

**IMMOBILIZED  $\beta$ -GALACTOSIDASE-  
MEDIATED CONVERSION OF LACTOSE:  
PROCESS, KINETICS AND MODELLING  
STUDIES**

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**IMMOBILIZED  $\beta$ -GALACTOSIDASE-MEDIATED CONVERSION OF  
LACTOSE: PROCESS, KINETICS AND MODELLING STUDIES**

**by**

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## LIST OF SYMBOLS

|                 |   |
|-----------------|---|
| $A$             | Permeation velocity constant due to gel layer                               |
| $A_{410}Blank$  | The absorbance of the blank sample (abs)                                    |
| $A_{410}Sample$ | The absorbance of the unknown or standard sample (abs)                      |
| $A_m$           | Surface area of the membrane ( $m^2$ )                                      |
| $C$             | Solute concentration (mol/L)  |
| $C_b$           | Concentration of solutes at the bulk solution (mol/L)                       |
| $C_{CP,i}$      | Concentration of solute $i$ at the concentration polarization layer (mol/L) |
| $C_{CM,i}$      | Concentration of solute $i$ at the membrane layer (mol/L)                   |
| $C_i$           | Concentration of solute $i$ (mol/L)   |
| $C_{o,i}$       | Initial concentration of solute $i$ (mol/L)                                 |
| $C_p$           | Concentration of product at the permeate (mol/L)                            |
| $C_{p,i}$       | Concentration of product $i$ at the permeate (mol/L)                        |
| $C_{p,s}$       | Concentration of the substrate at the permeate (mol/L)                      |
| $C_{PS,i}$      | Concentration of solute $i$ at the permeate side of membrane (mol/L)        |
| $C_{r,i}$       | Concentration of product $i$ at the retentate (mol/L)                       |
| $C_{r,s}$       | Concentration of the substrate at the retentate (mol/L)                     |
| $d$             | Hydrodynamic diameter of water  |
| $DF$            | Dilution factor   |
| $D_m$           | Diffusivity in the membrane pore channel ( $m^2/s$ )                        |
| $D_{m,i}$       | Diffusivity in the membrane pore channel for solute $i$ ( $m^2/s$ )         |
| $D_o$           | Diffusivity in the bulk solvent ( $m^2/s$ )                                 |

|             |  |
|-------------|--|
| $D_{o,i}$   | Diffusivity in the bulk solvent for solute $i$ ( $\text{m}^2/\text{s}$ )                 |
| $[E]$       | Enzyme concentration ( $\text{mol/L}$ )  |
| $[E']$      | Enzyme-galactosyl complex ( $\text{mol/L}$ )   |
| $[E'Gos_3]$ | $\beta$ -Gal-galactosyl-(6'-O- $\beta$ -Galactosyl-lactose) ( $\text{mol/L}$ )           |
| $[E'Lac]$   | $\beta$ -Gal-galactosyl-lactose ( $\text{mol/L}$ )                                       |
| $[EGal]$    | Enzyme-galactose complex ( $\text{mol/L}$ )  |
| $[ELac]$    | Enzyme-lactose complex ( $\text{mol/L}$ )  |
| $[EGos_3]$  | $\beta$ -Gal-6'-O- $\beta$ -Galactosyl-lactose ( $\text{mol/L}$ )                        |
| $[EGos_4]$  | $\beta$ -Gal-6'-O- $\beta$ -(6-galactobiosyl)-lactose ( $\text{mol/L}$ )                 |
| $[E_T]$     | Total mass balance of enzyme components (g)  |
| $F$         | The volume force vector ( $\text{N}/\text{m}^3$ )  |
| $[Gal]$     | Galactose concentration ( $\text{mol/L}$ )   |
| $[GAL]$     | Dimensionless galactose concentration  |
| $[Gbi]$     | Galactobiose concentration ( $\text{mol/L}$ )  |
| $[Glu]$     | Glucose concentration ( $\text{mol/L}$ )   |
| $[GLU]$     | Dimensionless glucose concentration  |
| $[Gos_3]$   | 6'-O- $\beta$ -Galactosyl-lactose ( $\text{mol/L}$ )                                     |
| $[GOS_3]$   | Dimensionless trisaccharide ( $Gos_3$ ) concentration                                    |
| $[Gos_4]$   | 6'-O- $\beta$ -(6-galactobiosyl)-lactose ( $\text{mol/L}$ )                              |
| $[GOS_4]$   | Dimensionless tetrasaccharide ( $Gos_4$ ) concentration                                  |
| $H$         | Scale at $z$ -axis ( $\text{m}^{-1}$ )   |
| $I$         | Identity matrixes  |
| $J$         | Flux ( $\text{kg}/(\text{m}^2.\text{s})$ or $\text{mol}/(\text{m}^2.\text{s})$ )         |
| $k$         | Diffusive mass transfer coefficient at concentration polarization layer ( $\text{m/s}$ ) |

