First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

EEE512 – Advanced Digital Signal and Image Processing

Duration : 3 hours

Please check that this examination paper consists of SEVEN (7) pages and appendix THREE (3) pages of printed material before you begin the examination.

Instructions: This question paper consists **SIX (6)** questions. Answer any FIVE (5) questions. All questions carry the same marks.

1. (a) Figure 1 shows a structure of a linear time invariant digital system. It is observed that the output sequence of the system is $y_1(n)$={10,19,16,9,2} when the given input is $x_1(n)=\delta(n)$.

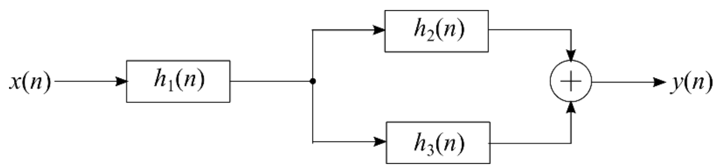


Figure 1

(i) Determine the impulse response $h_3(n)$, if $h_1(n)$ = {5,2} and $h_2(n)$ = {1,-1,2,0,-1}.
(20 marks)

(ii) Determine the output from this system $y_2(n)$ if the input is $x_2(n)$={2,5}.
(10 marks)

(b) Sketch all possible ROCs for the following system function $H(z)$. Then, get all the possible impulse response $h(n)$ from this system function:

$$H(z) = \frac{1}{1 - 2.3z^{-1} + 0.6z^{-2}}$$

(50 marks)

(c) Two discrete-time sequences are given as:

$$x(n) = \{1,2,3,4,5\}$$
$$y(n) = \{2,2,4,4,5\}$$

Determine the normalized cross-correlation of signal y with respect to signal x, $\rho_{yx}(l)$.
(20 marks)

2. (a) By using pole-zero plot, propose a third order high pass filter. The filter should be stable. Provide $H(z)$ and $H(\omega)$.

(30 marks)

(b) Your research project requires you to design a lowpass finite impulse response (FIR) filter. The specifications of the filter are as follows:

- Passband edge frequency Fp = 2.2kHz
- Stopband edge frequency Fs = 2.3kHz
- Peak passband ripple α_p = 0.02dB
- Minimum stopband attenuation α_s = 45 dB
- Sampling rate F_T = 5kHz

The first step of designing this FIR filter is to estimate the filter's order. Estimate the order of the filter by using:

(i) Kaiser's formula.
(ii) Ballenger's formula.
(iii) Hermann's formula.

Which formula gives the lowest order of the filter?

(40 marks)

(c) We want to design a digital lowpass Butterworth filter $G(z)$ with the passband edge frequency ω_p at 0.35π, with a passband ripple not exceeding 0.25dB. The minimum stopband attenuation is 10dB at the stopband edge frequency ω_s of 0.55π. Assume $|G(e^{j0})|=1$. By using the following formula, determine the order N of the filter.

$$N = \frac{1}{2}\frac{\log_{10}\left[(A^2-1)/\varepsilon^2\right]}{\log_{10}(\Omega_s/\Omega_p)} = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)}$$

(30 marks)

3. Figure 3 shows a parallel form I realization of a digital system. From this figure,

   a) Obtain the system function $H(z)$.

   (10 marks)

   b) Draw the equivalent cascade realization of the system.

   (40 maks)

   c) Draw the equivalent parallel form II realization of the system.

   (50 marks)

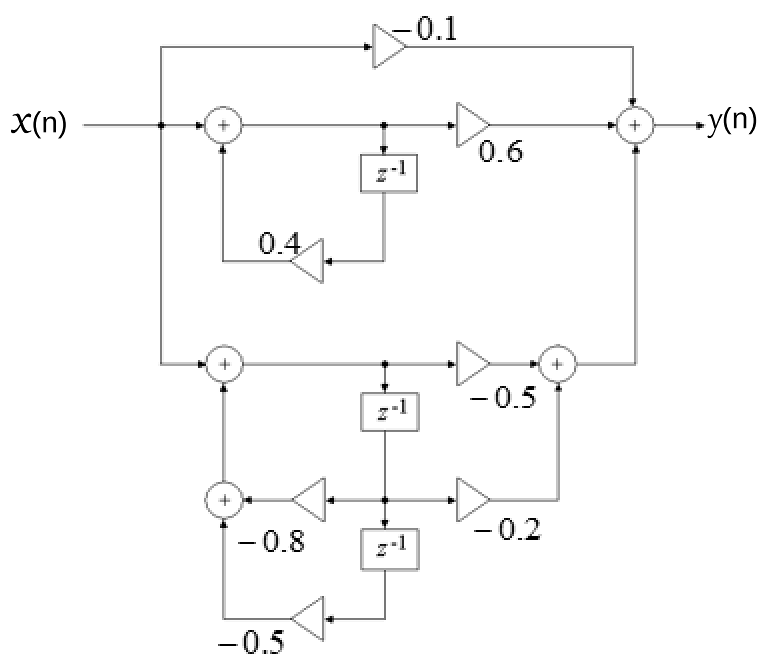
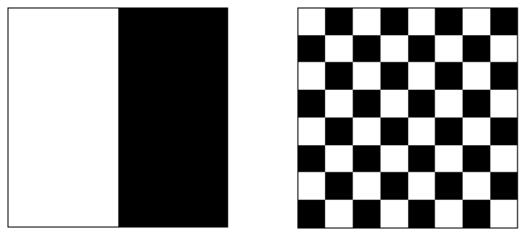


