First Semester Examination
2018/2019 Academic Session

December 2018/January 2019

EEE443 – DIGITAL SIGNAL PROCESSING
(Pemprosesan Isyarat Digit)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **SEVEN (7)** pages and appendices **SEVEN (7)** pages of printed appendices material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat dan **TUJUH (7)** muka surat lampiran yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry the same marks.

[***Arahan:*** *Kertas soalan ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. You are given a task to analyze one simple system that is initially at rest (i.e., it is relaxed). This linear time-invariant system is recursive and causal. This system is implemented as a direct form II structure as shown in Figure 1.

Anda telah ditugaskan untuk menganalisa satu sistem ringkas yang pada asalnya dalam keadaan rehat (i.e., iaitu santai). Sistem masa tak-berubah lelurus ini adalah jadi semula dan kausal. Sistem ini telah dilaksanakan sebagai stuktur bentuk terus II seperti ditunjukkan dalam Rajah 1.

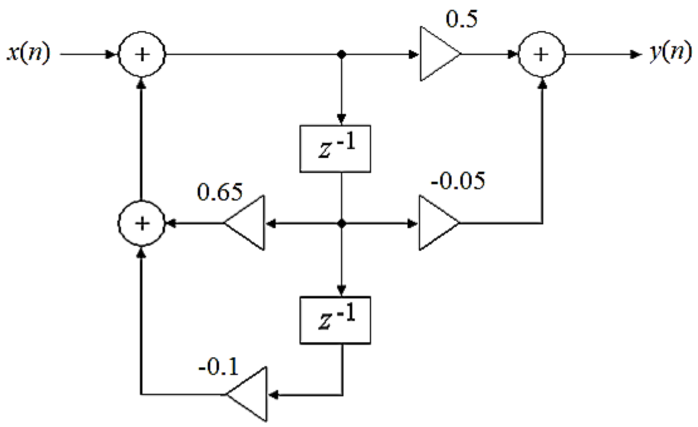

Figure 1
Rajah 1

(a) Based on the structure given in Figure 1, find the equation to present $y(n)$.

Berdasarkan struktur yang ditunjukkan dalam Rajah 1, cari persamaan untuk mewakili $y(n)$.

(5 marks/*markah*)

(b) Determine the equation for the system function $H(z)$. Then, sketch the pole-zero plot for the system with its region of convergence (ROC). Base on the pole-zero plot, investigate whether the system is BIBO stable or not. Give the justification for your answer.

Tentukan persamaan untuk fungsi sistem $H(z)$. Seterusnya, lakaran plot kutub-sifar bagi sistem tersebut berserta kawasan penumpuannya. Berdasarkan plot kutub-sifar tersebut, siasat samaada sistem tersebut stabil BIBO atau tidak. Berikan justifikasi bagi jawapan anda.

(10 marks/*markah*)

(c) Derive the equation for the impulse response $h(n)$ of the system.
Terbitkan persamaan bagi sambutan dedenyut $h(n)$ *sistem tersebut.*

(10 marks/*markah*)

(d) If the input to this system is a causal signal $x(n) = 2cos((\pi n)/4)u(n)$, find the z-transform for that signal $X(z)$, and give the pole-zero plot for the system with its region of convergence (ROC).

Sekiranya masukkan kepada sistem ini adalah isyarat kausal $x(n) = 2cos((\pi n)/4)u(n)$*), cari jelmaan-z bagi isyarat tersebut* $X(z)$*, dan berikan plot kutub-sifar bagi sistem bersama kawasan penumpuannya.*

(10 marks/*markah*)

(e) With the input signal as indicated in part (d), derive the equation for $Y(z)$. Expand it into partial fractions.
Dengan isyarat masukan seperti yang dinyatakan dalam bahagian (d), terbitkan persamaan bagi $Y(z)$*. Kembangkannya kepada pecahan separa.*

(35 marks/*markah*)

(f) Base on part (e), find the natural response of the system, $y_{nr}(n)$.
Berdasarkan bahagian (e), dapatkan sambutan asli sistem, $y_{nr}(n)$*.*

(10 marks/*markah*)

(g) Base on part (e), find the forced response of the system, $y_{fr}(n)$.
Berdasarkan bahagian (e), dapatkan sambutan paksaan sistem, $y_{fr}(n)$*.*

(10 marks/*markah*)

(h) Base on part (e), find the total response $y(n)$ of the system for the given input.
Berdasarkan bahagian (e), dapatkan sambutan keseluruhan $y(n)$ *sistem tersebut bagi masukan yang diberikan.*

(10 marks/*markah*)

2. (a) Design a low pass filter by using a zero-polar plot. The filter you are about to design should have three poles. By drawing the appropriate plot, obtain the equation for system function $\mathrm{H(z)}$ and frequency response $\mathrm{H}(\omega)$ for the filter.