|                     |   |
|---------------------|---|
| $\kappa$            | Permeability ( $\text{m}^2$ )   |
| $\mathcal{K}$       | Denominator of rate of reaction   |
| $\hat{\mathcal{K}}$ | Dimensionless denominator of rate of reaction   |
| $k_1$               | Rate formation of $[ELac]$ ( $1/(\text{M.h})$ )   |
| $k_2$               | Rate formation of $[E' Lac]$ ( $1/(\text{M.h})$ )   |
| $k_3$               | Rate formation of $[EGos_3]$ ( $1/(\text{M.h})$ )   |
| $k_4$               | Rate formation of $[E' Gos_3]$ ( $1/(\text{M.h})$ )   |
| $k_5$               | Rate formation of $[EGos_4]$ ( $1/(\text{M.h})$ )   |
| $k_6$               | Rate formation of $[EGal]$ ( $1/(\text{M.h})$ )   |
| $k_{-1}$            | Rate deformation of $[ELac]$ ( $1/\text{h}$ )   |
| $k_{-2}$            | Rate deformation of $[E' Lac]$ ( $1/\text{h}$ )   |
| $k_{-3}$            | Rate deformation of $[EGos_3]$ ( $1/\text{h}$ )   |
| $k_{-4}$            | Rate deformation of $[E' Gos_3]$ ( $1/\text{h}$ )   |
| $k_{-5}$            | Rate deformation of $[EGos_4]$ ( $1/\text{h}$ )   |
| $k_{-6}$            | Rate deformation of $[EGal]$ ( $1/\text{h}$ )   |
| $K_B$               | Boltzman constant ( $1.3806 \times 10^{-23} \text{ m}^2.\text{kg}/(\text{s}^2 \text{ K})$ )           |
| $k_{h1}$            | Rate constant for hydrolysis of 6'-O- $\beta$ -Galactosyl-lactose ( $Gos_3$ )                         |
| $k_{h2}$            | Rate constant for hydrolysis of 6'-O- $\beta$ -(6-galactobiosyl)-lactose ( $Gos_4$ ) ( $1/\text{h}$ ) |
| $k_{hn}$            | Rate constant for hydrolysis ( $n=1,2$ ) ( $1/\text{h}$ )   |
| $\check{k}_{hn}$    | Dimensionless rate constant for hydrolysis ( $n=1,2$ )  |
| $k_m$               | Diffusive mass transfer coefficient at membrane layer ( $\text{m/s}$ )                                |
| $K_n$               | Dissociation constant ( $n=1,2,3,\dots$ )   |
| $k_{p1}$            | Rate constant for production of glucose ( $1/\text{h}$ )  |

