

**CONSTRAINT LOSS ESTIMATION SCHEMES IN
DEEP AND SHALLOW THREE-DIMENSIONAL
CRACK TIP FIELDS**

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**CONSTRAINT LOSS ESTIMATION SCHEMES IN DEEP AND SHALLOW
THREE-DIMENSIONAL CRACK TIP FIELDS**

by

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LIST OF ABBREVIATIONS

BLF	Boundary layer formulation
CCP	Center cracked tension panel
CT	Compact tension specimen
EPFM	Elastic-plastic fracture mechanics
EDI	Equivalent domain integral
HPC	High performance computing
HRR	Hutchinson, Rice & Rosengren
LEFM	Linear elastic fracture mechanics
LGC	Large geometry change
LSY	Large scale yielding
MBLF	Modified boundary layer formulation
SENB	Single edge notched bend bar
SENT	Single edge notched tension specimen
SSY	Small scale yielding
VCE	Virtual crack extension

LISTS OF SYMBOLS

a	Crack length
a_{eff}	Effective crack length
B	Physical specimen thickness
c	Uncracked ligament length
C_{ijkl} ($i, j, k, l=1,2,3$)	Stiffness tensor
E	Young's Modulus/Modulus of elasticity
$f_{ij}(i, j= r, \theta, z)$	Angular stress function in (r, θ, z) cylindrical coordinate system
$g_{ij}(i, j= r, \theta, z)$	Angular stress function for corner singularity fields
G	Shear modulus
\mathcal{G}	Energy released to propagate a crack
H	Specimen height
I (subscript)	Designation for mode I
$I(s)$	Interaction integral
I_n	Dimensionless function in HRR fields and $J - T_z$ fields
J	J -integral
J_{loc}	Local J -integral along a crack front
k	Yield stress in shear
K	Stress Intensity Factor
\mathcal{K}	Amplitude coefficient of stress dominant term
M	Global bending moment per unit thickness
n	Strain hardening exponent/rate
P	Applied load

P_0	Plastic limit load
r	Radial distance ahead of a crack tip
r_p	Plastic zone size
s	Order of stress singularity
S	Span between support of bend specimen
S_i ($i=1,2,3$)	Principal deviatoric stress components
t	Physical specimen thickness
T	T -stress
u_i ($i=1,2,3$)	Displacement components in (x_1, x_2, x_3) Cartesian coordinate system
ν	Poisson's ratio
w	Strain energy density
W	Specimen width
W_s	Work required to create new crack surfaces
Y	Crack calibration factor
z	Distance measured from the free surface of a specimen
σ_{app}	Remotely applied stress
σ_{cr}	Critical stress for fracture to occur
σ_0	Yield strength/stress
$\sigma_{ij}(i, j=1,2,3)$	Stress components in (x_1, x_2, x_3) Cartesian coordinate system
$\sigma_{ij}(i, j= r, \theta, z)$	Stress components in (r, θ, z) cylindrical coordinate system
$\tilde{\sigma}_{ij}(i, j= r, \theta, z)$	Dimensionless stress functions for HRR fields and $J - T_z$ fields
$\sigma_e, \bar{\sigma}$	von Mises stress
σ_{kk}	Volumetric stress
σ_m	Mean stress

ε_{ij} ($i, j=1,2,3$)	Strain components in (x_1, x_2, x_3) Cartesian coordinate system
ε_0	Yield strain
α	Material constant
β	Stress biaxiality ratio
β^{thin}	Stress biaxiality ratio for thin specimen
β_c	Corner stress intensity factor
γ_s	Surface energy per unit area
γ_p	Plastic work done per unit area of crack surface area created
$\gamma_{T_z}, \gamma_\sigma$	Slope constants in the $J - \Delta\sigma$ approach
λ	Strength of corner singularity field
μ	Plastic deformation level
δ_{ij} ($i, j=1,2$)	Kronecker delta
Φ	Airy stress function
Π	Potential energy
Γ	Arbitrary contour around a crack tip

SKEMA ANGGARAN KEHILANGAN KEKANGAN DALAM MEDAN HUJUNG RETAKAN TIGA DIMENSI YANG DALAM DAN CETEK

ABSTRAK

Matlamat utama kajian ini adalah untuk memahami ciri-ciri kehilangan kekangan tiga dimensi dan melanjutkan pencirian skema anggaran kehilangan kekangan tiga dimensi seperti kaedah $J - T_z$ dan $J - \Delta\sigma$ dalam retakan. Skema anggaran kehilangan kekangan tiga dimensi dalam medan di hujung retakan elastik plastik telah disiasat dalam kajian ini dengan menggunakan bar retak bawah beban lenturan (SENB) dan plat retak tengah bawah beban tegangan (CCP). Model tersebut telah ditakrifkan dengan sifat bahan pengerasan terikan, $n = 3,6,13$ dan sifat bahan tanpa pengerasan ($n \rightarrow \infty$). Kehilangan kekangan dalam medan tegasan di hujung retakan didapati berubah dalam model dengan panjang retakan yang berlainan, $a/W = 0.1, 0.2, 0.3, 0.5$ and ketebalan yang berbeza, $B/(W - a) = 0.05, 1$.