Rekabentuk satu penuras laluan rendah dengan menggunakan plot kutub-sifar. Penuras yang anda reka perlu mempunyai tiga kutub. Dengan melukis plot yang bersesuaian, dapatkan persamaan bagi fungsi sistem $H(z)$ dan sambutan frequensi $H(\omega)$ bagi penuras tersebut.

(30 marks/*markah*)

(b) You have designed a non-uniform linear time-invariant system with impulse response $\mathrm{h(n)} = \{1, -1\}$. You want to determine the output from this system by using discrete Fourier transform (DFT) and inverse DFT (IDFT), for input $\mathrm{x(n)} = \{4,2\}$. Confirm your answer by using circular convolution.

Anda telah merekabentuk satu sistem masa tak-berubah lelurus yang mempunyai sambutan dedenyut $h(n) = \{1, -1\}$. Anda ingin menentukan keluaran daripada sistem ini dengan menggunakan jelmaan Fourier diskret (DFT) dan songsangan DFT, bagi masukan $x(n) = \{4,2\}$. Sahkan jawapan anda dengan menggunakan pelingkaran bulat.

(50 marks/*markah*)

(c) Direct computational of N-point Discrete Fourier Transform (DFT) may results a high computational load, especially when N is large. To overcome this, fast Fourier transform (FFT) has been introduced. Design an FFT structure that can be used to obtain an 8-point DFT.

Pengiraan secara terus jelmaan Fourier diskret (DFT) N-titik boleh mengakibatkan beban pengiraan yang tinggi, terutamanya apabila N adalah besar. Untuk mengatasinya, jelmaan Fourier yang cepat (FFT) telah diperkenalkan. Reka satu struktur yang boleh digunakan untuk mendapatkan DFT 8-titik.

(20 marks/*markah*)

3. (a) Design a bandstop filter that satisfies the following specification and estimate the order of the equiripple filter required to meet these specifications.

Reka bentuk penapis jalur henti yang memenuhi spesifikasi berikut dan anggarkan tertib penapis sama riak yang diperlukan untuk memenuhi spesifikasi ini.

$$0.98 \leq |H(e^{j\omega})| \leq 1.02 \qquad 0 \leq \omega \leq 0.2\pi$$
$$|H(e^{j\omega})| < 0.001 \qquad 0.22\pi \leq \omega \leq 0.78\pi$$
$$0.95 \leq |H(e^{j\omega})| \leq 1.05 \qquad 0.8\pi \leq \omega \leq \pi$$

(20 marks/*markah*)

(b) Design a linear phase digital FIR low-pass filter with the following specifications;

Reka bentuk satu fasa lelurus penapis digital FIR laluan rendah dengan spesifikasi berikut:

$$0.99 \leq |H(e^{j\omega})| \leq 1.01 \qquad 0 \leq \omega \leq 0.3\pi$$
$$|H(e^{j\omega})| \leq 0.01 \qquad 0.35\pi \leq \omega \leq \pi$$

using windows design method.
menggunakan kaedah tetingkap.

(30 marks/*markah*)

(c) Determine the coefficients of an IIR filter which are obtained using bilinear transformation from a second-order Butterworth analog prototype filter with a 3-dB cut-off frequency of 3 kHz. The sampling rate for the digital filter is 30,000 samples per second.

Tentukan pekali penapis IIR yang diperolehi dengan menggunakan transformasi dwilelurus daripada tertib-kedua prototaip penapis analog Butterworth dengan 3-dB frekuensi potong 3 kHz. Kadar pensampelan untuk penapis digital ialah 30,000 sampel sesaat. Tentukan pekali penapis IIR ini.

(50 marks/*markah*)

4. (a) Given an IIR digital filter transfer functions below. Compute its' parallel realization and show the parallel structure diagram.

