construction Scheme for Fuzzy Systems
EER	Error-Evolving Rate
EFS	Evolving Fuzzy System
EIG	Evolving Information Granule
eFSM	Evolving Neural-Fuzzy Semantic Memory
EOCFS	Evolving Output-Context Fuzzy System
ESFS	Evolving Structural Fuzzy System
FCM	Fuzzy C-Means
FIG	Fuzzy Information Granule
FRBS	Fuzzy Rule Base System
FST	Fuzzy Set Theory
GrC	Granular Computing
GFRBS	Granular Fuzzy Rule-Based System
IRT	Infrared Thermography
JG	Justifiable Granularity

LFM	Linguistic Fuzzy Modeling
LPA	Localized Parameter Adaptation
LSM	Least Square Method
MF	Membership Function
MLP	Multi-Layer Perceptron
MPG	Miles Per Gallon
MSE	Mean Square Error
MSB	Main Switch Board
NR	Number of Rules
NFS	Neural Fuzzy System
OAIG	Optimal Allocation of Information Granularity
PFM	Precise Fuzzy Modeling
RCFS	Recursive Construction of Fuzzy System
RCFS	Recursively Constructed Fuzzy System
RGB	Red, Blue and Green
RMSE	Root of MSE
RPE	Rational Partition Error
RSA	Rule Selection Algorithm
SaFIN	Self-Adaptive Fuzzy Inference Network
SC	Semantic Cointention
SM	Similarity Measure
SONFIN	Self-Constructing Neural Fuzzy Inference Network
SSE	Sum of Square Error
SSEM	Simplified Structure Evolving Method
SVM	Support Vector Machine

TFIG	Theory of Fuzzy Information Granule
UoD	Universe of Discourse
VSANF	Variable Selection Algorithm for the Neuro-fuzzy

## LIST OF SYMBOLS

$[\mathbf{x}, d]_i$	Evidence of the $i$ th Training Data where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is an input vector, and $d$ is the corresponding output
$i$	$i$ th Training Data Where $i = \{1, 2, \dots, N\}$
$n$	Number of Features (Inputs)
$N$	Number of Training Data
$x_p$	$p$ th input feature
$r$	A Rule
$t$	An Evolving Stage
$R^{t,r}$	$r$ th IF-THEN Mamdani-type Fuzzy Rule at $t$ th Evolving Stage Where $r = 1, 2, \dots, R^t$
$R^t$	Number of Rules at $t$ th Evolving Stage and $\{r = 1, 2, \dots, R^t\}$
$\mu$	Gaussian Membership Function
$c$	Center of the Linguistic Levels
$\sigma$	Width (or variance) of the Linguistic Levels
$\alpha$	Minimum Membership Value or Distinguishability Factor for UoD
$C^{t,(r)}$	$r$ th Consequent Part at $t$ th Evolving Stage
$c^{t,(r)}$	Center of the $r$ th Consequent Part at $t$ th Evolving Stage
$\sigma^{t,(r)}$	Width (or variance) of the $r$ th Consequent Part at $t$ th Evolving Stage
$A_p^{t,(r)}$	$r$ th Antecedent Part at $t$ th Evolving Stage Associated with the $p$ th Input Variable
$c_p^{t,(r)}$ or $c_p^{t,(r)}(x_p)$	Center of the $r$ th Antecedent Part at $t$ th Evolving Stage Associated with the $p$ th Input Variable
$\sigma_p^{t,(r)}$ or $\sigma_p^{t,(r)}(x_p)$	Width (or variance) of the $r$ th Antecedent Part at $t$ th

	Evolving Stage Associated with the $p$ th Input Variable
$E(t)$	Error Function at $t$ th Evolving Stage Using SSE, MSE or RMSE
$E(t - 1)$	Error Function at $(t - 1)$ th Evolving Stage Using SSE, MSE or RMSE
$E_{global}$	global threshold error
$E$	Error, Without Considering Any Stage
$E_i$	Error for $i$ Training Data Where $i = \{1, 2, \dots, N\}$
$o(x_i)$	Model's output of the $i$ th Training Data
$d_{des_i}$ or $d_i$	Desired Output of the $i$ th Training Data
$c_{1p}$	Center of the First Antecedent Part ( $A_{1p}$ ) (used in SAFIN)
$\sigma_{1p}$	Width of the First Antecedent Part ( $A_{1p}$ ) (used in SAFIN)
$Q$	Optimization Factor
$y_i$	Desired Output data Where $i = \{1, 2, \dots, N\}$
$T_i(y_i)$	Granular Output of $y_i$
$\hat{y}_i$	Model Output Where $i = \{1, 2, \dots, N\}$
$\Delta w$	Gradient Vector
$w_{new}$	New Gradient Point
$w_{old}$	Current Gradient Point
$\eta$	Iteration Rate
$u_j$	Output Parameter with Rule Weight
$\Omega$	An Information Granule
$D$	Experimental Evidence (data) where $D = \mathbf{x} = \{x_1, x_2, \dots, x_N\}$ (used in Pedrycz et al. (2013))
$\text{card } \Omega$	Cardinality of $\Omega$