First Semester Examination
2018/2019 Academic Session

December 2018 / January 2019

EEE382 - PROBABILITY & ENGINEERING STATISTIC
(Kebarangkalian dan Statistik Kejuruteraan)

Duration 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **TEN (10)** pages and before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEPULUH (10)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry the same marks.

*[Arahan: Kertas soalan ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) You are in a game of Russian Roulette with a revolver that has 3 bullets placed in three consecutive chambers. The cylinder of the gun will be spun once at the beginning of the game. Then, the gun will be passed between two players until it fires. Would you prefer to go first or second and what is the probability of surviving for your choice?

Anda berada dalam permainan Roulette Rusia dengan revolver yang mempunyai 3 peluru ditempatkan dalam tiga ruang silinder pistol berturut-turut. Silinder pistol akan berputar sekali pada permulaan permainan. Kemudian, pistol itu akan dilepaskan di antara dua pemain hingga penembakan. Adakah anda lebih gemar pergi dulu atau kemudian dan apakah keberangkalian hidup bagi keputusan anda?

(10 marks/*markah*)

(b) A privately owned café operates both a drive-in facility and walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

Kafe persendirian mengendalikan kedua-dua kemudahan pandu-lalu dan kemudahan berjalan kaki. Pada hari yang dipilih secara rawak, biarkan X dan Y, masing-masing, menjadi perkadaran masa yang digunakan oleh pemandu dan berjalan masuk, dan mengandaikan bahawa fungsi kepadatan bersama bagi pembolehubah rawak ini adalah

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 \le x \le 1, 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

(i) Find the marginal density of X

Cari ketumpatan marginal X

(5 marks/*markah*)

(ii) Find the marginal density of Y
Cari ketumpatan marginal Y

(5 marks/*markah*)

(iii) Find the probability that drive-in facility is used at most half of the time given that walk-in facility is in used all the time
Cari kebarangkalian kemudahan pandu-lalu digunakan sekurang-kurangnya separuh masa jika kemudahan berjalan masuk digunakan sepanjang masa.

(5 marks/*markah*)

2. (a) Ten alkalinity readings are made in the upper reaches of a river in the region and fifteen in the lower reaches of the same river with the following results in Table 2(a).
Sepuluh bacaan kealkalian dibuat di kawasan hulu sungai di rantau ini dan lima belas di muara sungai yang sama dengan keputusan berikut dalam Jadual 2(a).

Table 2(a)

Jadual 2(a)

Upper reaches	Lower reaches
91	86
75	95
91	135
88	121
94	68
63	64
86	113
77	108
71	79
69	62
	143
	108
	121
	85
	97

Investigate, at 1% level of significance, the claim that the true mean alkalinity of water in the lower reaches of this river is greater than that in the upper reaches.

Selidik pada aras keyakinan 1%, dakwaan bahawa kealkalian min sebenar air di muara sungai ini adalah lebih tinggi daripada yang di hulu.

(9 marks/*markah*)

(b) Mr. Brown is the owner of a small bakery in a large town. He believes that the smell of fresh baking will encourage customers to purchase goods from his bakery. To investigate this belief, he records the daily sales for 10 days when all the bakery's windows are open, and the daily sales for another 10 days when all the windows are closed. The following sales, in RM are recorded in Table 2(b). Assuming that these data may be deemed to be random samples from normal populations with the same variance, investigate the baker's belief.

Encik Brown adalah pemilik sebuah kedai roti kecil di bandar besar. Beliau percaya bahawa bau roti segar akan menggalakkan pelanggan untuk membeli barangan dari kedai roti beliau. Untuk menyiasat kepercayaan ini, beliau merekodkan jualan harian selama 10 hari apabila semua tingkap kedai roti adalah terbuka, dan jualan setiap hari untuk 10 hari lagi apabila semua tingkap ditutup. Jualan berikut, dalam RM direkodkan dalam Jadual 2(b) Dengan mengandaikan bahawa data ini boleh disifatkan sebagai sampel rawak daripada populasi normal dengan varians yang sama, tentukan dakwaan pemilik kedai roti ini.

(10 marks/*markah*)

Table 2 (b)
Jadual 2(b)

Windows open	Windows closed
202.0	193.5
204.5	192.2
207.0	199.4
215.5	177.6
190.8	205.4
215.6	200.6
208.8	181.8
187.8	169.2
204.1	172.2
185.7	192.8

(c) Two gauges that measure tire tread depth are being compared. Ten different locations on a tire are measured once by each gauge. The results, in mm, are presented in Table 2(c).

Dua tolok yang mengukur kedalaman bunga tayar dibandingkan. Sepuluh lokasi berbeza pada tayar diukur sekali oleh setiap tolok. Hasilnya, dalam mm, dibentangkan dalam Jadual 2(c).

Table 2(c)

Jadual 2(c)

Locations	Gauges 1	Gauges 2
1	17.6	16.8
2	19.4	20.0
3	19.5	18.2
4	17.1	16.4
5	15.3	16.0
6	15.9	15.4
7	16.3	16.5
8	18.4	18.0
9	17.3	16.4
10	19.1	20.1
11	17.8	16.7
12	18.2	17.9

Use the Wilcoxon signed-rank test to determine whether there is difference between the two gauges. Use $\alpha = 0.05$.