|                     |  |
|---------------------|--|
| $k_{P2}$            | Rate constant for production of galactose (1/h)  |
| $k_{P3}$            | Rate constant for production of 6'-O- $\beta$ -Galactosyl-lactose (Gos <sub>3</sub> ) (1/h)        |
| $k_{P4}$            | Rate constant for production of 6'-O- $\beta$ -(6-galactobiosyl)-lactose (Gos <sub>4</sub> ) (1/h) |
| $k_{Pn}$            | Rate constant for the production of saccharides ( $n=1,2,3,4$ ) (1/h)                              |
| $\check{k}_{Pn}$    | Dimensionless rate constant for the production of saccharides ( $n=1,2,3,4$ )                      |
| $[Lac]$             | Lactose concentration (mol/L)  |
| $[LAC]$             | Dimensionless lactose concentration  |
| $[Lac]_o$           | Initial lactose concentration (mol/L)  |
| $M_T$               | Total mass of solute in the permeate side (kg)   |
| $MW_i$              | Molecular weight of the solute $i$ (g/mol)   |
| $N_{f,i}$           | Inward molar flux of solute $i$ (mol/(m <sup>2</sup> .s))  |
| $N_i$               | Molar flux of solute $i$ (mol/(m <sup>2</sup> .s))   |
| $p$                 | Pressure (bar)   |
| $Pe_L$              | Peclet number at the concentration polarization layer  |
| $Pe_m$              | Peclet number at the membrane layer  |
| $P_{lumen}$         | Initial pressure at the lumen side (bar)   |
| $P_{lumen\_outlet}$ | Pressure at lumen outlet (bar)   |
| $P_{membrane}$      | Initial pressure at the membrane (bar)   |
| $P_P$               | Average applied pressure at the permeate (bar)   |
| $P_R$               | Average applied pressure at the retentate (bar)  |
| $P_{shell}$         | Initial pressure at the shell side (Pa)  |
| $P_{shell\_outlet}$ | Pressure at shell outlet (Pa)  |

|                             |  |
|-----------------------------|--|
| $Q_{lumen}$                 | Lumen inlet flow rate (mL/s)   |
| $Q_{ms}$                    | Mass source term (kg/(m <sup>3</sup> .s))  |
| $Q_{shell}$                 | Shell inlet flow rate (mL/s)   |
| $r$                         | Radial coordinate from the centre line of membrane (m)   |
| $R$                         | Universal gas constant (8.31415 J/(mol.K))   |
| $r_1$                       | Inner radius of lumen (m)  |
| $r_2$                       | Outer diameter of lumen (m)  |
| $r_3$                       | Radius between the centre line of lumen to the inner wall of membrane shell (or open boundary layer at shell side) (m) |
| $r_h$                       | Hydrodynamic radius of solutes   |
| $R_i$                       | Reaction rate of solute $i$ (mol/s)  |
| $r_p$                       | Pore radius  |
| $R_{p,i}$                   | Rejection coefficient of the product $i$   |
| $r_s$                       | Stoke's radius of solutes (m)  |
| $R_s$                       | Rejection coefficient of the substrate   |
| $R_T$                       | Total membrane resistance coefficient  |
| [SAC]                       | Dimensionless saccharides (product and substrate) concentration  |
| $[SAC]_{\Gamma_{n+1,calc}}$ | Calculated value of dimensionless concentration of saccharide at $\Gamma=n+1$ ( $n=1,2,3\dots$ )                       |
| $[SAC]_{\Gamma_{n+1,exp}}$  | Experimental value of dimensionless concentration of saccharide at $\Gamma=n+1$ ( $n=1,2,3\dots$ )                     |
| $S_f$                       | Separation factor  |
| $S_{f,i}$                   | Separation factor of solute $i$  |
| $t$                         | time (s)   |