Diberikan fungsi pemindahan penapis digital IIR di bawah. Hitungkan perlaksanaan selarinya dan tunjukkan gambarajah struktur selari tersebut.

$$H(z) = \frac{3(2z^2 + 5z + 4)}{(2z + 1)(z + 2)}$$

(40 marks/*markah*)

(b) Consider the parallel three causal first-order LTI discrete-time systems as shown in Figure 4.1, where the overall transfer function $H(z)$ is given as

Pertimbangkan satu penggabungan secara selari tiga LTI sistem isyarat diskret yang ditunjukkan dalam Rajah 4.1 di mana keseluruhan fungsi pindah $H(z)$ *adalah seperti berikut:*

$$H(z) = \frac{4z^{-2} + z^{-1} + 1}{z^{-3} + 7z^{-2} + 15z^{-1} + 9}$$

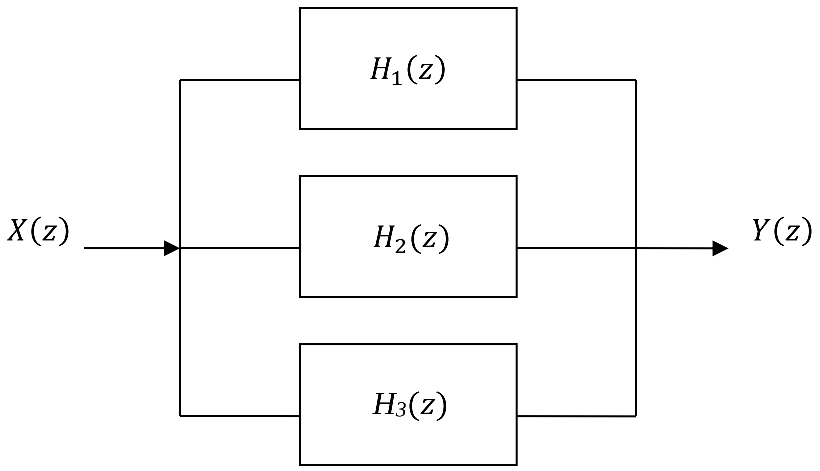

Figure 4.1

Rajah 4.1

(i) Find each of the transfer function $H_1(z)$, $H_2(z)$ and $H_3(z)$.

Cari fungsi-pindah bagi setiap satu $H_1(z)$, $H_2(z)$ and $H_3(z)$.

(15 marks/*markah*)

(ii) Develop the realization of the overall parallel system with each section is realized in the canonic direct form II. Show the working (drawing) for each transfer function.

Bangunkan kerealisasian untuk sistem selari keseluruhan dengan setiap bahagian dalam bentuk langsung II. Tunjukkan jalan kerja (lukisan) bagi setiap fungsi-pindah

(20 marks/*markah*)

(iii) Determine the impulse response of the transfer function $H(z)$ for the overall parallel system.

Tentukan sambutan dedenyut untuk fungsi-pindah $H(z)$ *untuk keseluruhan sistem selari tersebut.*

(25 marks/*markah*)

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APPENDIX

LAMPIRAN

Course Outcomes (CO)- Programme Outcomes (PO) Mapping

Pemetaan Hasil Pembelajaran Kursus – Hasil Program

Question	CO	PO
1	1	4
2	2	3
3	3	7
4	4	7

i. Information on continuous-time signal and discrete-time signal

Table A.1: Summary of analysis and synthesis formulas

		Continuous-time signal		Discrete-time signals	
		Time-domain	Frequency-domain	Time-domain	Frequency-domain
Periodic signals	Fourier series	$c_k = \frac{1}{T_p}\int_{T_p} x_a(t)e^{-j2\pi kF_0t}dt$	$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kF_0t}$	$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$	$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$
		Continuous and periodic	Discrete and aperiodic	Discrete and periodic	Discrete and periodic
Aperiodic signal	Fourier transform	$X_a(F) = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}dt$	$x_a(t) = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft}dF$	$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$	$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}d\omega$
		Continuous and aperiodic	Continuous and aperiodic	Discrete and aperiodic	Continuous and periodic

Discrete Fourier Transform (DFT):

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0,1,2,\ldots,N-1$$

Inverse Discrete Fourier Transform (IDFT):

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \qquad n = 0,1,2,\ldots,N-1$$

Table A.2: Some common z-transform pairs.

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$(\cos\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$(\sin\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$(a^n\cos\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$(a^n\sin\omega_0 n)u(n)$	$\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Table A.3: Properties of the z-transform.