Figure 3

4. (a) The two texture images shown below are quite different, but their histograms are identical. Both images have size 80 × 80 pixels, with black (0) and white (255).



(i) (ii)

Figure 4(a)

Suppose that both images are blurred with a 3×3 smoothing filter, would the resultant histograms still be the same? Draw an approximation of the histograms of both images. Explain your answer.

Note: the black lines are used to signify the boundaries of the two images but not part of them.

(30 marks)

(b) The following figures shows (i) a 3-bit image of size 5-by-5 image in the square, with x and y coordinates specified, (ii) a Laplacian filter and (iii) a low-pass filter.

Image

y \ x	0	1	2	3	4
0	7	3	5	4	0
1	1	0	7	5	0
2	4	6	2	4	1
3	4	3	4	1	2
4	6	1	7	4	3

(i)

Laplacian filter

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(ii)

Low pass filter

$$\begin{pmatrix} 0.01 & 0.1 & 0.01 \\ 0.1 & 0.56 & 0.1 \\ 0.01 & 0.1 & 0.01 \end{pmatrix}$$

(iii)

Figure 4(b)

Compute the following:

(i) The output of a 3 × 3 mean filter at (2,2).

(ii) The output of a 3 × 3 median filter at (2,2).

(iii) The output of the 3 × 3 Laplacian filter shown above at (2,2).

(iv) The output of the 3 × 3 low-pass filter shown above at (2,2).

(v) The histogram of the whole image.

(70 marks)

5. (a) A filtered function in spatial domain is given by:

$$g(x,y) = f(x,y) - f(x+1,y) + f(x,y) - f(x,y+1)$$

(i) Obtain the filter transfer function $H(u,v)$ in frequency domain,

(30 marks)

(ii) Show that $H(u,v)$ is a high pass fitter.

(20 marks)

(b) The convolution theorem of two dimensional variables $f(x,y)$ and $h(x,y)$is given by:

$$f\,(x,y)\ \otimes\ h(x,y)\ =\ F\,(u,v)\,H(u,v)$$

where $F(u,v)$ and $H(u,v)$ are two dimensional Fourier transform of $f(x,y)$ and $h(x,y)$ respectively.

Prove the validity of this theorem.

(50 marks)

Given:

$$\Im f(x-x_0, y-y_0) = F(u,v)e^{-j2\pi\left(\frac{ux_0}{M}+\frac{vy_0}{N}\right)}$$
$$2j\sin x = e^{jx} - e^{-jx}$$
$$2\cos x = e^{jx} + e^{-jx}$$

6. (a) Write an expression for a wavelet $\Psi_{1,4}(x)$ in terms of the Haar scaling function. Hence plot $\Psi_{1,4}(x)$.

(40 marks)

(b) Consider a 4×4 image as follow

$$f(x,y) = \begin{pmatrix} 4 & 8 \\ 8 & 4 \end{pmatrix}$$

(i) Draw the required filter bank to implement a first-scale two-dimensional fast wavelet transform (FWT) of $f(x,y)$. Label all inputs and outputs with the proper arrays.

(40 marks)

(ii) Draw the synthesis filter bank of FWT for reconstructing the *f(x,y)*.

(20 marks)

Given:

The wavelet functions are defined as:

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi\left(2^j\, x - k\right)$$

$$\psi(x) = \sum_n h_\psi(n)\sqrt{2}\,\varphi(2x-n)$$

$$\psi(x) = \begin{cases} 1 & ; \quad 0 \le x < 0.5 \\ -1 & ; \quad 0.5 \le x < 1 \\ 0 & \quad elsewhere \end{cases}$$

The Haar scaling functions are defined as:

$$\varphi(x) = \begin{cases} 1 & ; \quad 0 \le x < 1 \\ 0 & ; \quad elsewhere \end{cases}$$

$$\varphi_{j,k}(x) = 2^{\frac{j}{2}}\,\varphi\left(2^j\, x - k\right)$$

The scaling function coefficients for the Haar function are given by:

$h_\varphi(n) = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$ for $n = 0,1$

The scaling function coefficients for the Haar wavelet are given by:

$h_\psi(n) = \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}$ for $n = 0,1$

