

**PARTIAL POSE ESTIMATION OF RIGID
OBJECT SYSTEM USING CAD DATABASE**

ZULIANI BINTI ZULKOFFLI

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**PARTIAL POSE ESTIMATION OF RIGID OBJECT SYSTEM USING CAD
DATABASE**

by

ZULIANI BINTI ZULKOFFLI

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LIST OF ABBREVIATIONS

AVI	Automated Visual Inspection
CAVI	Computer Aided Visual Inspection
CAD	Computer Aided Design
1D FD	1 Dimension Fourier Descriptor
2D FD	2 Dimension Fourier Descriptor
2D	2 Dimensional
3D	3 Dimensional
PRI	Projection Real Image
CAIP	Computer Aided Inspection Planning
CAIM	Computer Aided Integrated Manufacturing
CCD	Charge Coupled Devices
CWF	Cool White Fluorescent
CCT	Correlated color temperature
CRI	Color rendering index
LED	Light emitting diode

LIST OF SYMBOLS

a	BoxMin coordinate
b	BoxMax coordinate
b_c	Is the baseline distance between the two cameras
c	Hypotenuse length
A	Region area
A_c	An intrinsic parameters
$AreaPRI$	Area in pixel ² of PRI object
$AreaCAD$	Area in pixel ² of CAD object
\mathbf{A}	Column i vectors of unit length for Subspace Matrix
\mathbf{B}	Column j vectors of unit length for Subspace Matrix
$c(m, n)$	Matching value of (m,n) image to its database image
c_{CAD}	Hypotenuse length CAD object
c_{PRI}	Hypotenuse length PRI object
d	Length distance
d_c	Distance of camera and object
d_x	Displacement transformation at x axis
d_y	Displacement transformation at y axis
f_c	Camera focal length
$f(u, v)$	2D Fourier Transform spectrum
$g(Inspected)$	Inspected image
d_z	Displacement transformation at z axis
D_c	Extrinsic parameters matrix
$E(m, n)$	Matching value of template matching method

$t(\text{Template})$	Template image
k_u	Scale factors
k_v	Scale factors
m_c	Point coordinate in calibration image
(m, n)	Template image matrix
(i, j)	Coordinates and the grayscale index value of each pixel
$(\nabla^2 f)$	Twice partial differential of function
(x_i, y_i, z_i)	Surface point
M_{ch}	Chessnut pattern board size $[x_c, y_c]$
$I_T(i, j)$	Threshold value
$I_{T_1}(i, j)$	Modified threshold value
i	Imaginary axis
\bar{i}	Integration function for i area calculation
j	Integration function for j area calculation
n	Surface points
N	Periodic constant value
P	Coordinate matrix of the view
P'	View after being rotated about one of the coordinate axes
P''	Translation point
P_1	Physical point
P_c	Camera matrix
$P_p \begin{bmatrix} I_c \\ J_c \\ Z_c \\ 1 \end{bmatrix}$	Perspective projection matrix
R	Radius length

R	Correlation coefficient value
R	Coordinate axes
R_c	3 x 3 rotation matrix
R_x	Rotation angle at x axis
R_y	Rotation angle at y axis
R_z	Rotation angle at z axis
(R_i, R_j)	Center coordinate of hypotenuse length in outer box method
(R_{imin}, R_{jmin})	minimum coordinate of the hypotenuse length
(R_{imax}, R_{jmax})	maximum coordinate of the hypotenuse length
$S(r, \theta)$	Polar coordinate
S_c	A scale of parameter
$S_\theta(r)$	1D Fourier Transform function
$S(r)$	1D Fourier Transform Spectrum frequency
$S(\theta)$	1D Fourier Transform Spectrum angle
u_0	Principal points
v_0	Principal points
v	2D Fourier Magnitude spectrum
X_1	Point in image 1
X_2	Point in image 2
X	Plane axis x
Y	Plane axis y
Z	Plane axis z
Z_0	Center of gravity of the shape
Z_k	Center of gravity of the modified shape
∂^2	Twice partial differential of function of i value

∂_j^2	Twice partial differential of function of j value
$\partial^2 f$	Twice partial differential of function
α	Transformation scale factor x axis
β	Transformation scale factor y axis
γ	Transformation scale factor z axis
θ	Rotation angle
θ_c	Angle between retina axes
$\emptyset(u, v)$	Phase spectrum
$[i_c, j_c, z_c]^t$	Geometry model maps
$[i'_c, j'_c, 1]$	2D point in image
$ F(u, v) $	Magnitude spectrum