Kehilangan kekangan di hujung retakan telah dikaji melalui perbandingan antara medan tegasan asimptotik bersifat tanpa pengerasan dengan penyelesaian hujung retakan terikan satah medan Prandtl dan penyelesaian hujung retakan tegasan satah Sham & Hancock. Kehilangan kekangan dalam satah didapati bertambah dengan tegasan T negatif apabila nisbah a/W dikecilkan. Penurunan ketebalan model juga didapati mengurangkan kehilangan kekangan dalam satah kerana tegasan T meningkat dalam model yang nipis. Kehilangan kekangan luar satah diperhatikan di kawasan dari satah tengah ke permukaan bebas dalam semua model. Medan tegasan di permukaan bebas tidak dapat mencapai keadaan tegasan satah penuh kerana dipengaruhi oleh medan singulariti penjuru. Medan tegasan deviatorik adalah unik dalam semua model dan tidak bergantung pada kehilangan kekangan dalam satah dan

luar satah. Skema anggaran kehilangan kekangan juga dikemukakan untuk tegasan lingkaran di depan retakan dengan menghubungkan kehilangan kekangan dengan magnitud tegasan T .

Keberkesanaan kaedah $J - T_z$ dan kaedah $J - \Delta\sigma$ dalam mencirikan medan di hujung retakan tiga dimensi juga dibincangkan. Pemerolehan terperinci dan algoritma untuk mengira kaedah $J - T_z$ telah ditunjukkan. Kaedah $J - T_z$ didapati bahawa gagal menggambarkan medan di hujung retakan model yang menunjukkan kehilangan kekangan dalam satah. Kaedah $J - T_z - Q$ juga dikesahkan dengan menggunakan parameter Q terikan satah. Kaedah $J - T_z - Q$ didapati bahawa membuat anggaran berlebihan tentang kehilangan kekangan dalam satah dalam model nipis yang menunjukkan tegasan T negatif seperti model CCP nipis. Manakala, kaedah $J - \Delta\sigma$ adalah lebih bermanfaat kerana dapat menyifatkan kehilangan kekangan dalam dan luar satah secara bersepadu dengan memplotkan tegasan paksi terhadap $J_{loc}/z\sigma_0$ parameter. Penggunaan kaedah $J - T_z$ memerlukan pertaburan T_z di depan retakan. Sebaliknya, aplikasi kaedah $J - \Delta\sigma$ adalah lebih mudah kerana kehilangan kekangan sepanjang retakan dapat dianggarkan melalui satu lengkungan unik untuk model yang mempunyai ketebalan yang berbeza.

CONSTRAINT LOSS ESTIMATION SCHEMES IN DEEP AND SHALLOW THREE-DIMENSIONAL CRACK TIP FIELDS

ABSTRACT

The primary goal of this study is to determine the three-dimensional constraint loss behavior and further extend the three-dimensional constraint loss estimation schemes of $J - T_z$ and $J - \Delta\sigma$ approaches in three-dimensional crack tip fields consisting of various crack configurations. The three-dimensional constraint loss estimation schemes in elastic-plastic crack tip fields were examined for a single edge notched bend bar (SENB) and a center cracked panel in tension (CCP). The finite element models were characterized with a strain hardening material, $n = 3, 6, 13$ and a non-hardening material, $n \rightarrow \infty$. The crack tip constraint loss was found to vary in the models with various crack length, $a/W = 0.1, 0.2, 0.3, 0.5$ and different thicknesses, $B/(W - a) = 0.05, 1$.

Crack tip constraint loss was studied by comparing the non-hardening crack tip asymptotic fields with the plane strain Prandtl's crack tip fields solutions and the plane stress Sham & Hancock's crack tip solutions. The in-plane constraint loss increased with a more negative T -stress following the reduction of a/W ratio. The thin model exhibited smaller the in-plane constraint loss as T -stress was less negative. The out-of-plane constraint loss occurred in all models at the region away from the midplane to the free surface. The radial and angular distribution of deviatoric stress field ahead of the crack tip was also found to be unique in all models and independent of the in-plane and the out-of-plane constraint loss. A constraint estimation loss scheme at $\theta = 0^\circ$ was proposed for the hoop stress along a crack front by correlating the constraint loss to the magnitude of the T -stress.

A detailed derivation and an algorithm to compute the $J - T_z$ approach were shown. The $J - T_z$ approach was unable to characterize the crack tip fields in the models that feature in-plane constraint loss and at the free surface due to a corner singularity field. The $J - T_z - Q$ approach using a plane strain Q parameter was evaluated. It was found that the $J - T_z - Q$ approach overestimated the in-plane constraint loss in a thin model with negative T -stress as seen in the thin CCP model. New equations were developed to extend the $J - \Delta\sigma$ approach in strain hardening models. The extended $J - \Delta\sigma$ approach offered a unified characterization of the in-plane and out-plane constraint loss along a crack front by plotting the normal stresses against a dimensionless $J_{loc}/z\sigma_0$ parameter. Unlike the $J - T_z$ approach that required an exact distribution of T_z along a crack front, the $J - \Delta\sigma$ approach is more advantageous as it can be applied immediately to approximate the constraint loss along a crack front by using a unified curve for the models with different thicknesses.