Property	Time domain	*z*-domain	ROC
Notation	$x(n)$ $x_1(n)$ $x_2(n)$	$X(z)$ $X_1(z)$ $X_2(z)$	ROC: $r_2 < \|z\| < r_1$ ROC_1 ROC_2
Linearity	$ax_1(n)+bx_2(n)$	$aX_1(z)+bX_2(z)$	At least the intersection of ROC_1 and ROC_2
Time-shifting	$x(n-k)$	$z^{-k}X(z)$	That of $X(z)$ except z=0 if k>0 and $z=\infty$ if k<0.
Scaling in the *z*-domain	$a^n x(n)$	$X(a^{-1}z)$	$\|a\|r_2 < \|z\| < \|a\|r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$(1/r_1) < \|z\| < (1/r_2)$
Conjugation	$x^*(n)$	$X^*(z^*)$	ROC
Real part	$\text{Re}\{x(n)\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Includes ROC
Imaginary part	$\text{Im}\{x(n)\}$	$\frac{1}{2}j[X(z)-X^*(z^*)]$	Includes ROC
Differentiation in the *z*-domain	$nx(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \|z\| < r_1$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)*X_2(z)$	At least the intersection of ROC_1 and ROC_2
Correlation	$r_{x_1x_2}(l)=x_1(l)*x_2(-l)$	$R_{x_1x_2}(z)=X_1(z)*X_2(z^{-1})$	At least the intersection of ROC of $X_1(z)$ and ROC of $X_2(z^{-1})$
Initial value theorem	If $x(n)$ causal	$x(0)=\lim_{z\to\infty} X(z)$	
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi j}\oint_C X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	At least, $r_{1l}\, r_{2l} < \|z\| < r_{1v}\, r_{2v}$
Parseval's relation	$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n)=\frac{1}{2\pi j}\oint_C X_1(v)X_2^*\left(\frac{1}{v^*}\right)v^{-1}dv$		

ii. List of formulae:

- Transformasi jalur rendah ke jalur lulus:
  Low-pass to band-pass transformation

$$s \longrightarrow \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}$$

- Ω_l dan Ω_u masing-masing adalah frekuensi terendah dan teratas bagi frekuensi-frekuensi jalur tepi dalam penuras jalur lulus.
  Ω_l and Ω_u are the lower and upper band-edge frequencies of the band-pass filter, respectively.

- Jelmaan-z Biliear:
  Bilinear z-Transform:

$$H(z) = H(s)\big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

- Frekuensi analog pra-sampel $\Omega = \frac{2}{T}\tan(\frac{\omega}{2})$; T ialah jangkamasa sample dan ω adalah frekuensi digit dalam domain digit.

The pre-warped analog frequency $\Omega = \frac{2}{T}tan(\frac{\omega}{2})$*; T is the sampling period and ω is the frequency of the digital domain.*

- Sambutan dedenyut ideal, $h_D(n)$ untuk penuras lulus rendah adalah seperti berikut:
  Ideal impulse response, $h_D(n)$ for a lowpass filter is given by:

  $h_D(n) = 2f_c\frac{\sin(n\omega_c)}{n\omega_c}$ for $n \neq 0$, and

  $h_D(n) = 2f_c$ for $n = 0$.

- f_c adalah frekuensi potong (tepi) bagi penuras tersebut dan $\omega = 2\pi f$.
  f_c is the cutoff (edge) frequency of the filter and $\omega = 2\pi f$.

iii. Information on FIR filter design using common windows

Table A.4 Some common windows

Rectangular	$w(n) = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hanning[1]	$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hamming	$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Blackman	$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{N}\right) + 0.08\cos\left(\frac{4\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$

Table A.5 The Peak side-lobe amplitude of some common windows and the approximate transition width and stopband attenuation of an Nth-Order low-pass filter designed using the given window

Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	−13	$0.9/N$	−21
Hanning	−31	$3.1/N$	−44
Hamming	−41	$3.3/N$	−53
Blackman	−57	$5.5/N$	−74