|                   |  |
|-------------------|--|
| $T$               | Absolute temperature (K)   |
| $t_f$             | Duration of reaction at steady state (h)                                     |
| $u$               | Fluid velocity (m/s)   |
| $V$               | Partial molar volume of solute   |
| $V_{Gal}$         | Rate formation of galactose (M/h)  |
| $\dot{u}_{GAL}$   | Dimensionless rate formation of galactose                                    |
| $V_{Gbi}$         | Rate formation of galactobiose (M/h)   |
| $V_{Glu}$         | Rate formation of glucose (M/h)  |
| $\dot{u}_{GLU}$   | Dimensionless rate formation of glucose                                      |
| $V_{Gos_3}$       | Rate formation of trisaccharide (Gos <sub>3</sub> ) (M/h)                    |
| $\dot{u}_{GOS_3}$ | Dimensionless rate formation of trisaccharide (Gos <sub>3</sub> )            |
| $V_{Gos_4}$       | Rate formation of tetrasaccharide (Gos <sub>4</sub> ) (M/h)                  |
| $\dot{u}_{GOS_4}$ | Dimensionless rate formation of tetrasaccharide (Gos <sub>4</sub> )          |
| $V_{h1}$          | Hydrolysis rate of trisaccharide (Gos <sub>3</sub> ) (M/g-enzyme.h)          |
| $\dot{u}_{h1}$    | Dimensionless hydrolysis rate of trisaccharide (Gos <sub>3</sub> )           |
| $V_{h2}$          | Hydrolysis rate of tetrasaccharide (Gos <sub>4</sub> ) (M/g-enzyme.h)        |
| $\dot{u}_{h2}$    | Dimensionless hydrolysis rate of tetrasaccharide (Gos <sub>4</sub> )         |
| $V_{Lac}$         | Rate consumption of lactose (M/h)  |
| $\dot{u}_{LAC}$   | Dimensionless rate consumption of lactose                                    |
| $V_{mean}$        | Nominal velocity at the lumen or shell sides of membrane (m/s)               |
| $V_{m,Gal}$       | Maximum rate formation of galactose (M/g-enzyme.h)                           |
| $V_{m,Glu}$       | Maximum rate formation of glucose (M/g-enzyme.h)                             |
| $V_{m,Gos_3}$     | Maximum rate formation of trisaccharide (Gos <sub>3</sub> ) (M/g-enzyme.h)   |
| $V_{m,Gos_4}$     | Maximum rate formation of tetrasaccharide (Gos <sub>4</sub> ) (M/g-enzyme.h) |

|                      |   |
|----------------------|---|
| $v_o$                | Initial solvent convective velocity (m/s)                       |
| $v_{o,cp}$           | Solvent permeation velocity in the presence of cake layer (m/s) |
| $v_{o,i}$            | Initial velocity of solute $i$ (m/s)                            |
| $V_T$                | The total reaction volume for activity assay (mL)               |
| $v_x$                | Fluid velocity at $x$ direction (m/s)                           |
| $v_y$                | Fluid velocity at $y$ direction (m/s)                           |
| $v_z$                | Fluid velocity at $z$ direction (m/s)                           |
| $V_z$                | Nominal solvent velocity at the lumen (m/s)                     |
| $V_{z,lumen}$        | Fluid velocity at the lumen side of membrane (m/s)              |
| $V_{z,shell}$        | Fluid velocity at the shell side of membrane (m/s)              |
| $V_{\beta-Gal\ sol}$ | The volume of $\beta$ -galactosidase during activity assay (mL) |
| $x$                  | Space coordinate perpendicular to the membrane interface (m)    |
| $Y$                  | Dimensionless space coordinate ( $x/\delta$ )                   |
| $[6Gos_3]$           | 6'-O- $\beta$ -(6-galactobiosyl)-galactose (mol/L)              |
| $\alpha$             | Convective hindrance constant                                   |
| $\alpha_i$           | Convective hindrance constant for solute $i$                    |
| $\varepsilon$        | Membrane porosity   |
| $\varepsilon_{ext}$  | Milimolar extinction coefficient of ONP                         |
| $[\mathcal{E}_T]$    | Dimensionless total mass balance of enzyme components           |
| $\xi_c$              | Hindrance factor related to hydrodynamic coefficient            |
| $\xi_{bs}$           | Dimensionless solidosity at the bulk solution                   |
| $\xi_d$              | Hindrance factor related to lag coefficient                     |
| $\xi_{gl}$           | Dimensionless solidosity at the gel layer                       |
| $\delta$             | Concentration polarization layer thickness (m)                  |