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Appendix/Lampiran

Kaiser's formula:

$$N \cong \frac{-20\log_{10}\left(\sqrt{\delta_p \delta_s}\right) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

Ballenger's formula:

$$N \cong \frac{-2\log_{10}\left(10\delta_p \delta_s\right)}{3(\omega_s - \omega_p)/2\pi} - 1$$

Hermann's formula:

$$N \cong \frac{D_\infty\left(\delta_p, \delta_s\right) - F\left(\delta_p, \delta_s\right)\left[\left(\omega_s - \omega_p\right)/2\pi\right]^2}{(\omega_s - \omega_p)/2\pi}$$

$$D_\infty\left(\delta_p, \delta_s\right) = \left[a_1\left(\log_{10}\delta_p\right)^2 + a_2\left(\log_{10}\delta_p\right) + a_3\right]\log_{10}\delta_s - \left[a_4\left(\log_{10}\delta_p\right)^2 + a_5\left(\log_{10}\delta_p\right) + a_6\right]$$

$$F\left(\delta_p, \delta_s\right) = b_1 + b_2\left[\log_{10}\delta_p - \log_{10}\delta_s\right]$$

$a_1 = 0.005309$ $\quad a_2 = 0.07114$ $\quad a_3 = -0.4761$

$a_4 = 0.00266$ $\quad a_5 = 0.5941$ $\quad a_6 = 0.4278$

$b_1 = 11.01217$ $\quad b_2 = 0.51244$

Table 1: Summary of analysis and synthesis formulas

		Continuous-time signal		Discrete-time signals	
		Time-domain	Frequency-domain	Time-domain	Frequency-domain
Periodic signals	Fourier series	$c_k = \frac{1}{T_p}\int_{T_p} T_p x_a(t)e^{-j2\pi kF_0 t}dt$	$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kF_0 t}$	$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$	$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$
		Continuous and periodic	Discrete and aperiodic	Discrete and periodic	Discrete and periodic
Aperiodic signal	Fourier transform	$X_a(F) = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}dt$	$x_a(t) = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft}dF$	$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$	$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}d\omega$
		Continuous and aperiodic	Continuous and aperiodic	Discrete and aperiodic	Continuous and periodic

Discrete Fourier Transform (DFT):

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0,1,2,\ldots,N-1$$

Inverse Discrete Fourier Transform (IDFT):

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \qquad n = 0,1,2,\ldots,N-1$$

Table 2: Some common z-transform pairs.

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
4	$na^n u(n)$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2}$	$\|z\|>\|a\|$
5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
6	$-na^n u(-n-1)$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2}$	$\|z\|<\|a\|$
7	$(\cos\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\|>1$
8	$(\sin\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\|>1$
9	$(a^n\cos\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\|>\|a\|$
10	$(a^n\sin\omega_0 n)u(n)$	$\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\|>\|a\|$

Table 3: Properties of the *z*-transform.

Property	Time domain	*z*-domain	ROC
Notation	$x(n)$ $x_1(n)$ $x_2(n)$	$X(z)$ $X_1(z)$ $X_2(z)$	ROC: $r_2 < \|z\| < r_1$ ROC_1 ROC_2
Linearity	$ax_1(n)+bx_2(n)$	$aX_1(z)+bX_2(z)$	At least the intersection of ROC_1 and ROC_2
Time-shifting	$x(n-k)$	$z^{-k}X(z)$	That of $X(z)$ except z=0 if k>0 and $z=\infty$ if k<0.
Scaling in the *z*-domain	$a^n x(n)$	$X(a^{-1}z)$	$\|a\|r_2 < \|z\| < \|a\|r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$(1/r_1) < \|z\| < (1/r_2)$
Conjugation	$x^*(n)$	$X^*(z^*)$	ROC
Real part	$\mathrm{Re}\{x(n)\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Includes ROC
Imaginary part	$\mathrm{Im}\{x(n)\}$	$\frac{1}{2}j[X(z)-X^*(z^*)]$	Includes ROC
Differentiation in the *z*-domain	$nx(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \|z\| < r_1$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)*X_2(z)$	At least the intersection of ROC_1 and ROC_2
Correlation	$r_{x_1x_2}(l)=x_1(l)*x_2(-l)$	$R_{x_1x_2}(z)=X_1(z)*X_2(z^{-1})$	At least the intersection of ROC of $X_1(z)$ and ROC of $X_2(z^{-1})$
Initial value theorem	If $x(n)$ causal	$x(0)=\lim_{z\to\infty}X(z)$	
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi j}\oint_C X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	At least, $r_{1l}\, r_{2l} < \|z\| < r_{1v}\, r_{2v}$
Parseval's relation	$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n)=\frac{1}{2\pi j}\oint_C X_1(v)X_2^*\left(\frac{1}{v^*}\right)v^{-1}dv$		