iv. Information on FIR and IIR filter design

Table A.6Equations used to design FIR and IIR filters

Unit sample response for an ideal lowpass filter	$$h_d(n) = \frac{\sin(n-\alpha)\omega_c}{\pi(n-\alpha)}$$
Equiripple filter	$$N = \frac{-10\log(\delta_s\delta_p) - 13}{14.6\Delta f}$$
Discrimination Factor	$$d = \left[\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}\right]^{1/2} = \frac{\epsilon}{\sqrt{A^2-1}}$$
Selectivity Factor	$$k = \frac{\Omega_p}{\Omega_s}$$
Butterworth	$$\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1+(\frac{j\Omega}{j\Omega_c})^{2N}}$$ $$\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1+\epsilon^2(\frac{j\Omega}{j\Omega_p})^{2N}}$$ where, $$\epsilon = (\frac{\Omega_p}{\Omega_c})^N$$ $$\lvert H_a(j\Omega)\rvert^2 = H_a(s)H_a(-s)\rvert_{s=j\Omega}$$ The poles: $$s_k = (-1)^{1/2N}(j\Omega_c) = \Omega_c \exp\left[j\frac{(N+1+2k)\pi}{2N}\right], \quad k = 1, 2, \ldots, 2N-1$$ The order: $$N \geq \frac{\log d}{\log k}$$
Chebyshev	$$\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1+\epsilon^2 T_N^2(\frac{\Omega}{\Omega_p})}$$ System function of a type I Chebyshev filter $$H_a(s) = H_a(0)\prod_{k=0}^{N-1}\frac{-s_k}{s-s_k}$$ where, $$H_a(0) = (1-\epsilon^2)^{-1/2}, \quad N : \texttt{even}$$ $$H_a(0) = 1, \quad N : \texttt{odd}$$ The order: $$N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$$
Elliptic	$$\lvert H_a(\Omega)\rvert^2 = \frac{1}{1+\epsilon^2 U_N^2(\frac{\Omega}{\Omega_p})}$$ where $$U_N(\frac{1}{\Omega}) = \frac{1}{U_N(\Omega)}$$

	The order: $$N \geq \frac{\log(\frac{16}{d^2})}{\log(\frac{1}{q})}$$ where $$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}$$ where $$q_0 = \frac{1}{2}\frac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}$$

v. Information on bilinear and frequency transformation

Table A.7 Information used for bilinear and frequency transformation

Bilinear Z-transform	$H(z) = H(s)\|_{s=\frac{2}{T}(\frac{z-1}{z+1})}$
Pre-warped analog frequency	$\Omega = \frac{2}{T}\tan(\frac{\omega}{2})$

Table A.8 The transformation of an analog low-pass filter with a 3-dB cutoff frequency Ω_p to other frequency selective filters

Transformation	Mapping	New Cutoff Frequencies
Low-pass	$s \to \frac{\Omega_p}{\Omega'_p}s$	Ω'_p
High-pass	$s \to \frac{\Omega_p \Omega'_p}{s}$	Ω'_p
Bandpass	$s \to \Omega_p \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \to \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u}$	Ω_l, Ω_u

Table A.10 The transformation of a digital low-pass filter with a cutoff frequency ω_c to other frequency selective filters

Filter Type	Mapping	Design Parameters
Low-pass	$z^{-1} \to \dfrac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$	$\alpha = \dfrac{\sin[(\omega_c - \omega_c')/2]}{\sin[(\omega_c + \omega_c')/2]}$ ω_c' = desired cutoff frequency
High-pass	$z^{-1} \to -\dfrac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$	$\alpha = -\dfrac{\cos[(\omega_c + \omega_c')/2]}{\cos[(\omega_c - \omega_c')/2]}$ ω_c' = desired cutoff frequency
Bandpass	$z^{-1} \to -\dfrac{z^{-2} - [2\alpha\beta/(\beta+1)]z^{-1} + [(\beta-1)/(\beta+1)]}{[(\beta-1)/(\beta+1)]z^{-2} - [2\alpha\beta/(\beta+1)]z^{-1} + 1}$	$\alpha = \dfrac{\cos[(\omega_{c2} + \omega_{c1})/2]}{\cos[(\omega_{c2} - \omega_{c1})/2]}$ $\beta = \cot[(\omega_{c2} - \omega_{c1})/2]\tan(\omega_c/2)$ ω_{c1} = desired lower cutoff frequency ω_{c2} = desired upper cutoff frequency
Bandstop	$z^{-1} \to \dfrac{z^{-2} - [2\alpha/(\beta+1)]z^{-1} + [(1-\beta)/(1+\beta)]}{[(1-\beta)/(1+\beta)]z^{-2} - [2\alpha/(\beta+1)]z^{-1} + 1}$	$\alpha = \dfrac{\cos[(\omega_{c1} + \omega_{c2})/2]}{\cos[(\omega_{c1} - \omega_{c2})/2]}$ $\beta = \tan[(\omega_{c2} - \omega_{c1})/2]\tan(\omega_c/2)$ ω_{c1} = desired lower cutoff frequency ω_{c2} = desired upper cutoff frequency