Guna Ujian Tanda Wilcoxon untuk menentukan samada terdapat perbezaan bagi kedua-dua tolok dengan $\alpha = 0.05$.

(6 marks/*markah*)

3. (a) A new postsurgical treatment is being compared with a standard treatment. Seven subjects receive the new treatment (x), while seven others receive the standard treatment (y). The recovery times, in days, given in Table 3(a):

Satu rawatan selepas pembedahan baru sedang dibandingkan dengan rawatan piawai. Tujuh pesakit menerima rawatan yang baharu, manakala tujuh lagi menerima rawatan yang standard. Masa pemulihan, dalam hari, diberikan dalam Jadual 3(a):

Table 3(a)

Jadual 3(a)

Treatment (*X*)	12	13	15	19	20	21	27
Control(*Y*)	18	23	24	30	32	35	40

Using Wilcoxon rank-sum test, can you conclude that the mean rate differs between the treatment and control? Use $\alpha = 0.05$.

Menggunakan ujian pangkat Wilcoxon-sum, uji samada bahawa kadar min berbeza di antara rawatan baharu dan standard? Diberi $\alpha = 0.05$.

(6 marks/*markah*)

(b) Table 3(b) gives experimental values of the pressure P of a given mass of gas corresponding to various values of the volume V. According to thermodynamic principles a relationship having the form $PV^{\alpha} = C$, where α and C are constants, should exist between the variables.

Jadual 3(b) memberikan nilai eksperimen tekanan P bagi suatu jisim gas tertentu yang bersamaan dengan pelbagai nilai isipadu V. Menurut prinsip termodinamik hubungan mempunyai bentuk $PV^{\alpha} = C$, di mana α dan C adalah pemalar, harus wujud antara pembolehubah.

Table 3(b)

Jadual 3(b)

Volume (m^3)	54.3	61.8	72.4	88.7	118.6	194.0
Pressure (Pa)	61.2	49.5	37.6	28.4	19.2	10.1

(i) Find the values of constant α, constant C and the equation of a least square line fitting the data using natural by transforming the data to natural logarithms, using Volume V as the independent variable.

Cari nilai-nilai pemalar α, pemalar C dan persamaan least square line dengan mengubah data kepada logaritma asli, gunakan isipadu V sebagai pembolehubah bebas.

(10 marks/*markah*)

(ii) Estimate P when V = 100.0 m^3.

Anggarkan P apabila V = 100.0 m^3.

(3 marks/*markah*)

(iii) Calculate the value of the F statistic for the hypothesis H_0:β_1=0, at α=0.05. Does this hypothesis can be rejected?

Kirakan nilai statistik F untuk hipotesis H_0: β_1 = 0, pada α = 0.05. Adakah hipotesis ini boleh ditolak?

(6 marks/*markah*)

4. An engineer is investigating the porosity finish of ceramics coating produced on a turbine blade sample and its relationship to the sintering speed (in revolutions per second). The data are shown in Table 4. Note that the data have been collected using to two different types of sintering tools. Since the type of sintering tool likely affect the porosity finish, it can be fitted using multiple linear regression models as follows:

Seorang jurutera sedang menyiasat keliangan salutan seramik yang dihasilkan pada sampel bilah turbin dan hubungannya dengan kelajuan kemampatan semburan (dalam pusingan per saat). Data ditunjukkan dalam Jadual 4. Perhatikan bahawa data telah dikumpulkan menggunakan dua jenis alat penyembur. Oleh kerana jenis alat penyembur mungkin menjejaskan keliangan, ia boleh dihubung dengan menggunakan model regresi linear berganda seperti berikut:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

Where Y is the porosity, X_1 is the sintering speed in revolutions per second, and X_2 is an indicator variable denoting the type of sintering tool used that is:

Di mana Y adalah keliangan, X_1 adalah kelajuan kemampatan semburan dalam pusingan sesaat, dan X_2 adalah pembolehubah penunjuk yang menandakan jenis alat penyembur yang digunakan iaitu:

$$X_2 = \begin{cases} 0 & , for\ tool\ type\ 302 \\ 1 & , for\ tool\ type\ 416 \end{cases}$$

Table 4

Jadual 4

Number of sample, i	Porosity, Y_i	Sintering speed	Type of sintering tool
1	4.5	1.3	302
2	4.2	1.1	302
3	5.1	1.5	302
4	4.8	1.4	302
5	5.1	1.6	302
6	4.6	1.2	302
7	3.4	1.2	416
8	3.1	1.1	416
9	3.8	1.5	416
10	3.7	1.6	416
11	3.4	1.4	416
12	3.5	1.3	416

(a) Fit a multiple linear regression model.
Dapatkan model regresi linear berganda.

(15 marks/*markah*)

(b) Calculate the error sum of squares (SSE) and regression sum of squares (SSR).
Kirakan error sum of squares (SSE) dan regression sum of squares (SSR).

(6 marks/*markah*)

(c) Calculate the error variance estimate S^2 and the coefficient of determination R^2.
Kirakan variance estimate S^2 *dan coefficient of determination* R^2*.*

(4 marks/*markah*)

oooOooo